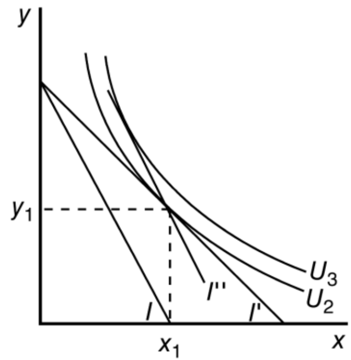
4.7 a.



b. $E(p_x, p_y, U) = 2p_x^{0.5}p_y^{0.5}U$. With $p_x = 1$ and $p_y = 4$, we have $U = 2$ and $E = 8$. To raise utility to 3 would require $E = 12$, that is, an income subsidy of 4.

c. Now we require $E = 8 = 2 \cdot p_x^{0.5} \cdot 4^{0.5} \cdot 3$ or $p_x^{0.5} = 8/12 = 2/3$. So $p_x = 4/9$; that is, each unit must be subsidized by $5/9$. At the subsidized price, this person chooses to buy $x = 9$. So total subsidy is 5, one dollar greater than in part c.

d. $E(p_x, p_y, U) = 1.84p_x^{0.3}p_y^{0.7}U$. With $p_x = 1$ and $p_y = 4$, we have $U = 2$ and $E = 9.71$. Raising U to 3 would require extra expenditures of 4.86. Subsidizing good x alone would require a price of $p_x = 0.26$, that is, a subsidy of 0.74 per unit. With this low price, the person would choose $x = 11.2$, so the total subsidy would be 8.29.

4.14 Altruism

a. When $a = 0$, $U_1 = c_1$, so Michele is completely self-interested. When $a = 1$, $U_1 = c_2$, so she cares only about others, not herself. Definitions of a "perfect altruist" may vary. According to the "Golden Rule" standard ("Regard others as you would have them regard you"), Michele would have a symmetric regard for the two consumption levels, corresponding to $a = 1/2$.

b. The choice problem is to maximize $c_1^{1-a}c_2^a$ subject to the budget constraint $c_1 + c_2 = I$. This is a standard Cobb-Douglas utility-maximization problem, having solutions $c_1^* = (1-a)I$ and $c_2^* = aI$. Michele's charity c_2^* is directly proportional to her altruism,a.

c. A proportional income tax just reduces her net income from I to $(1-t)I$. Substituting this new income into the solutions from part b, $c_1^* = (1-a)(1-t)I$ and $c_2^* = a(1-t)I$. Allowing a charitable deduction reduces the relative "price" of Sofia's consumption: p_1 is still 1 but p_2 falls to $1-t$. Solving the utility maximization problem with these new prices and income yields

$$c_1^* = \frac{(1-\alpha)(1-t)^2 I}{\alpha + (1-\alpha)(1-t)}, \quad c_2^* = \frac{\alpha(1-t)I}{\alpha + (1-\alpha)(1-t)}$$

Charitable contributions still fall compared to the no-tax case because of the income effect, but they rise relative to Michele's own consumption because of the change in relative prices.

d(1). Substituting Sofia's utility function into Michele's and solving for U_1 yields

$$U_1(c_1, c_2) = c_1^{1/(1+\alpha)} c_2^{\alpha/(1+\alpha)}$$

Solving the utility-maximization problem yields

$$c_1^* = \frac{1}{1+\alpha} I, \quad c_2^* = \frac{\alpha}{1+\alpha} I$$

For a given a, Michele reduces her charitable contributions compared to part b because she takes into account Sofia's benefit from Michele's consumption, leading Michele to keep her own consumption higher.

d(2). Substituting Sofia's utility into Michele's and solving for U_1 gives the same function as in part b.