National University of Singapore
Microeconomic Analysis III, EC4101 (gr.2)
Tutorial 7: Search Model
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We have discussed how people make decision under uncertainty in the class. We now see how it can be applied to the searching behavior of consumer.

Read the attached pages (pp. 2-6 of this file) taken from *Introduction to Economic Analysis* by R. Preston McAfee (2006) and answer the following question:

Suppose that there are two possible prices, $p_\ell = 1$ and $p_h = 2$, and that the probability of the lower price is q. Compute the consumer's reservation price, which is the expected cost of searching, as a function of q and the cost of search c. For what values of q and c should the consumer choose the following strategy:

(i) accept whatever price he finds on the first search;

(ii) continue searching until the lower price is found.

Search In most communities, every Wednesday grocery stores advertise sale prices in a newspaper insert, and these prices vary from week to week. Prices can vary a lot from week to week and from store to store. The price of gasolineGasoline varies as much as fifteen cents per gallon in a one mile radius. Decide you want a specific Sony television, and you may see distinct prices at Best Buy, Circuit City, and other electronics retailers. For many goods and services, there is substantial variation in prices, which implies that there are gains for buyers to search for the best price.

The theory of consumer search behavior is just a little bit arcane, but the basic insight will be intuitive enough. The general idea is that, from the perspective of a buyer, the price that is offered is random, and has a probability density function $f(p)$. If a consumer faces a cost of search (e.g. if you have to visit a store, in person, telephonically or virtually, the cost includes your time and any other costs necessary to obtain a price quote), the consumer will set a *reservation price*, which is a maximum price they will pay without visiting another store. That is, if a store offers a price below $p*$, the consumer will buy, and otherwise they will visit another store, hoping for a better price.

Call the reservation price $p*$ and suppose that the cost of search is c. Let $J(p*)$ represent the expected total cost of purchase (including search costs). Then J must equal

$$J(p^*) = c + \int_0^{p^*} pf(p)dp + \int_{p^*}^{\infty} J(p^*)f(p)dp.$$

This equation arises because the current draw (which costs c) could either result in a price less than $p*$, in which case observed price, with density f, will determine the price paid p, or the price will be too high, in which case the consumer is going to take another draw, at cost c, and on average get the average price $J(p*)$. It is useful to introduce the cumulative distribution function F, with $F(x) = \int_0^x f(p)dp$.

Note that something has to happen, so $F(\infty)$=1.

We can solve the equality for $J(p^*)$,

$$J(p^*) = \frac{\int_0^{p^*} pf(p)dp + c}{F(p^*)}.$$

This expression has a simple interpretation. The expected price $J(p^*)$ is composed of two terms. The first is the expected price, which is $\int_0^{p*} p\frac{f(p)}{F(p^*)}dp$. This has the interpretation of the average price conditional on that price being less than p^*. This is because $\frac{f(p)}{F(p^*)}$ is in fact the density of the random variable which is the price given that the price is less than p^*. The second term is $\frac{c}{F(p^*)}$. This is the expected search costs, and it arises because $\frac{1}{F(p^*)}$ is the expected number of searches. This arises because the odds of getting a price low enough to be acceptable is $F(p^*)$. There is a general statistical property underlying the number of searches. Consider a basketball player who successfully shoots a free throw with probability y. How many throws on average must he throw to sink one basket? The answer is $1/y$. To see this, note that the probability that exactly n throws are required is (1-y)$^{n-1}$ y. This is because n are required means n-1 must fail (probability (1-y)$^{n-1}$) and then the remaining one go in, with probability y. Thus, the expected number of throws is

$$
\begin{aligned}
& y + 2(1-y)y + 3(1-y)^2 y + 4(1-y)^3 y + \dots \\
=& y(1 + 2(1-y) + 3(1-y)^2 + 4(1-y)^3 + \dots) \\
=& y(1 + (1-y) + (1-y)^2 + (1-y)^3 + \dots) \\
& + (1-y)(1 + (1-y) + (1-y)^2 + (1-y)^3 + \dots). \\
& + (1-y)^2(1 + (1-y) + (1-y)^2 + (1-y)^3 + \dots) \\
& + (1-y)^3(1 + (1-y) + (1-y)^2 + \dots) + ... \\
=& y\left(\frac{1}{y} + (1-y)\frac{1}{y} + (1-y)^2\frac{1}{y} + (1-y)^3\frac{1}{y} + ...\right) \\
=& \frac{1}{y}
\end{aligned}
$$

Our problem has the same logic, where a successful basketball throw corresponds to finding a price less than p^*.

The expected total cost of purchase, given a reservation price: p^* is given by

$$J(p*) = \frac{\int_0^{p*} pf(p)dp + c}{F(p*)}.$$

But what value of p^* minimizes cost? Let's start by differentiating:

$$J'(p*) = p * \frac{f(p^*)}{F(p^*)} - \frac{f(p^*)\int_0^{p^*} pf(p)dp + c}{F(p*)^2}$$

$$= \frac{f(p^*)}{F(p^*)}\left(p^* - \frac{\int_0^{p*} pf(p)dp + c}{F(p^*)}\right) = \frac{f(p^*)}{F(p^*)}\left(p^* - J(p^*)\right).$$

Thus, if $p^* < J(p^*)$, J is decreasing, and it lowers cost to increase p^*. Similarly, if $p^* > J(p^*)$, J is increasing in p^*, and it reduces cost to decrease p^*. Thus, minimization occurs at a point where $p^* = J(p^*)$.

Moreover, there is only one such solution to the equation $p^* = J(p^*)$ in the range where f is positive. To see this, note that at any solution to the equation $p^* = J(p^*)$, $J'(p*) = 0$ and

$$J''(p*) = \frac{d}{dp^*}\left(\frac{f(p^*)}{F(p^*)}(p^* - J(p^*))\right)$$

$$= \left(\frac{d}{dp^*}\frac{f(p^*)}{F(p^*)}\right)(p^* - J(p^*)) + \frac{f(p^*)}{F(p^*)}(1 - J'(p^*)) = \frac{f(p^*)}{F(p^*)} > 0.$$

This means that J takes a minimum at this value, since its first derivative is zero and its second derivative is positive, and that is true about any solution to $p^*{=}J(p^*)$. Were there to be two such solutions, J' would have to be both positive and negative on the interval between them, since J is increasing to the right of the first (lower) one, and decreasing to the left of the second (higher) one. Consequently, the equation $p^*{=}J(p^*)$ has a unique solution that minimizes the cost of purchase. Consumer search to minimize cost dictates setting a reservation price equal to the expected total cost of purchasing the good, and purchasing whenever the price offered is lower than that level. That is, it is not sensible to "hold out" for a price lower than what you expect to pay on average, although this might be well useful in a bargaining context rather than in a store searching context.

Example (Uniform): Suppose prices are uniformly distributed on the interval $[a,b]$. For p^* in this interval,

$$J(p^*) = \frac{\int_0^{p^*} pf(p)dp + c}{F(p^*)} = \frac{\int_a^{p^*} p\frac{dp}{b-a} + c}{\frac{p^*-a}{b-a}}$$

$$= \frac{(p^{*2} - a^2) + c(b-a)}{p^* - a} = (p^* + a) + \frac{c(b-a)}{p^* - a}.$$

Thus, the first order condition for minimizing cost is

$0 = J'(p^*) = -\frac{c(b-a)}{(p^*-a)^2}$, implying $p^* = a + \sqrt{2c(b-a)}$.

There are a couple of interesting observations about this solution. First, not surprisingly, as $c{\to}0$, $p^*{\to}a$, that is, as the search costs go to zero, one holds out for the lowest possible price. This is sensible in the context of the model, but in the real search situations delay may also have a cost that isn't modeled here. Second, $p^* < b$, the maximum price, if $2c<(b-a)$. Put another way, if the *most* you can save by a search is twice the search cost, don't search, because the expected gains from search will be half the maximum gains (thanks to the uniform distribution) and the search unprofitable.

The third observation, which is much more general than the specific uniform example, is that the expected price is a concave function of the cost of search (second derivative negative). That is in fact true for any distribution. To see this, define a function

$$H(c) = \min_{p^*} J(p^*) = \min_{p^*} \frac{\int_0^{p^*} pf(p)dp + c}{F(p^*)}.$$

Since $J'(p^*) = 0$,

$$H'(c) = \frac{\partial}{\partial c} J(p^*) = \frac{1}{F(p^*)}.$$

It then needs only a modest effort to show p^* is increasing in c, from which it follows that H is concave. This means that the effects of an increase in c are passed on at a decreasing rate. Moreover, it means that a consumer should rationally be risk averse about the cost of search.