National University of Singapore
Microeconomic Analysis III, EC4101 (gr.2)
Tutorial 7 Solution: Search Model
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Question: Suppose that there are two possible prices, $p_\ell = 1$ and $p_h = 2$, and that the probability of the lower price is q. Compute the consumer's reservation price, which is the expected cost of searching, as a function of q and the cost of search c. For what values of q and c should the consumer choose the following strategy:

(i) accept whatever price he finds on the first search;

(ii) continue searching until the lower price is found.

Answer:

Expected total cost of purchase J is:

$$J(p^*) = c + p_\ell q + (1-q)J(p^*).$$

Since $p_\ell = 1$, we have

$$qJ(p^*) = c + q.$$

Hence,

$$\begin{aligned} J(p^*) &= \frac{c+q}{q} \\ &= 1 + \frac{c}{q}. \end{aligned}$$

Therefore reservation price p^* is then

$$p^* = 1 + \frac{c}{q}.$$

Method 1

Expected total cost of purchase if the reservation price $p^* \geq 2$ would be

$$\begin{aligned} J(p^*) &= c + qp_\ell + (1-q)p_h \\ &= c + 2 - q \end{aligned}$$

Expected total cost of purchase if the reservation price $p^* < 2$ would be

$$
\begin{aligned}
J(p^*) &= c + qp_\ell + (1-q)J(p^*) \\
&= 1 + \frac{c}{q}
\end{aligned}
$$

For (i), it must be

$$
\begin{aligned}
1 + \frac{c}{q} &\geq c + 2 - q \\
c &\geq cq + q - q^2 \\
q^2 - (1+c)\,q + c \geq 0 \\
(q-c)(q-1) \geq 0
\end{aligned}
$$

Since $q \leq 1$, we have $q - c \leq 0$ or $c \geq q$.

For (ii), the conditin is complementary to (i) and hence $c \leq q$.

Method 2

For (i), it is obvious that buying at p_ℓ is optimal. We need to have cost of purchasing now at p_h is weakly less than total expected cost of purchase:

$$2 \leq 1 + \frac{c}{q} \text{ or } c \geq q$$

For (ii), similarly, it is obvious that buying at p_ℓ is optimal and we need to have total expected cost of purchase weakly less than cost of purchasing now at p_h:

$$2 \geq 1 + \frac{c}{q} \text{ or } c \leq q$$

Method 3

For (i), since the decision problem is stationary, we can apply one-deviation principle from dynamic programming: the cost of following the rule (purchasing at high price) is weakly less than the cost of not following the rule this period (purchasing after one period of search):

$$\begin{aligned} p_h &\leq q\left(p_\ell\right) + (1-q)\left(p_h\right) + c \\ 2 &\geq q + (1-q)\,2 + c \\ c &\geq q. \end{aligned}$$

For (ii), similarly, the cost of following the rule (keeping searching until low price) is weakly less than the cost of not following the rule this period (pruchasing immediately):

$$\begin{aligned} q\left(p_\ell + c\right) + (1-q)\,q\left(p_\ell + 2c\right) + \ldots &\leq p_h \\ p_\ell + \frac{c}{q} &\leq p_h \\ c &\leq q \end{aligned}$$