Microeconomics EC4101
Tutorial 5

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Question

1. Imagine a two-consumer (A, B) and two-good (x_1, x_2) economy in which the total endowment of good 1 is 1 and the total endowment of good 2 is 2. Both consumers have utility functions given by

$$U(x_1, x_2) = \min\{x_1, x_2\}.$$

   (a) What is the set of Pareto efficient allocations in the economy?

   (b) If the endowment vector of each consumer is given by

$$(\omega_1, \omega_2) = (\frac{1}{2}, 1),$$

   what is the set of competitive equilibria? If the endowment vector of each consumer A is

$$(\omega_1^A, \omega_2^A) = (\frac{1}{2}, \frac{1}{2})$$

   and that of consumer B is

$$(\omega_1^B, \omega_2^B) = (\frac{1}{2}, \frac{3}{2}),$$

   what is the set of competitive equilibria?

Concepts Involved

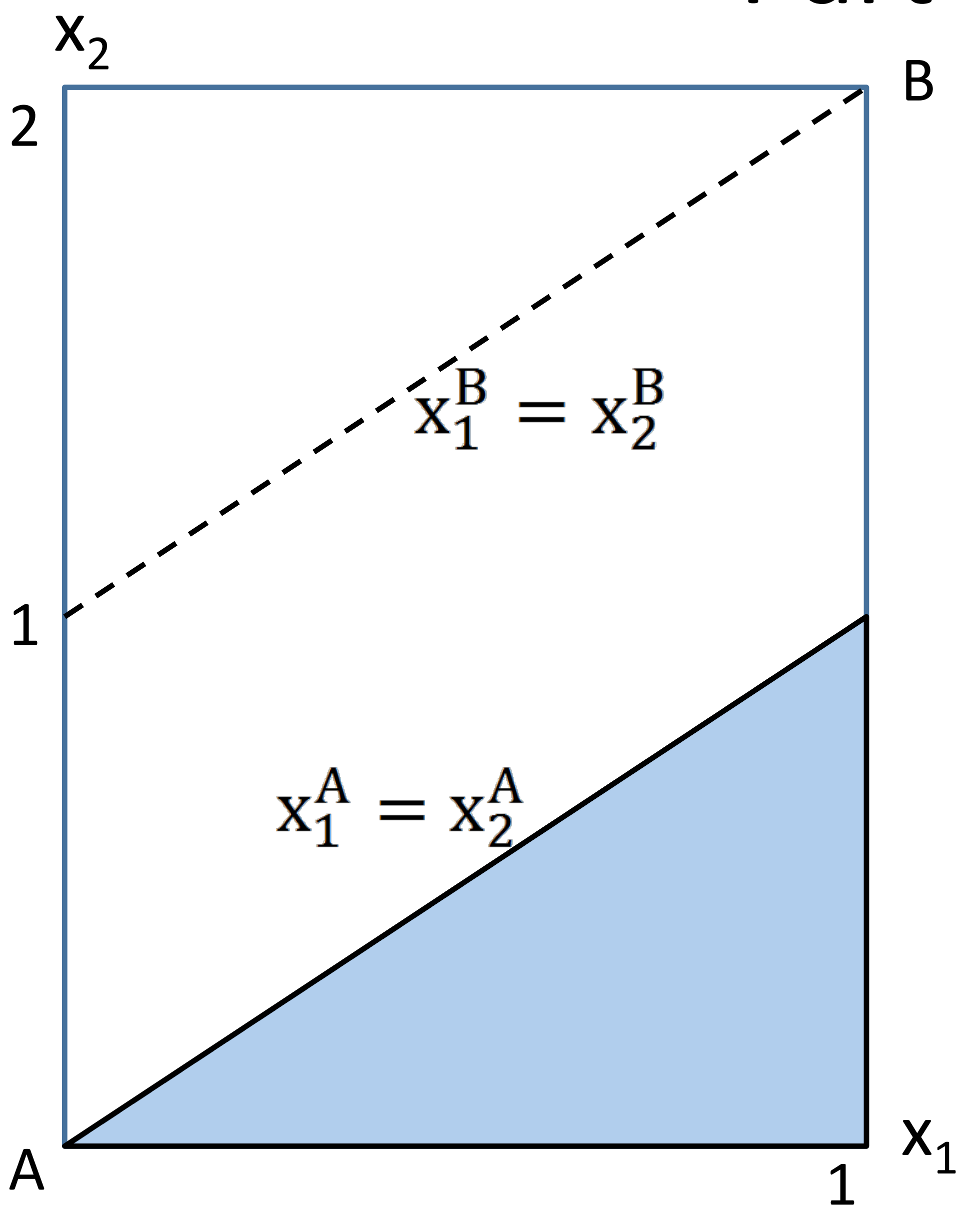
- Pareto efficiency
  - No alternative allocation in which at least one person is better off and no one is worse off
- 1st and 2nd Welfare Theorems
  - Every Walrasian (competitive) equilibrium is Pareto efficient
  - For any Pareto optimal allocation, there exists a set of initial endowments and a related price vector such that this allocation is also a Walrasian (competitive) equilibrium

Concepts Involved

- Walrusian (competitive) equilibria
  - A price vector *p** and allocation x
  - All markets clear (Demand equals supply in each market): $$\sum_{i=1}^{m} x^i(p^*, p^*\bar{x}^i) = \sum_{i=1}^{m} \bar{x}^i$$
  - Walras' law holds
    - The value of all quantities demanded must equal the value of all endowments

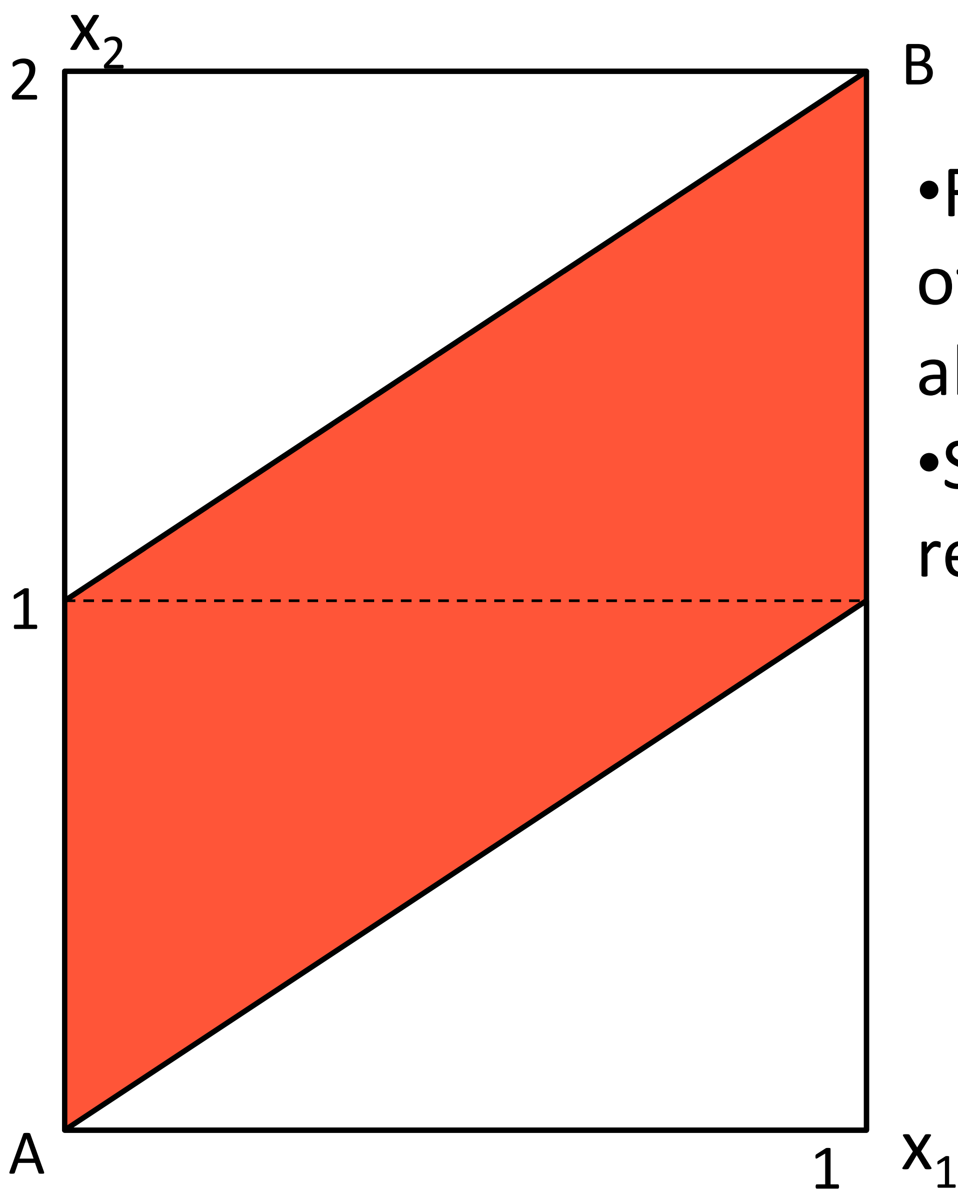
$$\sum_{i=1}^{m} px^i = \sum_{i=1}^{m} p\bar{x}^i$$

Part A



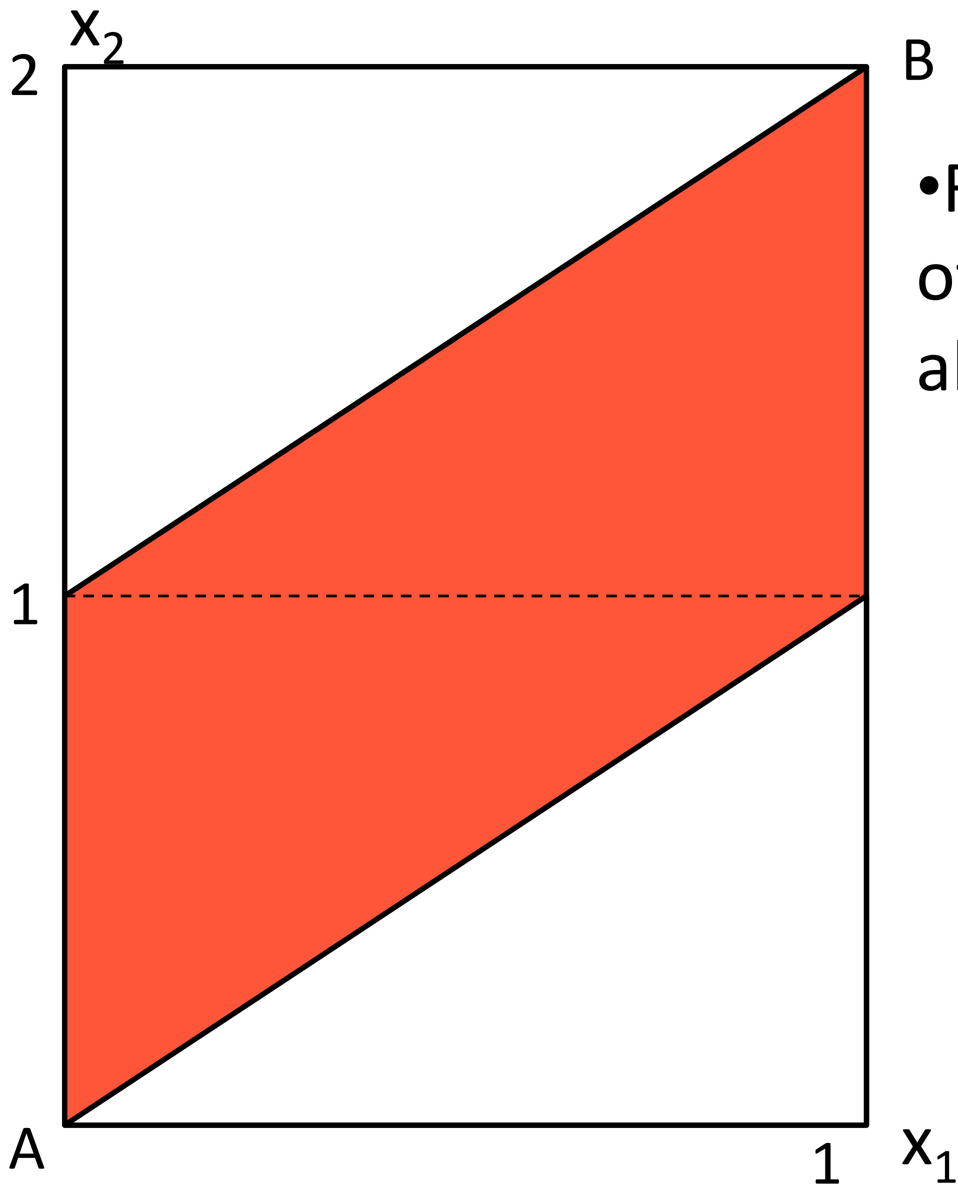
- Select a point within the region:
  - Pareto improvement possible for any allocation inside blue triangle by increasing consumption of x_2 for consumer A.
  - Utility of consumer A increases, while utility of consumer B remains unchanged.

Part A



- Red shaded region is the set of all pareto efficient allocations.
- Select a point within region:
  - Increasing consumption of x_2 will not make anyone better off.
  - Consider for all x_1

Part A

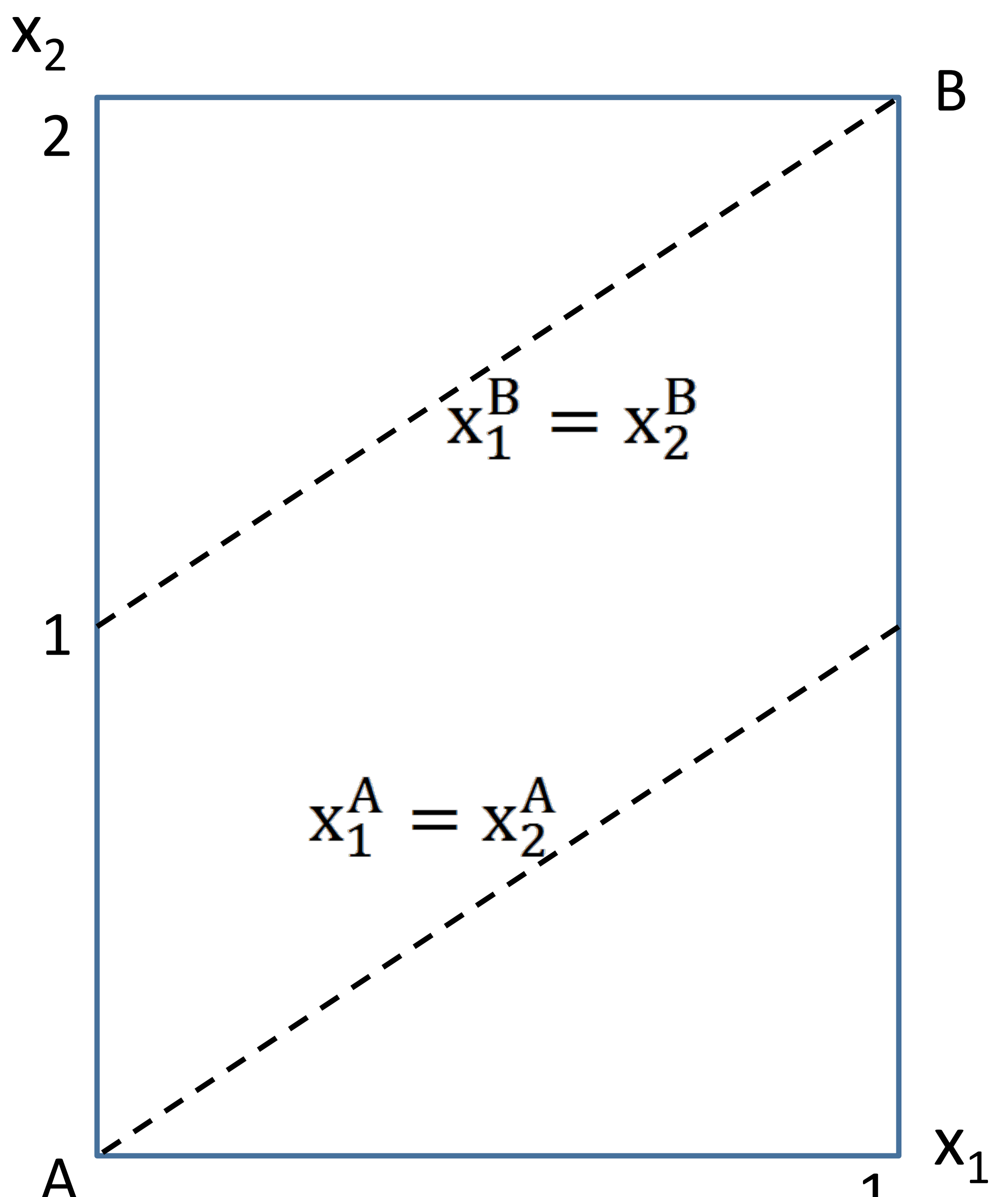


- Red shaded region is the set of pareto efficient allocations.
  - $\{X_1, X_2 \mid X_1 \in [0, 1], X_2 \in [X_1, (2-(1-X_1))]\}$

Approach to part B

- From definition of Walrasian (competitive) equilibrium, we need demand equals supply in each market ie. Excess demand for each good = 0
- Thus in this question, we will find the total excess demand of good 1 for consumer A and B which should equate to 0.
- Demand is derived from the Marshallian demand of consumer A and B, supply is derived from the endowments.

Approach to Part B



Part B: Finding equilibrium

- Utility maximization problem for the 2 consumer economy
  Maximise$_{X1A,X2A}$ $U^A = \min\{X_1^A, X_2^A\}$ subject to:
  - $U^B = \min\{X_1^B, X_2^B\}$
  - $X_1^A + X_1^B = 1$
  - $X_2^A + X_2^A = 2$

Part B

Deriving the Marshallian demand for consumer A and B,

$$\underset{x_1, x_2}{Max}\ U^i(x_1^i, x_2^i) = \min\{x_1^i, x_2^i\} \quad i \in A, B$$

$$s.t.\ P_1 x_1^i + P_2 x_2^i = P_1 \varpi_1^i + P_2 \varpi_2^i$$

Part B

Optimal consumption bundles:

$$\mathrm{x}_1^{\mathrm{A}}(\mathrm{P}_1, \mathrm{P}_2, \varpi_1^{\mathrm{A}}, \varpi_2^{\mathrm{A}}) = \mathrm{x}_2^{\mathrm{A}}(\mathrm{P}_1, \mathrm{P}_2, \varpi_1^{\mathrm{A}}, \varpi_2^{\mathrm{A}})$$

$$\mathrm{x}_1^{\mathrm{B}}(\mathrm{P}_1, \mathrm{P}_2, \varpi_1^{\mathrm{B}}, \varpi_2^{\mathrm{B}}) = \mathrm{x}_2^{\mathrm{B}}(\mathrm{P}_1, \mathrm{P}_2, \varpi_1^{\mathrm{B}}, \varpi_2^{\mathrm{B}})$$

Finding Marshallian demand for x_1:

$$\mathrm{P}_1 x_1^A + \mathrm{P}_2 x_2^A = \mathrm{P}_1 \varpi_1^A + \mathrm{P}_2 \varpi_2^A$$

$$\mathrm{P}_1 x_1^A + \mathrm{P}_2 x_1^A = \mathrm{P}_1 \varpi_1^A + \mathrm{P}_2 \varpi_2^A$$

$$\mathrm{x}_1^{\mathrm{A}} = \frac{\mathrm{P}_1 \varpi_1^A + \mathrm{P}_2 \varpi_2^A}{\mathrm{P}_1 + \mathrm{P}_2}$$

Part B

$$\text{Subst } (\varpi_1^A, \varpi_2^A) = \left(\frac{1}{2}, 1\right) \ in \ \ \mathrm{x}_1^{\mathrm{A}} = \frac{\mathrm{P}_1\varpi_1^A + \mathrm{P}_2\varpi_2^A}{\mathrm{P}_1 + \mathrm{P}_2}$$

$$\mathrm{x}_1^{\mathrm{A}} = \frac{2 + \frac{\mathrm{P}_1}{\mathrm{P}_2}}{2(1 + \frac{\mathrm{P}_1}{\mathrm{P}_2})}$$

Finding excess demand of good x_1:

$$\mathrm{z}_1^{\mathrm{A}}\left(\mathrm{P}_1, \mathrm{P}_2, \varpi_1^{\mathrm{A}}, \varpi_2^{\mathrm{A}}\right) = \mathrm{x}_1^{\mathrm{A}} - \ \varpi_1^A \ = \frac{1}{2(1 + \frac{\mathrm{P}_1}{\mathrm{P}_2})}$$

Part B

- As consumer A and B are identically the same, by symmetry,

$$z_1^B\left(P_1, P_2, \varpi_1^B, \varpi_2^B\right) = x_1^B - \varpi_1^B = \frac{1}{2(1+\frac{P_1}{P_2})}$$

- For competitive equilibrium, total excess demand for good 1= 0

$$z_1^A\left(P_1, P_2, \varpi_1^A, \varpi_2^A\right) + z_1^B\left(P_1, P_2, \varpi_1^B, \varpi_2^B\right) = 0$$

$$\frac{1}{1+\frac{P_1}{P_2}} = 0$$

This can only happen when:

$$P_1 = \infty, P_2 = (0, \infty)$$

$$\text{or } P_1 = (0, \infty), P_2 = 0$$

Part B

- We can perform similar steps to obtain total excess demand of good 2 .

$$x_1^A = x_2^A = \frac{P_1\varpi_1^A + P_2\varpi_2^A}{P_1 + P_2} = \frac{2 + \frac{P_1}{P_2}}{2(1 + \frac{P_1}{P_2})}$$

$$z_2^A\left(P_1, P_2, \varpi_1^A, \varpi_2^A\right) = x_2^A - \varpi_2^A$$

$$= \frac{2 + \frac{P_1}{P_2}}{2(1 + \frac{P_1}{P_2})} - 1 = \frac{-\frac{P_1}{P_2}}{2(1 + \frac{P_1}{P_2})}$$

$$\text{Similarly}, z_2^B\left(P_1, P_2, \varpi_1^B, \varpi_2^B\right) = x_2^B - \varpi_2^B = \frac{-\frac{P_1}{P_2}}{2(1 + \frac{P_1}{P_2})}$$

Part B

$$z_2^A(P_1, P_2, \varpi_1^A, \varpi_2^A) + z_2^B(P_1, P_2, \varpi_1^B, \varpi_2^B)$$

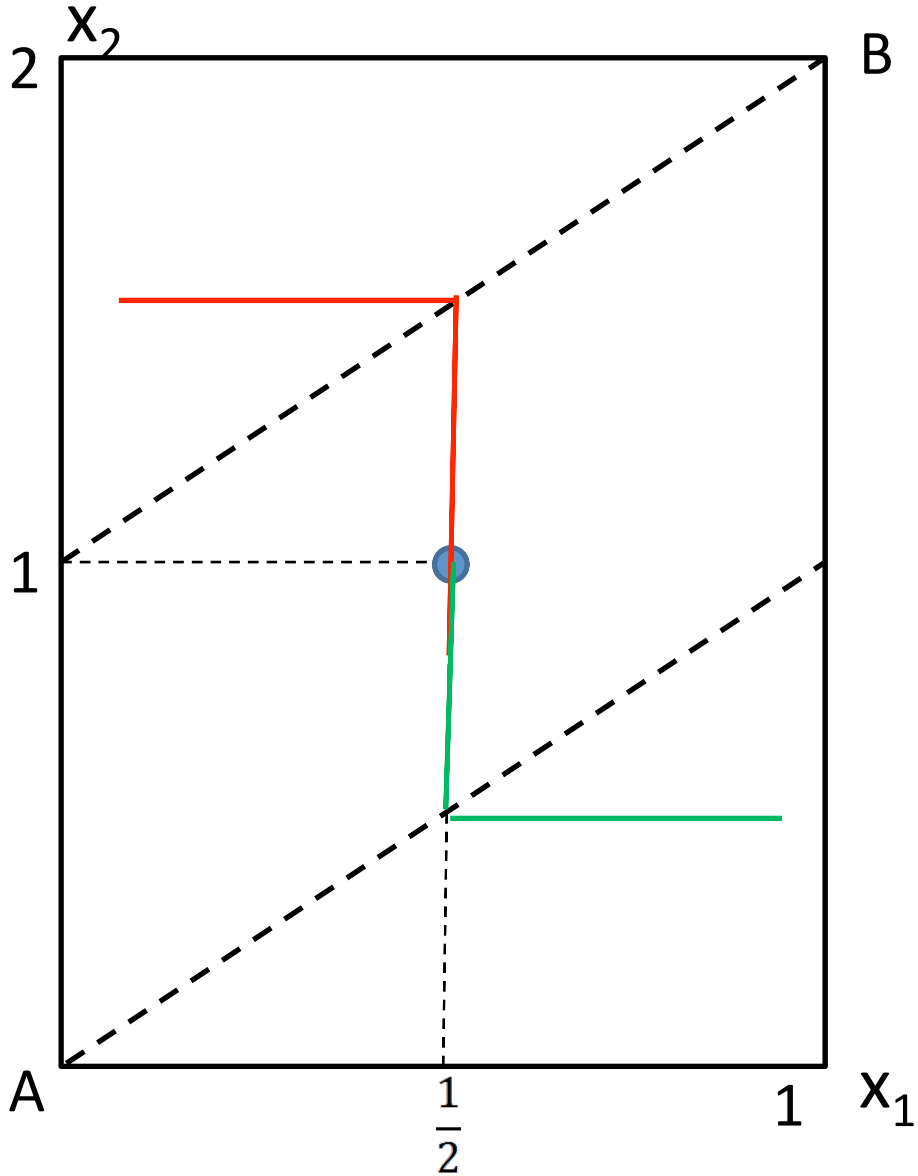
$$= \frac{-\frac{P_1}{P_2}}{1 + \frac{P_1}{P_2}} = \frac{-1}{1 + \frac{P_2}{P_1}}$$

When $P_1 = \infty, P_2 = (0, \infty)$ or $P_1 = (0, \infty), P_2 = 0$

Total excess demand for good 2 = -1 which violates condition for competitive equilibrium.

However, the demand function of each consumer for good 2 is irrelevant. As long as each consumer consumes at least $\frac{1}{2}$ units of good 2, he is indifferent with the number of extra units of good 2 they have.

Part B



Competitive equilibrium

$$P_1 = \infty, P_2 = (0, \infty)$$

$$\text{or } P_1 = (0, \infty), P_2 = 0$$

$$(x_1^A, x_2^A) = (\frac{1}{2}, 1)$$

$$(x_1^B, x_2^B) = (\frac{1}{2}, 1)$$

Part B

$$(\varpi_1^A, \varpi_2^A) = \left(\frac{1}{2}, \frac{1}{2}\right) \quad (\varpi_1^B, \varpi_2^B) = \left(\frac{1}{2}, \frac{3}{2}\right)$$

$$x_1^A = \frac{P_1\varpi_1^A + P_2\varpi_2^A}{P_1 + P_2} = \frac{1}{2} \quad x_1^B = \frac{P_1\varpi_1^B + P_2\varpi_2^B}{P_1 + P_2} = \frac{3 + \frac{P_1}{P_2}}{2(1 + \frac{P_1}{P_2})}$$

$$z_1^A(P_1, P_2, \varpi_1^A, \varpi_2^A) = \frac{1}{2} - \frac{1}{2} = 0$$

$$z_1^B(P_1, P_2, \varpi_1^B, \varpi_2^B) = \frac{3 + \frac{P_1}{P_2}}{2(1 + \frac{P_1}{P_2})} - \frac{1}{2} = \frac{1}{1 + \frac{P_1}{P_2}}$$

$$z_1^A(P_1, P_2, \varpi_1^A, \varpi_2^A) + z_1^B(P_1, P_2, \varpi_1^B, \varpi_2^B) = 0$$

For this to hold, $P_1 = \infty, P_2 = (0, \infty)$ or $P_1 = (0, \infty), P_2 = 0$

Part B

- Performing similar steps to obtain total excess demand of good 2

$$x_2^A = \frac{P_1\varpi_1^A + P_2\varpi_2^A}{P_1 + P_2} = \frac{1}{2} \qquad x_2^B = \frac{P_1\varpi_1^B + P_2\varpi_2^B}{P_1 + P_2} = \frac{3 + \frac{P_1}{P_2}}{2(1 + \frac{P_1}{P_2})}$$

$$z_2^A\left(P_1, P_2, \varpi_1^A, \varpi_2^A\right) = \frac{1}{2} - \frac{1}{2} = 0$$

$$z_2^B\left(P_1, P_2, \varpi_1^B, \varpi_2^B\right) = \frac{-\frac{P_1}{P_2}}{1 + \frac{P_1}{P_2}} = \frac{-1}{1 + \frac{P_2}{P_1}}$$

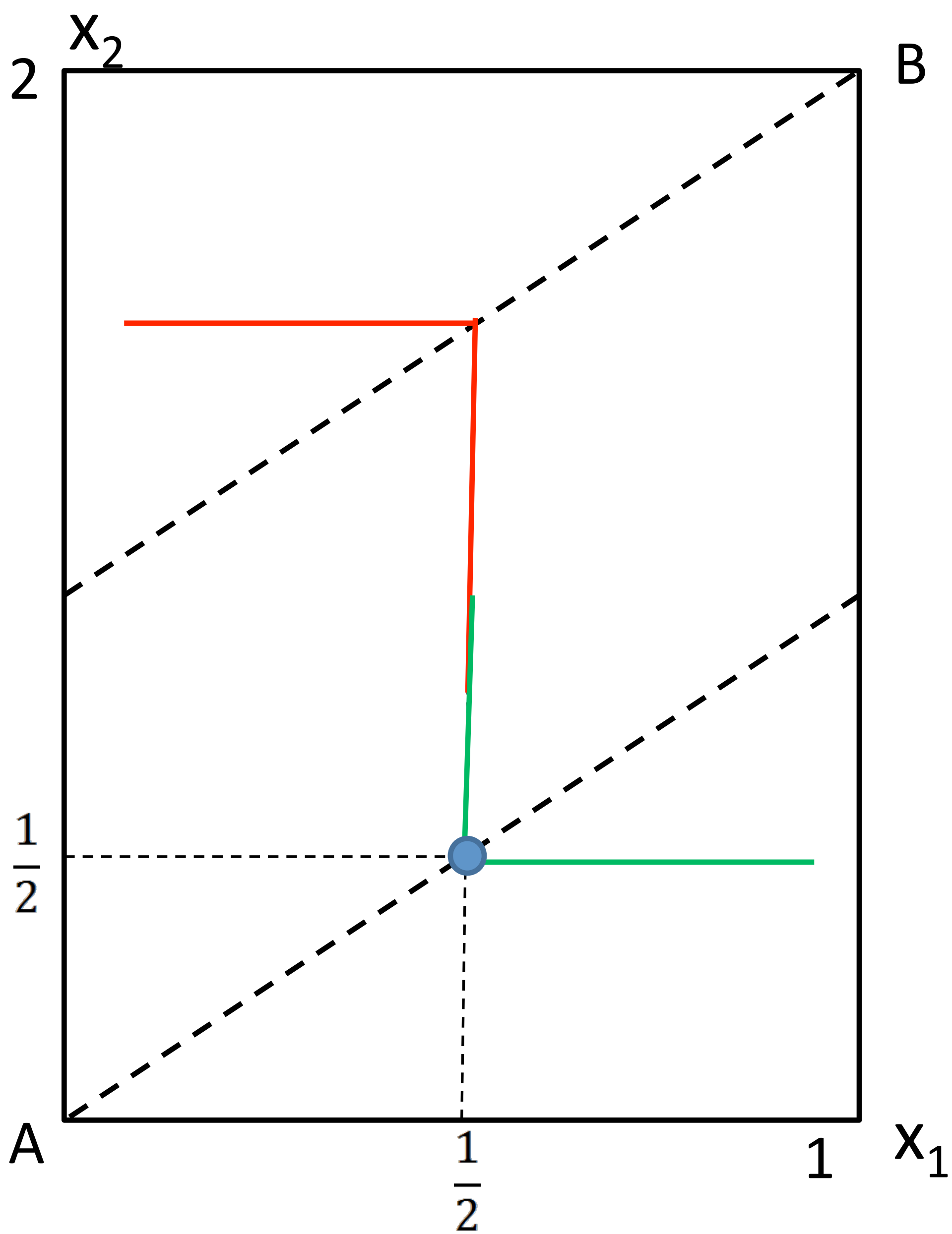
Part B

$$z_2^A(P_1, P_2, \varpi_1^A, \varpi_2^A) + z_2^B(P_1, P_2, \varpi_1^B, \varpi_2^B)$$
$$= \frac{-1}{1 + \frac{P_2}{P_1}}$$

When $P_1 = \infty, P_2 = (0, \infty)$ or $P_1 = (0, \infty), P_2 = 0$

Total excess demand for good 2 = -1 again which violates condition for competitive equilibrium. However, as mentioned in the previous part, the demand function of each consumer for good 2 is irrelevant. As long as each consumer consumes at least $\frac{1}{2}$ units of good 2, he is indifferent with the number of extra units of good 2 they have.

Part B



Competitive equilibrium

$P_1 = \infty, P_2 = (0, \infty)$

or $P_1 = (0, \infty), P_2 = 0$

$$(x_1^A, x_2^A) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

$$(x_1^B, x_2^B) = \left(\frac{1}{2}, \frac{3}{2}\right)$$