National University of Singapore
Microeconomics III, EC4101 (L1)
Tutorial 2 Solution: Labor supply, comparative statics
Lecturer: Parimal Bag

1. Suppose Emma has the following preferences

$$v(x, \ell) = (x-5)^{1/2}(\ell - 10)^{1/2},$$

where <u>five</u> units of the general consumption good, x, and <u>ten</u> hours of leisure, ℓ, are the minimal consumptions (of x and ℓ) that Emma must have to maintain her sanity. Let T be the maximum number of hours that Emma is allowed to work by her employer ($T > 24$). Interpret T also to be the maximum leisure hours that Emma can enjoy. Let w be the hourly wage rate, and p be the price of good x. Assume $w > p$.

(a) Write Emma's budget equation.

Answer:

$$px = w(T - \ell) + M, \tag{1}$$

where M is Emma's money endowment.

(b) Derive Emma's demand function for leisure as a function of w and p.

Answer: At the start set aside $5p + 10w$ as the minimal expenditure required to survive (i.e., buy $x_0 = 5$ and $\ell_0 = 10$ as a first-stage survival requirement). Then Emma is left with the residual budget

$$\mathcal{I} = wT + M - (5p + 10w) \underbrace{>}_{\text{assume}} 0,$$

which she must spend on $\tilde{x} = x - 5$ and $\tilde{\ell} = \ell - 10$ to maximize the utility:

$$v(\tilde{x}, \tilde{\ell}) = \tilde{x}^{(1/2)}\tilde{\ell}^{(1/2)}.$$

Utility maximization for Cobb-Douglas preferences yield

$$\tilde{x} = \frac{(1/2)\mathcal{I}}{p}, \qquad \text{and} \qquad \tilde{\ell} = \frac{(1/2)\mathcal{I}}{w}.$$

So, finally obtain

$$x^* = 5 + \tilde{x} = 5 + \frac{(1/2)\mathcal{I}}{p}, \qquad \text{and} \qquad \ell^* = 10 + \tilde{\ell} = 10 + \frac{(1/2)\mathcal{I}}{w}.$$

Writing out more elaborately,

$$\begin{aligned} \ell^* &= 10 + \frac{(1/2)[wT + M - (5p + 10w)]}{w} \\ &= 10 + (1/2)[T + \frac{(M - 5p)}{w} - 10], \end{aligned}$$

and

$$x^* = 5 + \frac{(1/2)[wT + M - (5p + 10w)]}{p}.$$

(c) Suppose the government offers Emma a wage-supplement $s > 0$ for every hour of work. What happens to Emma's optimal work choice?

Answer: With the wage-supplement, Emma's budget becomes

$$px = (w + s)(T - \ell) + M. \tag{2}$$

So her optimal leisure choice is altered to

$$\ell^*_{supp} = 10 + (1/2)[T + \frac{(M - 5p)}{(w + s)} - 10].$$

Comparing ℓ^*_{supp} with ℓ^* conclude that $\ell^*_{supp} < \ell^*$ if $M > 5p$, and $\ell^*_{supp} > \ell^*$ if $M < 5p$; if $M = 5p$ then $\ell^*_{supp} = \ell^*$.

So Emma's decision will be affected as follows:

- work <u>more</u> with the wage-supplement if $M > 5p$;
- work <u>less</u> with the wage-supplement if $M < 5p$;

- no change in work hours if $M = 5p$.

If $M = 0$ then the second case holds so that Emma would work less and take more time in leisure.

(d) Suppose the government imposes a tax $t > 0$ for every hour of work. What happens to Emma's work choice?

Answer: With the tax, Emma's budget becomes

$$px = (w - t)(T - \ell) + M. \tag{3}$$

So her optimal leisure choice is altered to

$$\ell^*_{tax} = 10 + (1/2)[T + \frac{(M - 5p)}{(w - t)} - 10].$$

It is easy to see that with the tax, Emma's work decision will be the exact opposite of the case of wage-supplement. For instance if $M = 0$, Emma would like to work more.

(e) Are x and ℓ substitutes or complements?

Answer: One way to see whether x and ℓ are substitutes or complements is to calculate the cross-price effect and conclude from the sign. So check that

$$\frac{\partial x^*}{\partial w} = (1/2)\frac{(T - 10)}{p} > 0, \qquad \text{and} \qquad \frac{\partial \ell^*}{\partial p} = (1/2)\frac{(-5)}{w} < 0.$$

Given that $\frac{\partial x^*}{\partial w} > 0$ suggests that if wage rate increases, demand for x will also increase. Since increase in w could be thought to have led to a decline in the demand for leisure (because leisure has become more expensive), increase in the demand for x can be interpreted as x and ℓ being *gross-substitutes*.

On the other hand, notice that $\frac{\partial \ell^*}{\partial p} < 0$, which can be interpreted as follows: $\uparrow p$ means $x \downarrow$, so if also $\ell \downarrow$ then it must be that x and ℓ are *gross-complements*.

Note that there have been other types of answers given based on the utility function which is also fine.