National University of Singapore

Microeconomics III, EC4101(L2)

Tutorial 3 Solution, Exercises from Chs 5-6

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Note: The solution provided by the publisher is a bit scratchy and incomplete. The following is by Prof. Bag. Please go over them carefully.

5.5 Suppose the utility function for goods x and y is given by

$$U(x, y) = xy + y.$$

(a) Calculate the uncompensated (Marshallian) demand functions for x and y, and describe how the demand curves for x and y are shifted by changes in I or the price of the other good.

Answer: Set up the Lagrangean as follows:

$$\mathcal{L} = xy + y + \lambda[I - p_x x - p_y y].$$

So are task is to solve:

$$\max_{x,y} \ \mathcal{L}.$$

The first-order conditions are as follows:

$$\frac{\partial \mathcal{L}}{\partial x} = y - \lambda p_x = 0 \tag{1}$$
$$\frac{\partial \mathcal{L}}{\partial y} = x + 1 - \lambda p_y = 0 \tag{2}$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = I - p_x x - p_y y = 0. \tag{3}$$

From (1) and (2), write:

$$\frac{y}{x+1} = \frac{p_x}{p_y},$$

which using in (3) yields:

$$x^* = \frac{I - p_x}{2p_x}, \qquad y^* = \frac{I + p_x}{2p_y}.$$

Now as I increases the demand curves for both x and y would shift to the right. As $p_x \uparrow$, demand for y goes up, indicating x and y are gross substitutes. On the other hand, as $p_y \uparrow$, demand for x remains unaffected. This shows that 'gross substitute' is not symmetric.

(b) Calculate the expenditure function of x and y.

Answer: First calculate the IUF:

$$\begin{aligned} V &= y^*(x^* + 1) \\ &= \frac{I + p_x}{2p_y}\left(\frac{I - p_x}{2p_x} + 1\right) \\ &= \frac{(I + p_x)^2}{4p_x p_y}. \end{aligned}$$

Now invert the IUF to obtain the expenditure function:

$$E = 2\sqrt{p_x p_y V} - p_x.$$

(c) Use the expenditure function calculated in part (b) to compute the compensated demand functions for goods x and y. Describe how the compensated demand curves for x and y are shifted by changes in income or by changes in the price of the other good.

Answer: By Shephard's Lemma (or equivalently, applying envelope theorem), obtain:

$$\begin{aligned} x^c &= \frac{\partial E}{\partial p_x} = \sqrt{\frac{V p_y}{p_x}} - 1 \\ y^c &= \frac{\partial E}{\partial p_y} = \sqrt{\frac{V p_x}{p_y}}. \end{aligned}$$

The compensated demand for both x and y increase as the price of the other good increases. This shows that x and y are **net substitutes**.

5.6 Over a three-year period an individual exhibits the following consumption behavior:

	p_x	p_y	x	y	
Year 1	3	3	7	4	A
Year 2	4	2	6	6	B
Year 3	5	1	7	3	C

Is this behavior consistent with the axiom of revealed preference?

Answer: Call year 1, year 2 and year 3 bundles as A, B, C. Now observe that

1. B RP A (why? in year 2 B costs the same as A – \$36 – yet B was chosen)
2. C RP B (why? in year 3 C costs \$38 and B costs \$36, yet C is chosen)
3. A RP C (why? in year 1 A costs \$49 and C costs \$30, yet A is chosen)

Using [1] and [3] and applying transitivity yields B RP C, contradicting [2], thus violating the revealed preference axiom (**Axiom: If a bundle X is revealed preferred to another bundle Y, then Y cannot be revealed preferred to X**).

6.5 Suppose an individual consumes three goods, x_1, x_2, and x_3, and that x_2 and x_3 are similar commodities (i.e., cheap and expensive restaurant meals) with $p_2 = kp_3$, where $k < 1$, that is, the goods' prices have a constant relationship to one another.

(a) Show that x_2 and x_3 can be treated as a composite commodity.

Answer: Composite commodity: $p_2x_2 + p_3x_3 = p_3(kx_2 + x_3)$.

(b) Suppose both x_2 and x_3 are subject to a transaction cost of t per unit. How will this transaction cost affect the price of x_2 relative to that of x_3? How will this effect vary with the value of t?

Answer: The relative price x_2 to x_3

$$= \frac{p_2 + t}{p_3 + t} = \frac{kp_3 + t}{p_3 + t} \uparrow 1 \qquad \text{as } t \to \infty.$$

Clearly, given that $k < 1$, relative price of good 2 increases in the transaction cost t.

(c) Can you predict how, following an increase in t, income compensation will affect expenditures on the composite commodity x_2 and x_3? Does the composite commodity theorem strictly apply to this case?

Answer: As t increases, between goods 2 and 3, good 2 becomes relatively more expensive and hence good 3 relatively cheaper. But both these goods have become dearer (i.e., more expensive) relative to good 1. So following an increase in t, an income compensation to keep the consumer on the same utility level means the consumer would shift towards more of good 1 (as x_1 is now cheaper relative to (x_2, x_3)), and this shift comes more at the expense of x_2 and relatively less at the expense of x_3 (x_2 has become relatively more expensive than x_3). As a package, overall spending on (x_2, x_3) may or may not increase – it depends on the substitutability between (x_2, x_3) and x_1.

The composite commodity theorem doesn't directly apply as prices of good 2 and 3 do not remain in constant proportion.

(d) How, following an increase in t, income compensation will affect the total spending on the composite commodity and its allocation between x_2 and x_3?

Answer: See the answer to the last part.

Note that I have slightly modified the questions in parts (c) **and** (d) **of Exercise 6.5 because the original sentences did not read well (incorrect english).**