EC4101
Microeconomics Analysis III
(Group 2)

Topic 3

General Equilibrium

Overview

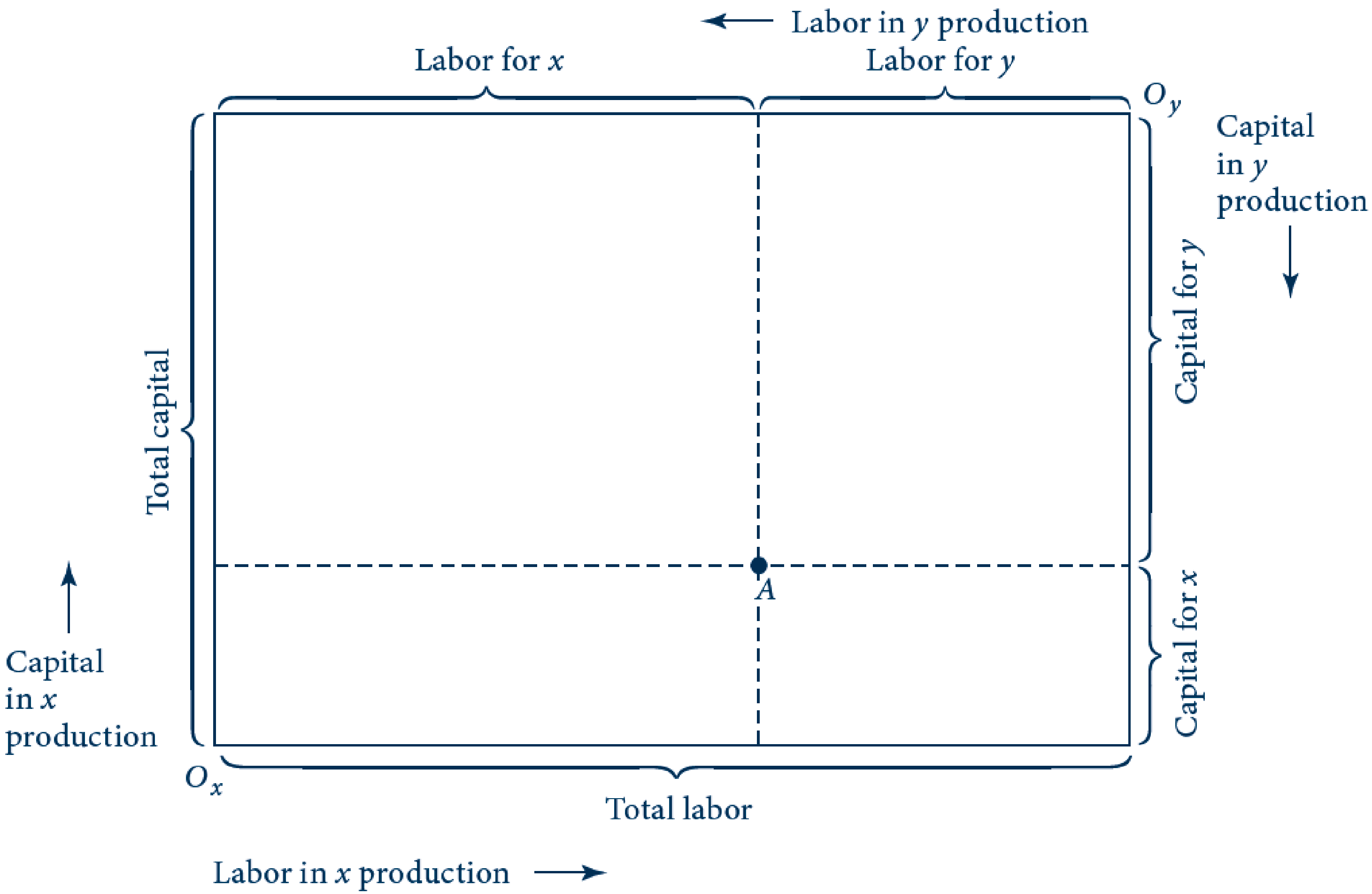
- Assumptions
- Edgeworth box diagram
  - Equilibrium; Comparative Statics
- Model of Exchange
  - Walra’s law, Walrasian equilibirum
  - Existence of equilibrium
  - Welfare theorems
- Model with production

Assumptions

- all markets are perfectly competitive
  - All consumers and firms are price takers
- Two goods: x and y
- Identical preferences
  - same indifference curve
- Identical technologies
  - same production functions

FIGURE 13.1

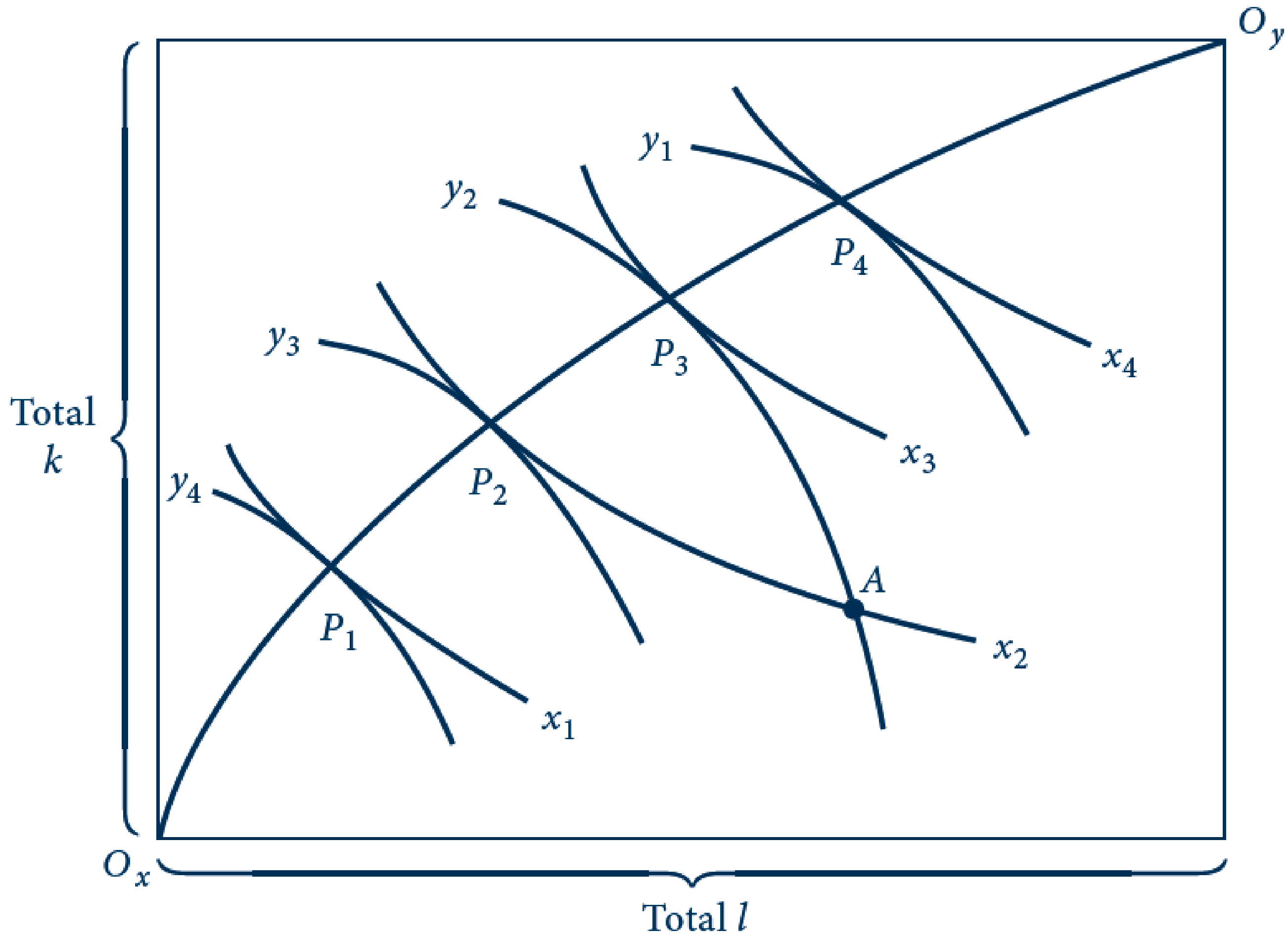
Construction of an Edgeworth Box Diagram for Production



The dimensions of this diagram are given by the total quantities of labor and capital available. Quantities of these resources devoted to x production are measured from origin O_x; quantities devoted to y are measured from O_y. Any point in the box represents a fully employed allocation of the available resources to the two goods.

FIGURE 13.2

Edgeworth Box Diagram of Efficiency in Production



This diagram adds production isoquants for x and y to Figure 13.1. It then shows technically efficient ways to allocate the fixed amounts of k and l between the production of the two outputs. The line joining O_x and O_y is the locus of these efficient points. Along this line, the RTS (of l for k) in the production of good x is equal to the RTS in the production of y.

Production Possibility Frontier

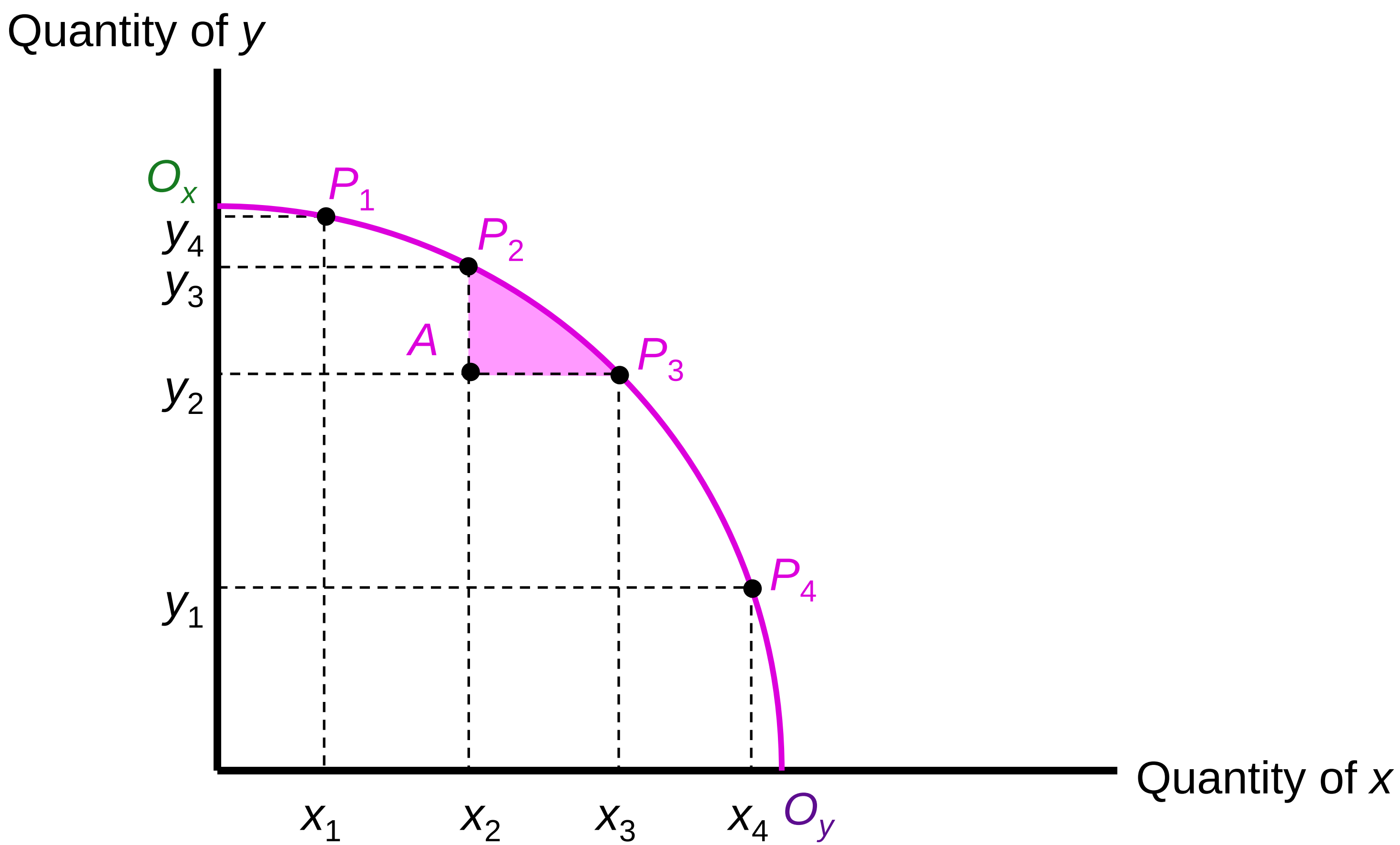
- How outputs and inputs are related
  - maximum output of *y* that can be produced for any level of *x*
- negative of the slope of the production possibility frontier

$$RPT\ (\text{of } x \text{ for } y) = -\begin{bmatrix}\text{slope of production} \\ \text{possibility frontier}\end{bmatrix}$$

$$RPT\ (\text{of } x \text{ for } y) = -\frac{dy}{dx}\ (\text{along } O_x O_y)$$

FIGURE 13.3

Production Possibility Frontier



The production possibility frontier shows the alternative combinations of x and y that can be efficiently produced by a firm with fixed resources. The curve can be derived from Figure 13.2 by varying inputs between the production of x and y while maintaining the conditions for efficiency. The negative of the slope of the production possibility curve is called the rate of product transformation (RPT).

Shape of the frontier

- concave shape: an increasing *RPT*
  - diminishing returns on both *x* and *y*
- Costs of any output combination: $C(x,y)$
  - Constant along the production possibility frontier,
- Total differential

$$RPT = -\left.\frac{dy}{dx}\right|_{C(x,y)-\bar{C}=0} = -\frac{C_x}{C_y} = -\frac{MC_x}{MC_y}$$

EXAMPLE 13.1 Concavity of the Production Possibility Frontier

- Production of *x* and *y*
  - Depends only on labor
  - Production functions: $x = f(l_x) = l_x^{0.5};\ y = f(l_y) = l_y^{0.5}$
  - Total labor supply = 100, $l_x + l_y = 100$
  - The production possibility frontier:

$$x^2 + y^2 = 100 \quad \text{for } x, y \geq 0$$

  - The *RPT* can be calculated:

$$RPT = -\frac{dy}{dx} = -\left(-\frac{f_x}{f_y}\right) = \frac{2x}{2y} = \frac{x}{y}$$

  - Concave

EXAMPLE 13.1 Concavity of the Production Possibility Frontier

- Two goods are produced under constant returns to scale:

$$x = f(k,l) = k_x^{0.5} l_x^{0.5} \quad \text{and} \quad y = g(k,l) = k_y^{0.25} l_y^{0.25}$$

  - Total labor and capital are constrained:

$$k_x + k_y = 100, \text{ and } l_x + l_y = 100$$

$$RTS_x = \frac{k_x}{l_x} = \kappa_x \quad \text{and} \quad RTS_y = \frac{3k_y}{l_y} = 3\kappa_y$$

where $\kappa_i = k_i / l_i$

On the production possibility frontier:

$$RTS_x = RTS_y \quad \text{or} \quad \kappa_x = 3\kappa_y$$

EXAMPLE 13.1 Concavity of the Production Possibility Frontier

- Capital-labor ratios:

$$\frac{k_x + k_y}{l_x + l_y} = \alpha\kappa_x + (1\text{-}\alpha)\kappa_y = 1$$

where $\alpha = l_x / (l_x + l_y)$

Input ratios:

$$\kappa_y = \frac{1}{1+2\alpha} \quad \text{and} \quad \kappa_x = \frac{3}{1+2\alpha}$$

EXAMPLE 13.1 Concavity of the Production Possibility Frontier

- Production possibility frontier in terms of the share of labor devoted to x production:

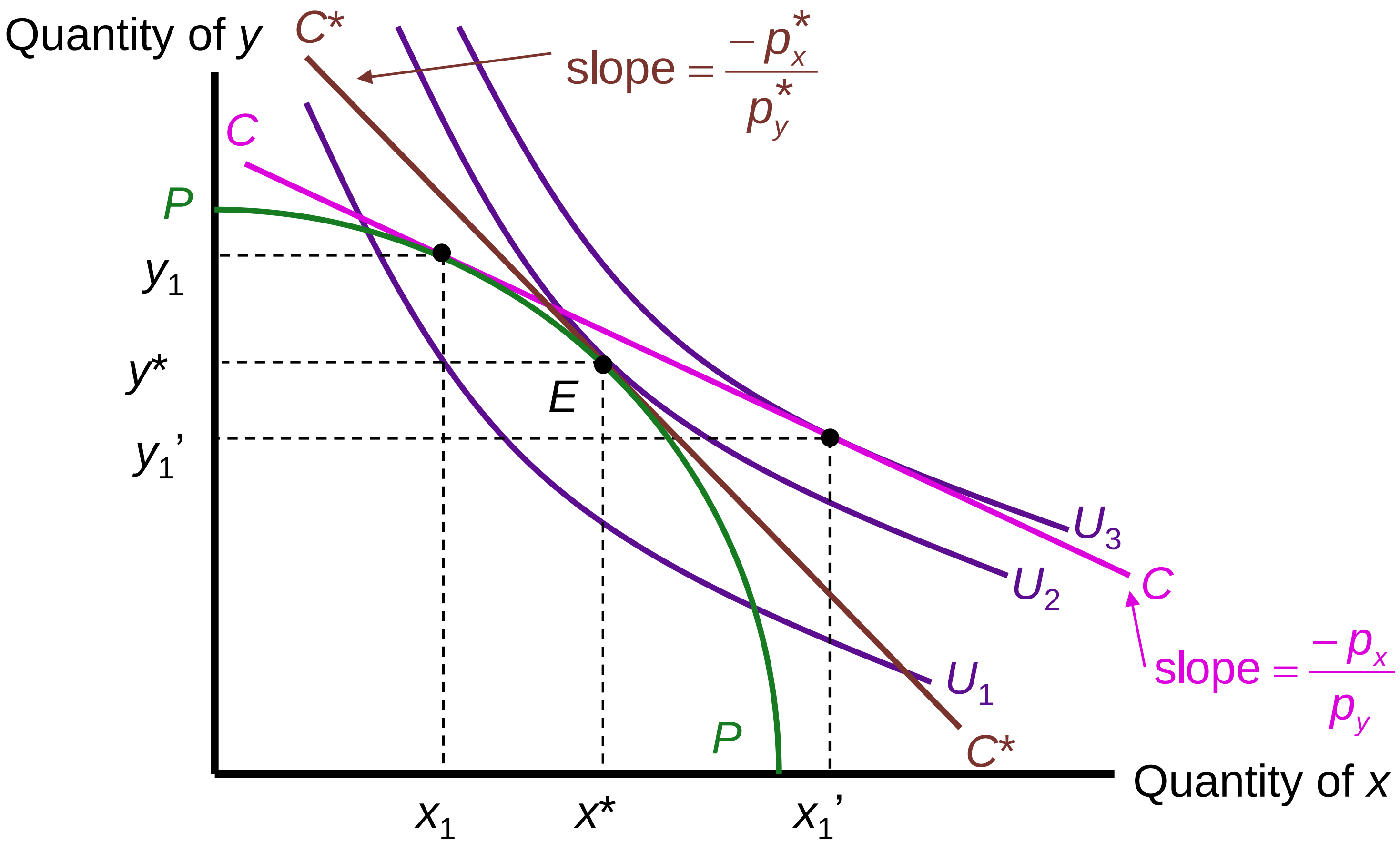
$$x = \kappa_x^{0.5} l_x = \kappa_x^{0.5} \alpha(100) = 100\alpha\left(\frac{3}{1+2\alpha}\right)^{0.5}$$

$$y = \kappa_y^{0.25} l_y = \kappa_y^{0.25}(1-\alpha)(100) = 100(1-\alpha)\left(\frac{1}{1+2\alpha}\right)^{0.25}$$

Equilibrium

- Price ratio = RPT = MRS

Determination of Equilibrium Prices

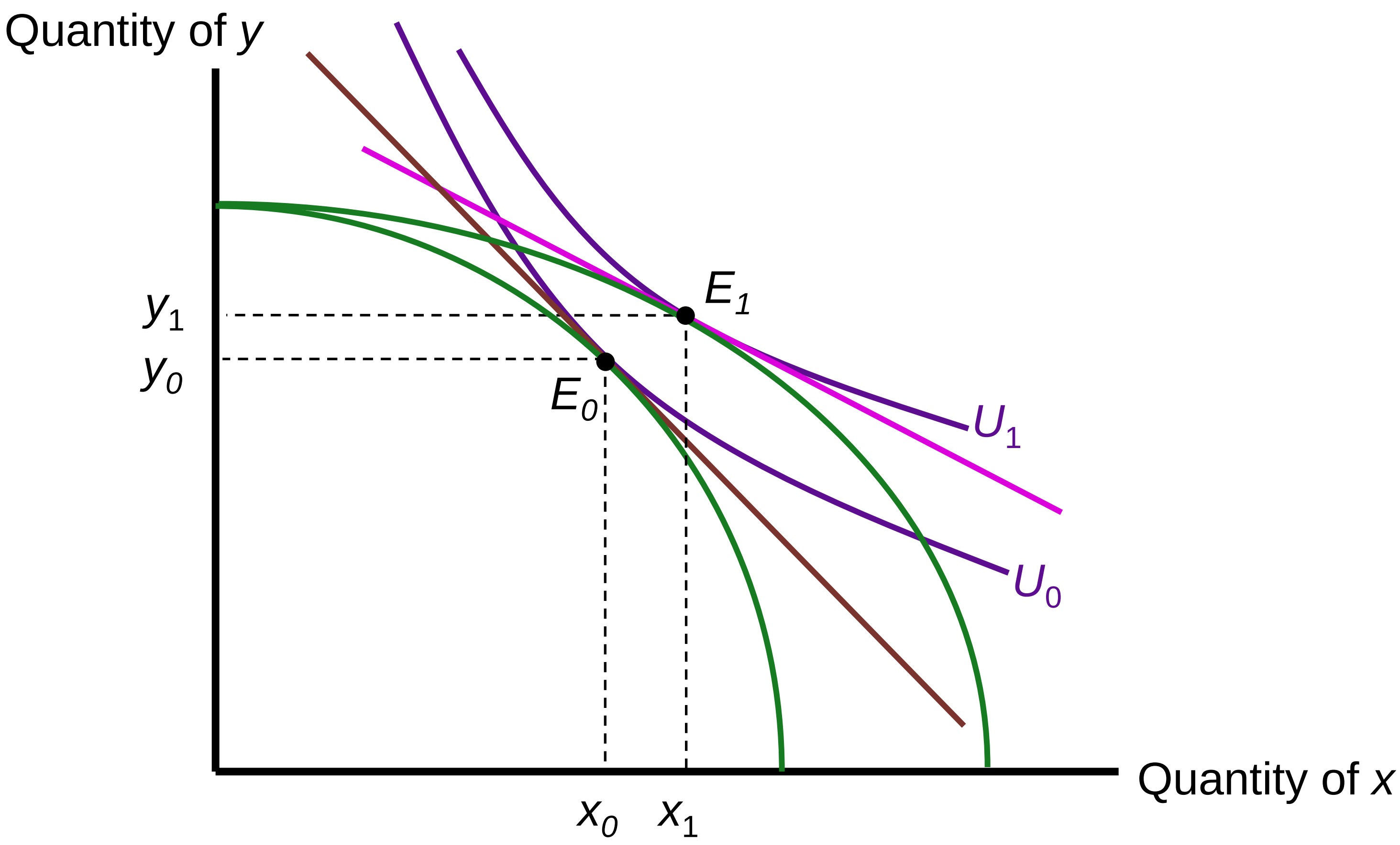


With a price ratio given by p_x/p_y, firms will produce x_1, y_1; society's budget constraint will be given by line C. With this budget constraint, individuals demand x'_1 and y'_1; that is, there is an excess demand for good x and an excess supply of good y. The workings of the market will move these prices toward their equilibrium levels p^*_x , p^*_y . At those prices, society's budget constraint will be given by line C, and supply and demand will be in equilibrium. The combination x*, y* of goods will be chosen.

Comparative Statics

- preferences were to shift toward good x
- technical progress in the production of good x

Effects of Technical Progress in x Production



Technical advances that lower marginal costs of x production will shift the production possibility frontier. This will generally create income and substitution effects that cause the quantity of x produced to increase (assuming x is a normal good). Effects on the production of y are ambiguous because income and substitution effects work in opposite directions.

EXAMPLE 13.2 Comparative Statics in a General Equilibrium Model

- Production possibility frontier: $x^2 + y^2 = 100$
- Utility function: $U(x,y) = x^{0.5}y^{0.5}$
- Demand functions:

$x = x(p_x,p_y,I) = 0.5I/p_x$ and $y = y(p_x,p_y,I) = 0.5I/p_y$

- Base-case equilibrium
  - Profit maximization by firms: $p_x/p_y = MC_x/MC_y = RPT = x/y$
  - Utility-maximizing demand : $p_x/p_y = y/x$
  - $x^*=y^*=7.07$ and $p_x/p_y = 1$

EXAMPLE 13.2 Comparative Statics in a General Equilibrium Model

- The budget constraint: total income = labor income + profits
  - Consider all prices in terms of the wage rate, w
  - Total labor income = $100w$
  - Profit for firm x: $\pi_x = (p_x - AC_x)x = 50w$
  - *total income = labor income + profits =* $100w+2(50w)=200w$

EXAMPLE 13.2 Comparative Statics in a General Equilibrium Model

- A shift in supply
  - Technical improvement in x production
  - New production function, $x = 2l_x^{0.5}$
  - Production possibility frontier: $x^2/4+y^2=100$
  - $RPT = x/4y$
  - Equilibrium: $x^*=2(50)^{0.5}$, $y^*=(50)^{0.5}$ $p_x/p_y = 1/2$

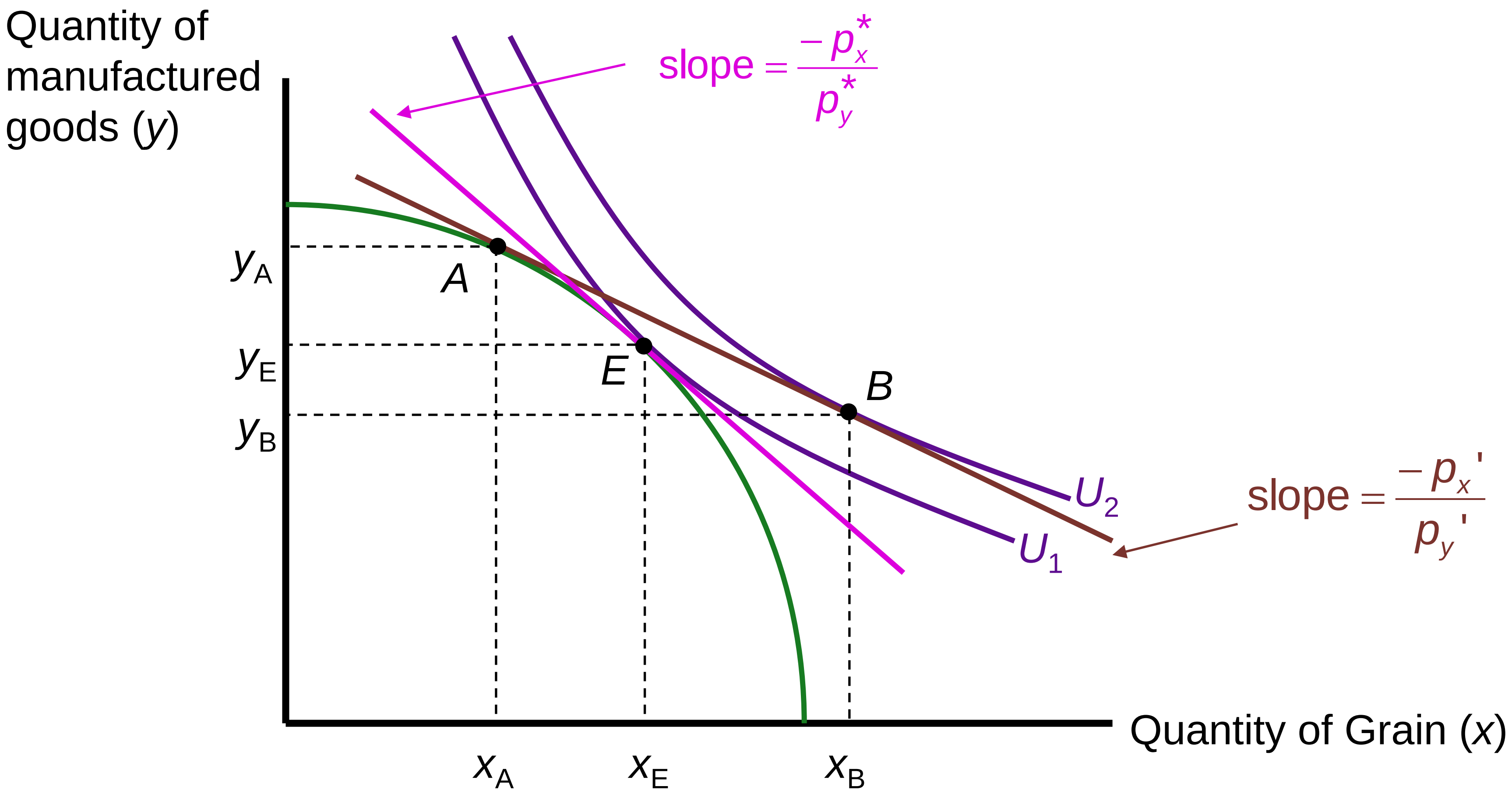
EXAMPLE 13.2 Comparative Statics in a General Equilibrium Model

- A shift in demand
  - Consumer preferences were to switch to favor good y , $U(x,y)=x^{0.1}y^{0.9}$
  - Demand functions: $x = 0.1I/p_x$; $y = 0.9I/p_y$
  - Equilibrium: $x^*=(10)^{0.5}$, $y^*=3(10)^{0.5}$ $p_x/p_y = 1/3$

Application: Trade Policy

- What happen if government impose tariff on agricultural product?
  - factor prices will change

Analysis of tariff



Reduction of tariff barriers on grain would cause production to be reallocated from point E to point A; consumption would be reallocated from E to B. If grain production is relatively capital intensive, the relative price of capital would decrease as a result of these reallocations.

Model of Exchange

- No production
- *n* goods
- *m* individuals
- Consumption bundle: $x^i=(x^i_1,x^i_2,\ldots,\ x^i_n)$
- Utility: $u^i(x^i)$ where $i = 1...m$
- Individuals are price-takers
- Initial endowments of the goods $\overline{x}^{\,i}$

Model of Exchange

- Consumers maximize utility under budget constraint
  - Total amount spent on consumption = Total value of his or her endowment

$$px^i = p\bar{x}^i$$

- Marshallian demand function: $x^i(p, p\bar{x}^i)$
  - Continuous
  - Homogeneous of degree 0 in all prices and income

Walrasian Equilibrium

- A price vector *p** and allocation **x**

$$\sum_{i=1}^{m} x^i(p^*, p^*\bar{x}^i) = \sum_{i=1}^{m} \bar{x}^i$$

- Demand equals supply in each market
- Walras' law
  - The value of all quantities demanded must equal the value of all endowments

$$\sum_{i=1}^{m} px^i = \sum_{i=1}^{m} p\bar{x}^i \quad \text{or} \quad \sum_{i=1}^{m} p(x^i - \bar{x}^i) = 0$$

Technical: Existence Proof

- Normalize these to form a new set of prices

$$p'_i = \frac{p_i}{\sum_{k=1}^{n} p_k}, \quad \text{with the properties:}$$

$$\sum_{k=1}^{n} p'_k = 1 \quad \text{and} \quad \frac{p'_i}{p'_j} = \frac{p_i / \sum p_k}{p_j / \sum p_k} = \frac{p_i}{p_j}$$

- Equilibrium Price

$$\sum_{i=1}^{m} x^i(p^*, p^* \bar{x}^i) - \sum_{i=1}^{m} \bar{x}^i = 0 \text{, or } z(p^*) = 0$$

Technical: Existence Proof

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- Equilibrium Price

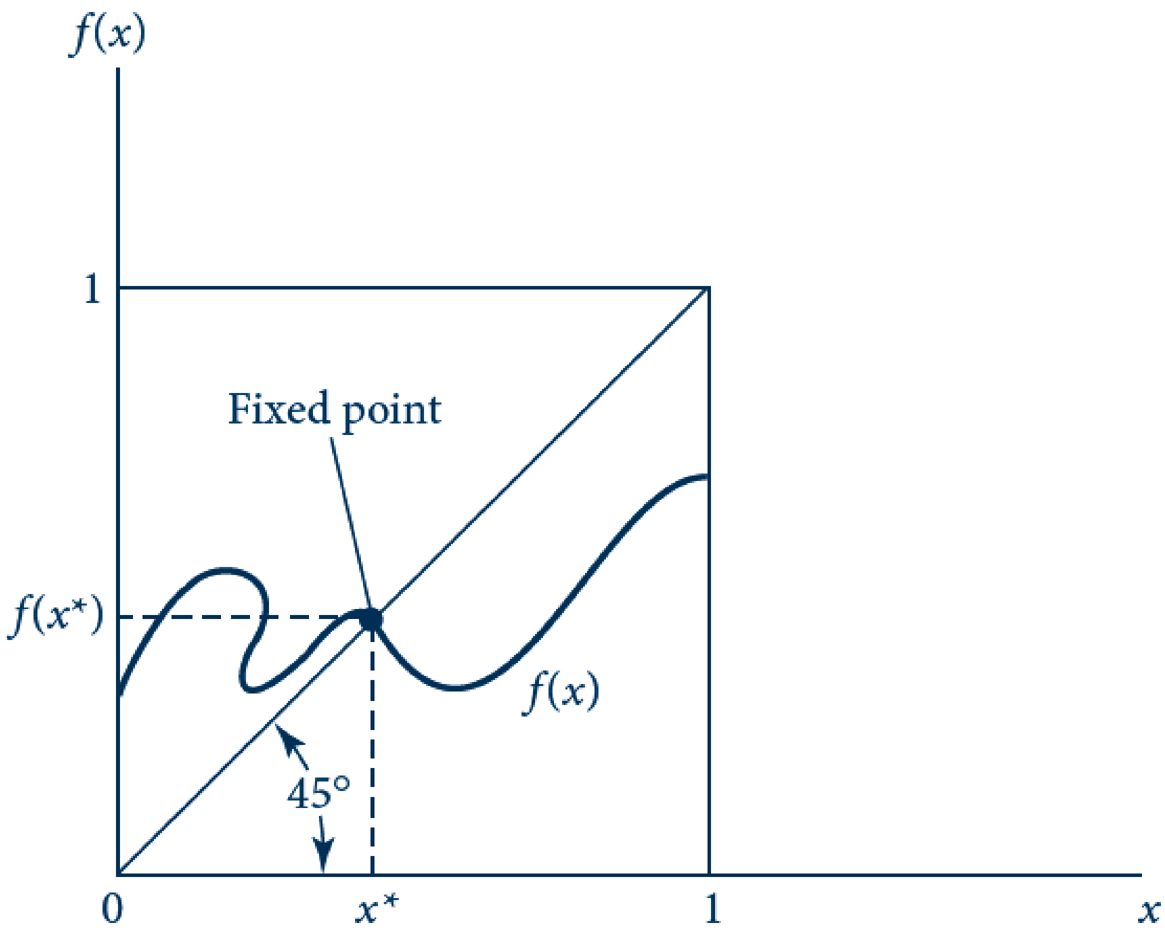
$$\sum_{i=1}^{m} x^i(p^*, p^* \bar{x}^i) - \sum_{i=1}^{m} \bar{x}^i = 0 \text{ , or } \quad z(p^*) = 0$$

Technical: Existence Proof

- Brouwer's fixed point theorem
  - Any continuous function from a closed compact set onto itself
  - Will have a "fixed point" such that $x=f(x)$
- New set of prices, $p_1=f(p_0)=p_0+kz(p_0)$
  - p_0 – arbitrary set of prices
  - k – positive constant
  - Continuous function
  - Will map one set of normalized prices into another
  - Meet the conditions of the Brouwer's fixed point theorem
- Fixed point: $p^*=f(p^*)=p^*+kz(p^*)$
  - We have: $z(p) = 0$ (excess demand)
  - Thus, p^* is an equilibrium price vector

FIGURE 13.7

A Graphical Illustration of Brouwer's Fixed Point Theorem

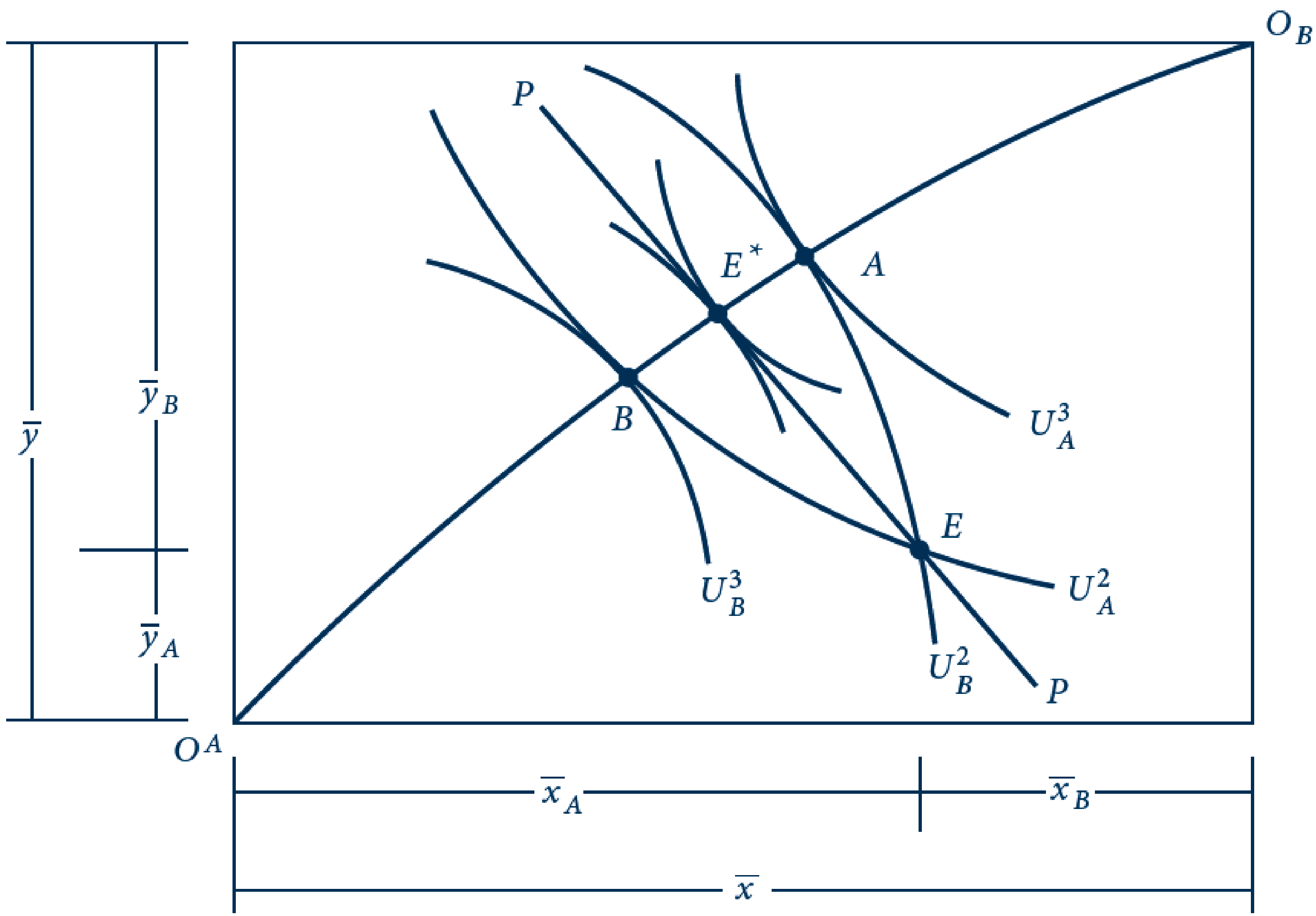


Because any continuous function must cross the 45 line somewhere in the unit square, this function must have a point for which f (x*) = x*. This point is called a fixed point.

First Welfare Theorem

- Pareto efficient allocation
  - No alternative allocation in which at least one person is better off and no one is worse off
- First theorem of welfare economics
  - Every Walrasian equilibrium is Pareto efficient

The First Theorem of Welfare Economics

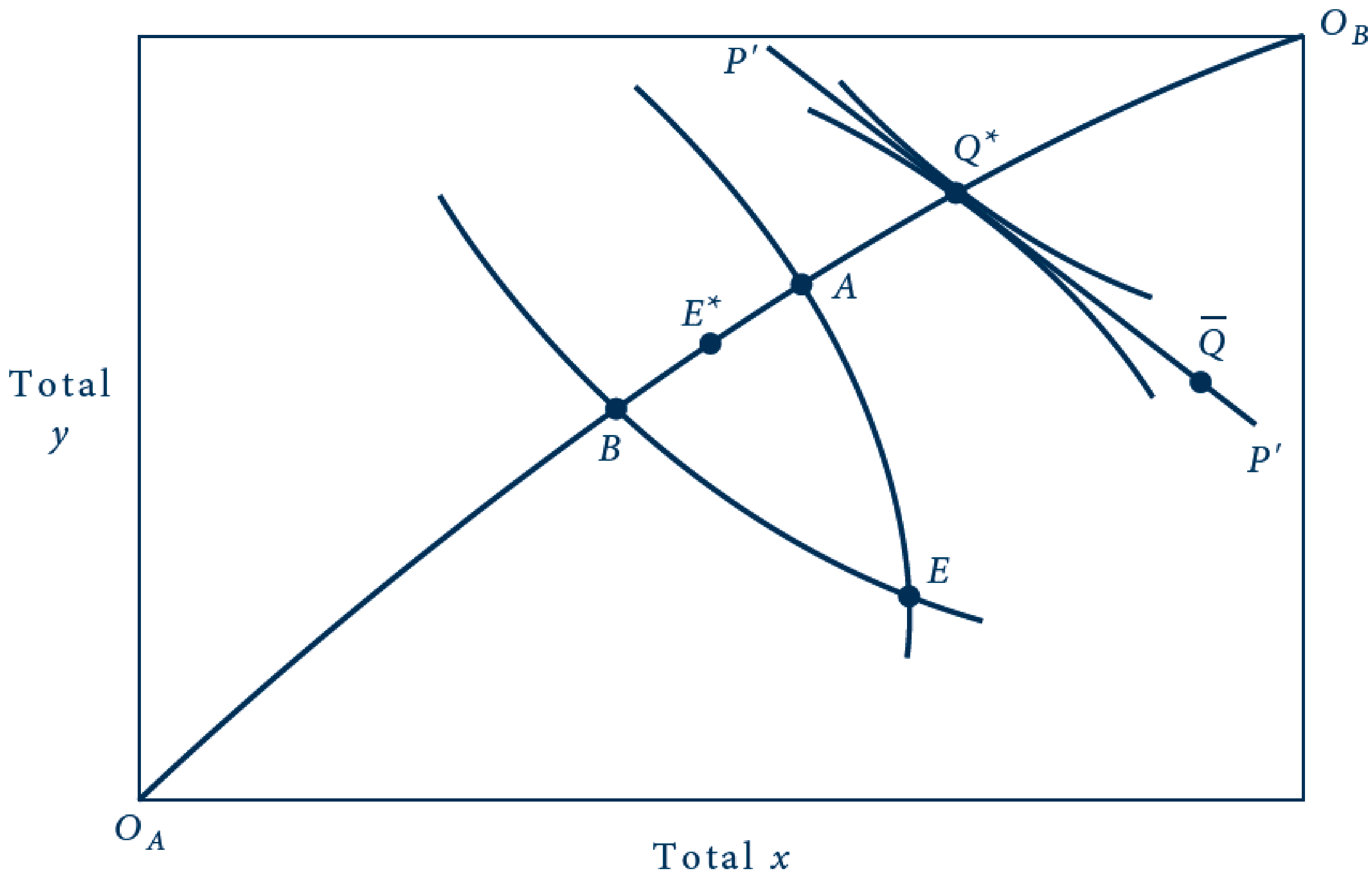


With initial endowments at point E, individuals trade along the price line PP until they reach point E*. This equilibrium is Pareto efficient.

Second Welfare Theorem

- The second theorem of welfare economics
  - For any Pareto optimal allocation, there exists a set of initial endowments and a related price vector such that this allocation is also a Walrasian equilibrium

The Second Theorem of Welfare Economics



If allocation Q* is regarded as socially optimal, this allocation can be supported by any initial endowments on the price line P'P'. To move from E to, say, $\bar{Q}$ would require transfers of initial endowments.

- A simple two-person, two-good exchange economy
  - Total quantities of the goods are fixed:

$$x = y = 1{,}000$$

  - Utility functions:

$U_A(x_A,y_A) = x_A^{2/3} y_A^{1/3}$ and $U_B(x_B,y_B) = x_B^{1/3} y_B^{2/3}$

  - Lagrangian expression

$$\mathcal{L}(x_A,y_A) = U_A(x_A,y_A) + \lambda[U_B(1{,}000-x_A, 1{,}000-y_A) - \bar{U}_B]$$

$$\mathcal{L}(x_A,y_A) = x_A^{2/3} y_A^{1/3} + \lambda[(1{,}000-x_A)^{1/3}(1{,}000-y_A)^{2/3} - \bar{U}_B]$$

EXAMPLE 13.3 A Two-Person Exchange Economy

- First order conditions:

$$\frac{\partial L}{\partial x_A} = \frac{2}{3}\left(\frac{y_A}{x_A}\right)^{1/3} - \frac{\lambda}{3}\left(\frac{1,000 - y_A}{1,000 - x_A}\right)^{2/3} = 0$$

$$\frac{\partial L}{\partial y_A} = \frac{1}{3}\left(\frac{x_A}{y_A}\right)^{2/3} - \frac{2\lambda}{3}\left(\frac{1,000 - x_A}{1,000 - y_A}\right)^{1/3} = 0$$

- We get:

$$\frac{x_A}{1,000 - x_A} = \frac{4y_A}{1,000 - y_A}$$

  - This equation allows us to identify all the Pareto optimal allocations in this exchange economy

- Equilibrium price ratio
  - We need to know the marginal rate of substitution

$$MRS_A = \frac{\partial U_A / \partial x_A}{\partial U_A / \partial y_A} = 2\frac{y_A}{x_A} = 0.8$$

$$MRS_B = \frac{\partial U_B / \partial x_B}{\partial U_B / \partial y_B} = 0.5\frac{y_B}{x_B} = 0.8$$

  - Therefore $p_x/p_y = 0.8$

Social Welfare Function

- ranking potential allocations of resources based on the utility they provide to individuals

$$Social\ welfare = SW[U_1(x^1), U_2(x^2), \ \ldots, \ U_m(x^m)]$$

- Utilitarianism:

$$SW(U_1,\ U_2,\ \ldots,\ U_m) = \frac{U_1^R}{R} + \frac{U_2^R}{R} + \ldots + \frac{U_m^R}{R}, \quad R \le 1$$

- Rawlsian:

$$SW(U_1, U_2, \ldots, U_m) = Min[U_1, U_2, \ldots, U_m]$$

Model of Production

- Includes factors of production
  - Whose prices also will be determined
- Firms (*r*) involved in production
  - Outputs: positive sign; Inputs take a negative sign
  - Maximize profits: firm's production plan
    - An $n \times 1$ column vector, y^j $(j = 1 \ldots r)$, which contains both positive and negative entries
  - Each individual owns a predefined share, s_i, of the profits of all firms $\sum_{i=1}^{m} s_i = 1$

Model of Production

- Production functions
  - Assumed to be sufficiently convex to ensure a unique profit maximum for any set of output and input prices
- Profits: $\pi_j(p) = py^j \quad \text{if} \quad \pi_j(p) \geq 0 \quad \text{and}$

  $y^j = 0 \quad \text{if} \quad \pi_j(p) < 0$

Model of Production

- Labor supply
  - Individuals are endowed with a certain number of potential labor hours

$$px_i = s_i \sum_{j=1}^{r} py^j + p\overline{x}^i \qquad i = 1 \ldots m$$

$$\text{Over all individuals:} \quad p\sum_{i=1}^{m} x_i(p) = p\sum_{j=1}^{r} y^j(p) + p\sum_{i=1}^{m} \overline{x}^i$$

$$\text{Let} \quad x(p) = \sum x_i(p), \quad \mathrm{y}(p) = \sum y^j(p), \quad x = \sum \overline{x}^i$$

$$\text{Walras' law: } px(p) = py(p) + p\overline{x}$$

Model of Production

- Walrasian equilibrium price vector ($p*$)
  - A set of prices at which demand equals supply in all markets simultaneously

$$x(p^*) = y(p^*) + \bar{x}$$

- Excess demand functions

$$z(p) = x(p) - y(p) - \bar{x}$$

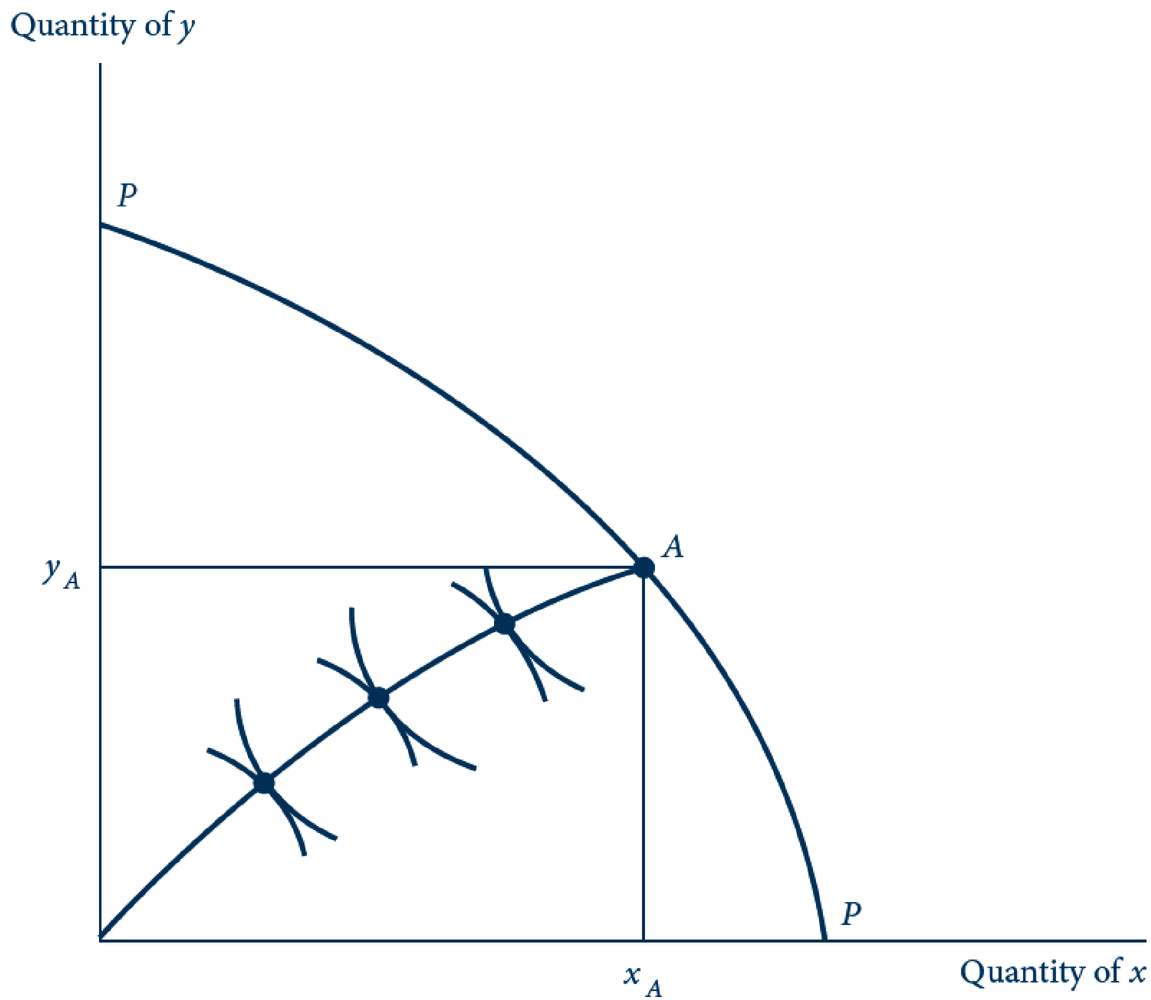
  - Are homogeneous of degree 0 in prices
    - Any price vector for which $z(p^*) = 0$ will also have the property that $z(tp^*) = 0$ and $t > 0$

Welfare Function

- The first theorem continues to hold

FIGURE 13.10

Production Increases the Number of Feasible Allocations



Any point on the production possibility frontier PP can serve as the dimensions of an Edgeworth exchange box.

Computable General Equilibrium Models

- Two advances - rapid development of general equilibrium models in recent years
  - The theory has been generalized to include many features of real-world economies
  - Expanding computer capacity has made it possible to study more complex models
- Procedure
  - Starts by defining the number of goods to be included in the model
  - The goal of the model:
    - To solve for the equilibrium prices for these goods and how they may change when conditions change

Insight from General Equilibrium

- Prices are endogenous in economic models
- All firms and productive inputs are owned by households
- Any model with a government sector
  - Is incomplete if it does not specify how tax receipts are used

Insight from General Equilibrium

- The "bottom line" in any policy evaluation
  - Is the utility of households
  - Firms and governments are only intermediaries in this accounting
- All taxes distort economic decisions along some dimension
  - The welfare costs of the distortions must be weighed against the benefits of such taxes

EXAMPLE 13.4 A Simple General Equilibrium Model

- Assume:
  - Two households, two consumer goods (*x* and *y*), and two inputs (*k* and *l*)
- Households
  - Each has an endowment of *k* and *l*
  - Obtain utility from consuming *x*, *y*, and leisure (l_r)
  - Simple Cobb-Douglas utility functions

$$U_1 = x_1^{0.5} y_1^{0.3} l_{r_1}^{0.2}; \qquad U_2 = x_2^{0.4} y_2^{0.4} l_{r_2}^{0.2}$$

- Production of *x* and *y*
  - Cobb-Douglas technologies

$$x = k_x^{0.2} l_x^{0.8}; \qquad y = k_y^{0.8} l_y^{0.2}$$

- Initial endowments

$$\bar{k}_1 = 40 \qquad \bar{l}_1 = 24$$
$$\bar{k}_2 = 40 \qquad \bar{l}_2 = 24$$

- Model without government
  - Solution will be four equilibrium prices

EXAMPLE 13.4 A Simple General Equilibrium Model

- Price normalization scheme

$$p_x + p_y + p_k + p_l = 1$$

- Solving for these prices yields

$$p_x = 0.363; \ p_y = 0.253; \ p_k = 0.136; \ p_l = 0.248$$

- Total production: $x = 23.7; \ y = 25.1$
- Utility-maximizing choices for household 1:

$$x_1 = 15.7; \ y_1 = 8.1; \ l_r = 9.2; \ U_1 = 13.5$$

- Utility-maximizing choices for household 2:

$$x_1 = 8.1; \ y_1 = 11.6; \ l_r = 5.9; \ U_1 = 8.75$$

EXAMPLE 13.5 The Excess Burden of a Tax

- The government
  - Imposes an ad valorem tax of 0.4 on good x
    - Wedge between what demanders pay for this good x (p_x)
    - And what suppliers receive for the good ($p'_x = (1 - t)p_x = 0.6p_x$)
- Revenues generated by this tax
  - Rebated to the households in a 50–50 split
- New equilibrium prices

$$p_x = 0.472;\ p_y = 0.218;\ p_k = 0.121;\ p_l = 0.188$$

- Total production: $x = 17.9$; $y = 28.8$
  - Allocation of resources has shifted significantly toward y production
- Utility-maximizing choices for household 1:

  $x_1 = 11.6$; $y_1 = 15.2$; $l_r = 11.8$; $U_1 = 12.7$

  - Worse off
- Utility-maximizing choices for household 2:

  $x_1 = 6.3$; $y_1 = 13.6$; $l_r = 7.9$; $U_1 = 8.96$

  - Better off

Computable general equilibrium models

- General methodology - Computable general equilibrium (CGE) models
  - Assume various forms for production and utility functions
  - Choose particular parameters of those functions based on empirical evidence
  - Generate general equilibrium solutions
    - Then compare with real-world data

Computable general equilibrium models

- General methodology - Computable general equilibrium (CGE) models
  - “Calibrate” the models to reflect reality
  - Vary various policy elements in the models
    - Provide general equilibrium estimates of the overall impact of those policy changes

Trade models

- The impact of trade barriers
  - Necessary to introduce a large degree of product differentiation into individuals' utility functions
  - Incorporate increasing returns-to-scale technologies into their production sectors
    - Capture one of the primary advantages of trade for smaller economies
- Impact of the North American Free Trade Agreement (NAFTA)

Tax and transfer models

- Evaluate potential changes in a nation’s tax and transfer policies
  - Considerable care must be taken in modeling the factor supply side
- The Dutch Micro Macro Model to Analyze the Institutional Context (MIMIC0 model

Environmental models

- Understanding the ways in which environmental policies may affect the economy
  - Production of pollutants - major side effect of the other economic activities
  - Specify environmental goals in terms of a given reduction in these pollutants
    - Economic costs of various strategies
  - Study the impact of environmental policies on income distribution

Environmental models

- Assessing CO_2 reduction strategies
  - The General Equilibrium Environmental (GREEN) model
    - Developed by the Organization for Economic Co-operation and Development (OECD)

Regional and urban models

- To examine economic issues that have important spatial dimensions
  - Widely used to examine the local impact of major changes in government spending policies