EC4101
Topic 6: Asymmetric Information

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1 Introduction

First welfare theorem says efficient outcomes can be achieved through competitive market. In this topics, we shall see how asymmetric information changes this conclusion. We will focus on principle-agent models and auction models.

1.1 Reading:

1. Snyder and Nicholoson, Chapter 18, Microeconomic Theory: Basic Principles and Extensions, 11th edition, 2012
2. The Market for "Lemons": Quality Uncertainty and the Market Mechanism, The Quarterly Journal of Economics, Vol. 84, No. 3. (Aug., 1970), pp. 488-500.

2 Principal-agent Problem

2.1 Hidden Action: Moral Hazard

Principal wants agent to take some action. Outcome of an action is observable. Action itself is NOT observable because there is random element between actions and outcomes. Agent might not adopt the best action from the principal perspective if their interests do not align. The solution is to sign a contract to link compensation to observable outcomes.

2.1.1 Owner-Manger

Three stage game:
1) Owner sets the incentive scheme (salary)
2) The manager decides whether or not to accept the contract
3) The manager decides how much effort to put forth (conditional on accepting the contract)
Firm's gross profit: $\pi_g = e+\varepsilon$ where $e \geq 0$ is effor by the manager and ε is unobservable having zero mean and variance σ^2. The next profit $\pi_n = \pi_g - s$ where s is salary of the manager. Owner is risk neutral. Hence, expected utility is

$$E\pi_n = E\left(e+\varepsilon - s\right) = e - E\left(s\right)$$

For manager, cost of effort is $c\left(e\right)$ where $c'\left(e\right) > 0$ and $c''\left(e\right) < 0$. Assuming constant risk averison, expected utility can be written as

$$EU = E\left(s\right) - \frac{A}{2}Var\left(s\right) - c\left(e\right)$$

where A is risk aversion.
First-best contract
A fixed salary s^* if he exerts a first-best level of effort e^* ; and nothing otherwise.
Participation constraint: Manager accepts contract

$$E(U) = s^* - c(e^*) \geq 0$$

Owner pays the lowest salary possible so that

$$s^* = c\left(e^*\right)$$

which gives net profit

$$\begin{aligned} E\pi_n &= e^* - E\left(s^*\right) \\ &= e^* - c\left(e^*\right) \end{aligned}$$

Hence, at optimum, we have $c'\left(e^*\right) = 1$.
Second-best contract.

We just consider linear contract:

$$s(\pi_g) = \alpha + b\pi_g$$

Then manager expected payoff is

$$\begin{aligned} & E(a + b\pi_g) - \frac{A}{2} Var(a + b\pi_g) - c(e) \\ = \; & a + be - \frac{A}{2} b^2 \sigma^2 - c(e) \end{aligned}$$

Hence, optimal condition is $c'(e) = b$

For participation constraint, we have

$$a + be - \frac{A}{2} b^2 \sigma^2 - c(e) \geq 0$$

The owner will maximize $E(\pi_n) = e(1-b) - a$ subject to $a + be - \frac{A}{2} b^2 \sigma^2 - c(e) = 0$ or

$$\begin{aligned} E(\pi_n) &= e - \frac{A}{2} b^2 \sigma^2 - c(e) \\ &= e - c(e) + \frac{A}{2} (c'(e))^2 \sigma^2 \end{aligned}$$

So that optimal condition is

$$c'(e^*) = \frac{1}{1 + A\sigma^2 c''(e^*)} = b$$

where a will be solved from the participation constraint.

Conclusion.

Second-best effort is less then first-best effort since owner cannot observe effort directly.

Manager is risk-averse so that risk-premium is needed to induce effort.

Numerical Example.

Suppose cost of effort $c(e) = \frac{e^2}{2}$ and $\sigma^2 = 1$.

First-best: $c'(e^*) = e^* = 1$, $s = 1/2$ and $\pi_n = 1/2$.

Second-best:

If $A = 1$, then $e^* = 1/2$, $b^* = 1/2$, $a^* = 0$ and $\pi_n = 1/4$

If $A = 2$, then $e^* = 1/3$, $b^* = 1/3$, $a^* = 1/18$ and $\pi_n = 1/6$

2.1.2 Insurance

Insurance company wants the policyholder to exert precautionary measures but cannot observe them.
Under first-best, we can have full insurance with socially efficient level of precaution.
Under second-best, no full insuarnce because individual needs to share some risk to induce some precautionary measure.

2.2 Hidden Information: Adverse Selection

Principal does not know the type of agent.
Since contract cannot be made specific to each type, principal can only offer different types of contract hoping different types of agent can be selected into differnet contracts.
Failure to do so might need to lemon's problem.

2.2.1 Price Discrimination

A monopolist (the principal): offers a nonlinear price schedule to consumers (agent). That is a menu of different-sized bundles at different prices. Usually a quantity discount: larger bundles sell for lower per-unit price.
First-best contract:
The monopolist observes types of consumer.
Type-θ consumer's preference is $\theta v(q) - T$ if q units are purchased and T is paid.
Participation constraint: $\theta v(q) - T \geq 0$. At the optimum: $\theta v'(q^*) = c$. Marginal benefit is equal to marginal cost. And monopolist sets $T = \theta v(q^*)$.
Numerical example:
Marginal cost of producton is \$5 per unit. Half of consumers are high type $\theta_H = 20$ and half of consumers are low type $\theta_L = 15$. Assume $v(q) = 2\sqrt{q}$.
Hence, $q = (\theta/c)^2$ so that

$$\begin{aligned} q_L^* &= 9,\ q_H^* = 16 \\ T_L^* &= 90,\ T_H^* = 160 \\ E\pi &= 62.5 \end{aligned}$$

Second-best contract:
We focus on an example here. For general analysis, see appendix.

For incentive compatibility, we need to have

$$\frac{\theta_L}{\sqrt{q_L}} = c + \frac{\theta_H - \theta_L}{\sqrt{q_L}}$$

so that

$$\begin{aligned} q_L^* &= 5 \\ T_L^* &= 60 \\ E\pi &= 50 \end{aligned}$$

2.2.2 Insurance

There are two types of policy holders: high risk and low risk.
First-best contract:
Insurance company can offer different contracts to different groups of policyholders.
Giving them full insurance and extract all the surplus.
Second-best contract:
If the same two contracts are offered, then high-risk would like to choose the low-risk contract.
Hence, we need to make the coverage to the low-risk unattractive to the high-risk.
Under competitive equilibrium, there is no pooling equilibrium.

2.2.3 Lemon Market

Lemon: Bad second-hand car (American slangs)
Sellers of used cars: have more information on the condition of the car
Hence, offering for sale can be bad signal of car quality because it must be that the value of car is below some threshold for the owner to keep it
Example. Under symmetric information (whether it is complete or incomplete), any type of cars are traded in the market but aymmetric information only the lowest quality of cars traded.
For a good car, the value to a seller is 90 and that to a buyer is 100
For a good car, the value to a seller is 10 and that to a buyer is 20
It is efficient for both type of cars to be traded.
Suppose supply of car finite and demand for car is infinite.
Under complete information, good cars are sold at 100 and bad cars are sold at 200.

Suppose the proportion of good cars to bad cars is 1:1.
Under two-sdied incomplete information (no one knows the quality of the car), every car is sold at $60 (= 0.5\,(100 + 20))$.
However, under asymmetric information, there will be any equilibrium that good cars are traded.
Sellers knows more about their cars but buyer does not. Hence, all cars are sold at the same price.
Case 1: price ≥ 90. Both cars are sold but buyers will pay no more than 60. Hence, not an equilibrium because high-quality car owners keep their own car.
Case 2: price < 90. Only lemons are sold but then price ≤ 20. Hence, high-quality car owners keep their own car.
Conclusion: only lemons are traded in equilibrium.
Parametric Example:
Buyers know quality of cars q are uniformly distributed from 0 to $\bar{q}$.
Let market price be p.
Sellers offer their cars for sale if and only if $q \leq p$.
The quality of a car offered for sale is also uniformly distributed between 0 and p.
Hence, expected quality:

$$\int_0^p q \left(\frac{1}{p}\right) dq = \frac{p}{2}$$

For cars with quality q, seller value them at q and buyers value them at $q + b$.
Buyers' expected surplus is

$$\begin{aligned} & q + b - p \\ = \ & \frac{p}{2} + b - p \\ = \ & b - \frac{p}{2} \end{aligned}$$

If supply of car finite and demand for car is infinite, then price will be $2b$.
If $2b < q^*$, all cars with qualities $q > 2b$ are out of the market.

3 Auction

A sellers has an divisible object for sale.
There are multiple potential buyers.

If the seller knows all of their willingness to pay, the seller can simply sell the object to the buyer with highest willingness to pay and extract all the surplus for the buyer. However, willingness to pay is private information of buyers so the seller has to use some way to extract surplus from buyers.

3.0.4 First-price sealed-bid auction

Timing:
1) All bidders simultaneously submit secret bids
2) The auctioneer unseals the bids and awards the object to the highest bidder
3) The highest bidder pays his own bid
Weakly dominated to bid $b = v$ since even if winning will leads to zero surplus.
Proposition. Under two-bidder first-price auction with independent private values uniformly distributed from 0 to 1, there is a symmetric equilibrium that every bidder with value v bids $b = v/2$.
Proof.
Suppose bidder 2 bids $b_2 = kv_2$ for some $k > 0$.
Then bidder 1 payoff is

$$\begin{aligned} & \Pr\left(b_1 > b_2\right)\left(v_1 - b_1\right) \\ = & \Pr\left(b_1 > kv_2\right)\left(v_1 - b_1\right) \\ = & \Pr\left(v_2 < \frac{b_1}{k}\right)\left(v_1 - b_1\right) \\ = & \frac{b_1}{k}\left(v_1 - b_1\right) \end{aligned}$$

First order condition: $\frac{1}{k}\left(v_1 - 2b_1\right) = 0$ or

$$b_1 = \frac{1}{2}v_1$$

3.0.5 Second-Price seal-bid auction

Timing:
1) All bidders simultaneously submit secret bids
2) The auctioneer unseals the bids and awards the object to the highest bidder
3) The highest bidder pays next highest bid
Proposition. Under second-price auction, Weakly dominant to bid $b = v$.

Proof.
Suppose $b > v$.
If $p \geq b > v$, then lose the object but it is the same outcome as $b = v$
If $b > p \geq v$, then win the object but it is better not to win under $b = v$.
If $b > v \geq p$, then win the object but it is the same outcome as $b = v$.
Suppose $b < v$.
If $p \geq v > b$, then lose the object but it is the same outcome as $b = v$
If $v \geq p > b$, then lose the object but it is better to win as $b = v$
If $v > b \geq p$, then win the object but it is the same outcome as $b = v$
Hence, $b = v$.

3.0.6 Revenue Equivalence

First-price auction and second-price auction give the same expected revenue.
Ordered Statistics: $X_{(k)}$ is the kth lowest draw from n independent draws made from the same distribution, arranged from smallest to largest
Expected value of the kth-order statistic for n draws taken from a uniform distribution between 0 and 1

$$E(X_{(k)}) = \frac{k}{n+1}$$

First-price auction

$$\begin{aligned} E\left(\max\left(b_1, b_2\right)\right) &= E\left(\max\left(\frac{v_1}{2}, \frac{v_2}{2}\right)\right) \\ &= \frac{1}{2} E\left(\max\left(v_1, v_2\right)\right) \\ &= \frac{1}{2}\frac{2}{3} = \frac{1}{3} \end{aligned}$$

Second-price auction

$$\begin{aligned} E\left(\min\left(b_1, b_2\right)\right) &= E\left(\min\left(v_1, v_2\right)\right) \\ &= \frac{1}{3} \end{aligned}$$

A Appendix

A.1 Contract Theory

A.1.1 Mechanism Design

A social choice problem.
There are n agent in the society.
Each agent i has type θ_i.
Social optimal allocation (despite Arrow's impossbility theorem) would be

$$y(\theta) = (y_1(\theta), \ldots, y_n(\theta))$$

where $\theta = (\theta_1, \ldots, \theta_n)$
Hard to implement $y(\theta)$ because each agent i has incentive to maniupulate θ_i to improve allocation.
A mechanism is $(y, M_1, \ldots, M_n)$ such that each agent i submits message $m_i \in M_i$ and $y(m)$ is the allocation rule. The informaiton set of agent i is I_i which may be include some subset of θ_{-i}.
An equilibrium is such that $y^*(I_1, \ldots, I_n) = y(m_1^*(I_1), \ldots, m_n^*(I_n))$.

A.1.2 Adverse Selection Problem

Principle-agent model: $n = 1$.
The problem becomes (y, M) and $I = \theta$ so the equilibirum is

$$m^*(\theta) \in \operatorname*{argmax}_{m \in M} u(y(m), \theta)$$

where the allocation is $y^*(\theta) = y(m^*(\theta))$.
A direct mechanism: report directly the type (message space is the type space) $M = \Theta$.
A truthful mechanism: report its own type

A.2 Revaluation Principle

Theorem. If $y^*(\theta)$ is implmented through some mechanism, then it can also be implemented through a direct truthfl mechanism where the agent reveals his information.
Proof.

1. Let (y, M) be the mechanism implement y^* and $m^*(\theta)$ be the equilibirum mesage. Hence $y^* = y(m^*(\theta))$.

2. Suppose a direct mechanism (y^*, Θ) but it is not truthful. This implies there exists $\theta' \neq \theta$ such that
$$u(y^*(\theta), \theta) < u(y^*(\theta'), \theta).$$

3. However, this implies
$$u(y(m^*(\theta)), \theta) < u(y(m^*(\theta')), \theta)$$
which contradicts our given condition. QED

See appendix for the general revelation principle.

A.2.1 Applicaton: Vertical Differentiation/Price Discrimination (Discrete)

Monopolist problem Discrete: (Mussa and Rosen 1998 JET)

1. Different qualities can be provided by a monopolist
2. multiple types of consumers
3. each consumer buys at most 1 unit
4. monopolist cannot observe consumer's type
5. monopolist offers a price schedule ever different quality goods

Setup:

1. Two consumers types: $T = \left\{\theta^H, \theta^L\right\}$ where $\theta^H > \theta^L > 0$
2. Belief: $p\left(\theta^H\right) = \alpha$ and $p\left(\theta^L\right) = 1 - \alpha$
3. Quantity: $q \in [0, \infty)$
4. marginal cost of producing type q: $c\left(q\right) > 0$, $c'\left(q\right) > 0$, $c''\left(q\right) > 0$ with $c\left(0\right) = 0$
5. Firm offers price for every quantity: $p(q)$
6. Consumer with type θ^t has utility $\theta^t q - p\left(q\right)$

A first-best solution: (First-order price discrimination)

1. Suppose θ^t is observable.
2. Firm just needs to provide two different price-quantity pairs $(p(\theta^t), \theta^t)$ for $t \in T$
3. Now the firm's problem becomes

$$\max_{p^t} \left[p\left(q\left(\theta^t\right)\right) - c\left(q\left(\theta^t\right)\right)\right]$$

such that

$$\theta^t q(\theta^t) - p\left(q\left(\theta^t\right)\right) \geq 0$$

4. The FOCs are

$$\begin{aligned} c'(q^*\left(\theta^t\right)) &= \theta^t \\ p\left(q^*\left(\theta^t\right)\right) &= \theta^t q^*\left(\theta^t\right) \end{aligned}$$

A second-best solution:

1. When θ^t is not observable, then θ^H will not choose $q^*\left(\theta^H\right)$ because

$$\begin{aligned} \theta^H q\left(\theta^L\right) - p\left(q\left(\theta^L\right)\right) &= \theta^H q\left(\theta^L\right) - \theta^L q\left(\theta^L\right) \\ &= \left(\theta^H - \theta^L\right) q\left(\theta^L\right) \\ &> 0 \\ &= \theta^H q\left(\theta^H\right) - p\left(q\left(\theta^H\right)\right) \end{aligned}$$

2. Firm decides $\left(p\left(q\left(\theta^L\right)\right), q\left(\theta^L\right), p\left(q\left(\theta^H\right)\right), q\left(\theta^H\right)\right)$ to maximize

$$\max_{p\left(q\left(\theta^L\right)\right), q\left(\theta^L\right), p\left(q\left(\theta^H\right)\right), q\left(\theta^H\right)} (1-\alpha)\left[p\left(q\left(\theta^L\right)\right) - c\left(q\left(\theta^L\right)\right)\right] + \alpha\left[p\left(q\left(\theta^H\right)\right) - c\left(q\left(\theta^H\right)\right)\right]$$

Two conditions have to be satisfied:

1. participation condition (PC)/Individual Rationality (IR)

$$(IR_1): \theta^L q\left(\theta^L\right) - p\left(q\left(\theta^L\right)\right) \geq 0$$
$$(IR_2): \theta^H q\left(\theta^H\right) - p\left(q\left(\theta^H\right)\right) \geq 0$$

2. incentive compatibility (IC)

$$
\begin{aligned}
(IC_1) &: \theta^L q\left(\theta^L\right) - p\left(q\left(\theta^L\right)\right) \geq \theta^L q\left(\theta^H\right) - p\left(q\left(\theta^H\right)\right) \\
(IC_2) &: \theta^H q\left(\theta^H\right) - p\left(q\left(\theta^H\right)\right) \geq \theta^H q\left(\theta^L\right) - p\left(q\left(\theta^L\right)\right)
\end{aligned}
$$

Preliminary results:

1. IR_1 is binding:

   From IC_2, $\theta^H q\left(\theta^H\right) - p\left(q\left(\theta^H\right)\right) \geq \theta^H q\left(\theta^L\right) - p\left(q\left(\theta^L\right)\right) \geq \theta^L q\left(\theta^L\right) - p\left(q\left(\theta^L\right)\right)$. Then if IR_1 is not binding, then IR_2 is not binding. Then firm can increase $p\left(q\left(\theta^L\right)\right)$ and $p\left(q\left(\theta^H\right)\right)$. Contradiction.

2. IC_2 is binding:

   Suppoe not. Then $\theta^H q\left(\theta^H\right) - p\left(q\left(\theta^H\right)\right) > \theta^H q\left(\theta^L\right) - p\left(q\left(\theta^L\right)\right) \geq \theta^L q\left(\theta^L\right) - p\left(q\left(\theta^L\right)\right) = 0$. Then increases $p\left(q\left(\theta^H\right)\right)$ will not violate IC_1, IC_2 and IR_2. Contradiction.

3. $q\left(\theta^H\right) \geq q\left(\theta^L\right)$

   Summing up IC_1 and IC_2, we have $\theta^H(q\left(\theta^H\right) - q\left(\theta^L\right)) \geq \theta^L(q\left(\theta^H\right) - q\left(\theta^L\right))$. Since $\theta^H > \theta^L$, we have $q\left(\theta^H\right) \geq q\left(\theta^L\right)$.

4. IC_1 is redundant

   Since IC_2 is binding, we have $p\left(q\left(\theta^H\right)\right) - p\left(q\left(\theta^L\right)\right) = \theta^H(q\left(\theta^H\right) - q\left(\theta^L\right)) \geq \theta^L(q\left(\theta^H\right) - q\left(\theta^L\right))$ because $\theta^H > \theta^L$ and $q\left(\theta^H\right) \geq q\left(\theta^L\right)$.

5. IR_2 is redundant

   Since IC_2 is binding, $\theta^H q\left(\theta^H\right) - p(q\left(\theta^H\right)) = \theta^H q\left(\theta^L\right) - p(q\left(\theta^L\right)) \geq \theta^L q\left(\theta^L\right) - p(q\left(\theta^L\right)) = 0$.

Hence, the maximization problem now becomes

$$
\max_{q\left(\theta^L\right), q\left(\theta^H\right)} \Pi = (1-\alpha)\left[\theta^L q\left(\theta^L\right) - c\left(q\left(\theta^L\right)\right)\right] + \alpha\left[\theta^H q\left(\theta^H\right) - \left(\theta^H - \theta^L\right) q\left(\theta^L\right) - c\left(q\left(\theta^H\right)\right)\right]
$$

with FOC

$$
\begin{aligned}
\frac{\partial \Pi}{\partial q\left(\theta^L\right)} &= 0 \Rightarrow \alpha\left[\theta^H - c'\left(q\left(\theta^H\right)\right)\right] = 0 \\
\frac{\partial \Pi}{\partial q\left(\theta^H\right)} &= 0 \Rightarrow (1-\alpha)\left[\theta^L - c'\left(q\left(\theta^L\right)\right)\right] - \alpha\left(\theta^H - \theta^L\right) = 0
\end{aligned}
$$

so that

$$
\begin{aligned}
\theta^H &= c'\left(q\left(\theta^H\right)\right) \\
\theta^L &= c'\left(q\left(\theta^L\right)\right) + \frac{\alpha}{1-\alpha}\left(\theta^H - \theta^L\right)
\end{aligned}
$$

See appendix for continuous case.