EC4101 Microeconomics Analysis III (Group 2)

Topic 6
Asymmetric Information

Asymmetric Information

- General equilibrium: efficient outcomes
  - Incomplete information might lead to inefficiency
- Principal-agent Problem
  - Hidden information:
    - Uninformed moved first: Moral Hazard
      - Owner-Manger, Insurance
    - Informed move first: Signaling
      - Applicaton: Job Market Signaling
  - Adverse Selection: Lemon market
- Auction: Second-Price Auction

Signaling

- Covered in Game Theory
- Pooling equilibrium: not efficient
- Separating equilibrium: wasteful education(?)

Principal-agent Problem

- Contract
  - Incentive scheme to avoid inefficiencies associated with asymmetric information
- Principal
  - The party who proposes the contract
- Agent
  - The party who decides whether to accept
  - And then performs under the terms of the contract
  - Typically the party with asymmetric information

Hidden Action or Information?

- Hidden-action: Moral hazard model
  - The agent's actions affect the principal, but the principal does not observe the actions directly
- Hidden-Information: Adverse selection model
  - The agent has private information before signing the contract (his type)

TABLE 18.1

Applications of the principal-agent

		Agent's Private Information	
Principal	**Agent**	**Hidden Type**	**Hidden Action**
Shareholders	Manager	Managerial skill	Effort, executive decisions
Manager	Employee	Job skill	Effort
Homeowner	Appliance repairer	Skill, severity of appliance malfunction	Effort, unnecessary repairs
Student	Tutor	Subject knowledge	Preparation, patience
Monopoly	Customer	Value for good	Care to avoid breakage
Health insurer	Insurance purchaser	Preexisting condition	Risky activity
Parent	Child	Moral fiber	Delinquency

First-Best and Second Best

- First-best: Full-information environment
- Second-best: with information constraint
- Third best…: adding further constraints

Hidden Action

- Principal wants agent to take some action
- Outcome of an action is observable
- Action itself is NOT observable
- Random element between action and outcome
- Agent might not adopt the best action if their interest does not align
- Contracts: link compensation to observable outcomes

Owner-Manager Example

- Owner wants manger to exert effort $e \geq 0$
  - Gross profit: $\pi_g = e + \varepsilon$
  - Where ε represents demand, cost, and other economic factors outside of the agent's control
    - Assume $\varepsilon \sim (0,\sigma^2)$
  - Net profit: $\pi_n = \pi_g - s$, s is the manager's salary
- Exerting effort is costly
  - c(e) is the manager's personal disutility from effort
    - Assume $c'(e) > 0$ and $c''(e) < 0$

Payoffs of Owner and Manager

- Risk-neutral owner: max E(profit)

$$E(\pi_n) = E(e + \varepsilon - s) = e - E(s)$$

- Risk adverse manager: Max EU
  - Constant risk aversion parameter, $A > 0$

$$E(U) = E(s) - \frac{A}{2}\mathrm{Var}(s) - c(e)$$

Three-stage game

- Owner sets the incentive scheme (salary)
- The manager decides whether or not to accept the contract
- The manager decides how much effort to put forth (conditional on accepting the contract)

First-Best

- Optimal salary contract
  - A fixed salary *s** if he exerts a first-best level of effort *e** ; and nothing otherwise
  - Participation constraint: Manager accepts contract

$$E(U) = s^* - c(e^*) \geq 0$$

  - Owner pays the lowest salary possible [$s^* = c(e^*)$]
  - Net profit: $E(\pi_n) = e^* - E(s^*) = e^* - c(e^*)$
  - At the optimum: $c'(e^*) = 1$

Second Best

- owner offers a salary: $s(\pi_g) = a + b\pi_g$
  - a = fixed salary; b = power of incentive scheme
- Manger: expected

$$E(a + b\pi_g) - (A/2)\, Var(a + b\pi_g) - c(e)$$

$$= a + be - (A\, b^2\sigma^2 /2) - c(e)$$

- Optimal: $c'(e) = b$
- Participation: expected utility is non-negative

$$a \geq c(e) + (A\, b^2\sigma^2 /2) - be$$

- Owner: max *e(1 – b) – a* subject to PC

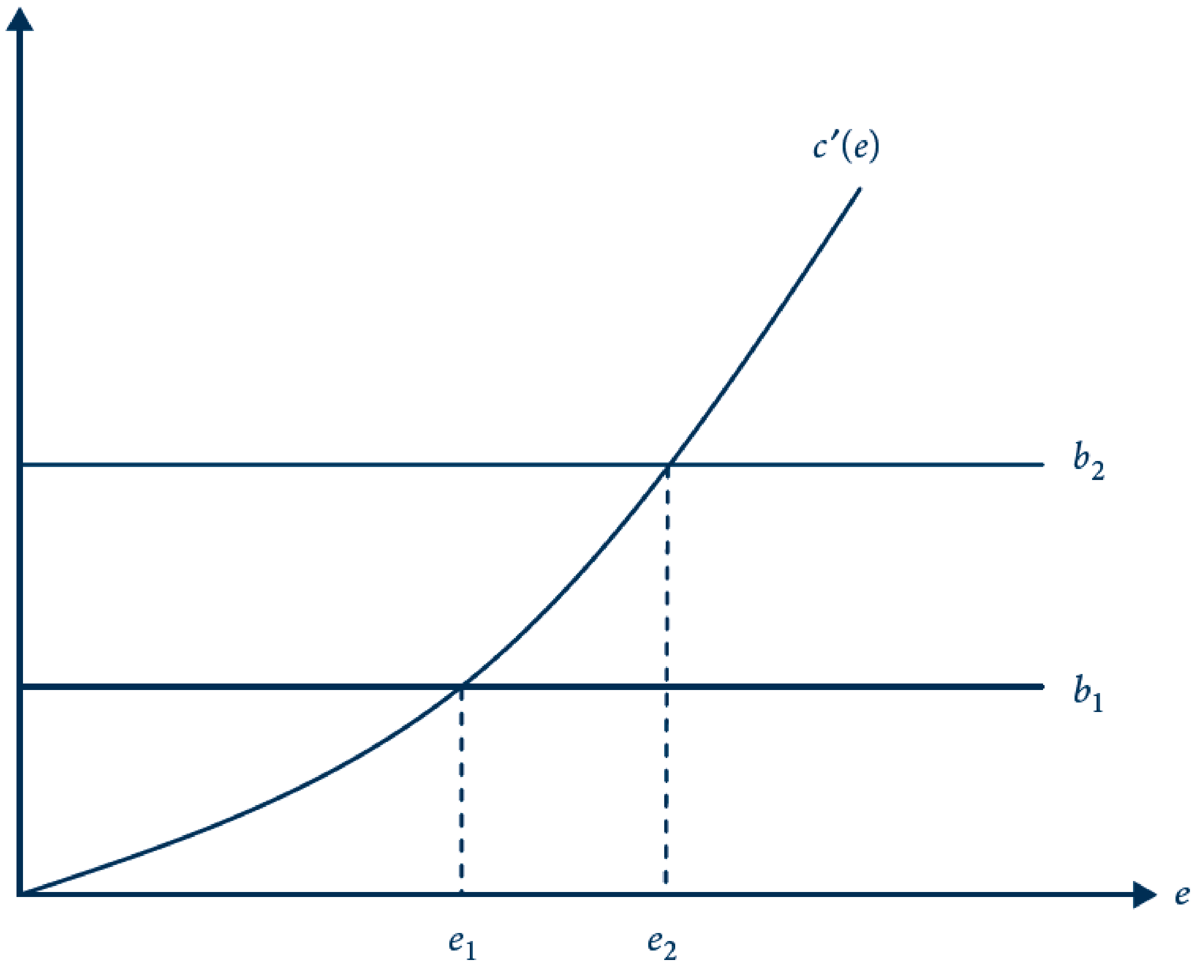
$$\text{Owner扭surplus?} = e - c\ \ e) - \frac{A\sigma^2[c'(e)]^2}{2}$$

- Optimal: $$c'(e^{**}) = \frac{1}{1 + A\sigma^2 c''(e^{**})} = b^{**}$$

- second-best effort will be less than the first-best effort,
  - The owner cannot observe *e* directly
  - The manager is risk-averse
  - Risk premium adds to the owner’s cost of inducing effort

FIGURE 18.2

Manager’s Effort Responds to Increased Incentives



Because the manager’s marginal cost of effort, c’(e), slopes upward, an increase in the power of the incentive scheme from b_1 to b_2 induces the manager to increase his effort from e_1 to e_2.

EXAMPLE 18.1 Owner-Manager Relationship

- Assume
  - Manager's cost of effort: $c(e) = e^2/2$
  - $\sigma^2 = 1$
- First best
  - $c'(e^*) = e^* = 1$
  - First-best effort $e^* = 1$
  - Manager's fixed salary = ½
  - Owner's net profit = ½

EXAMPLE 18.1 Owner-Manager Relationship

- Second best, assume $A = 1$
  - Then $e^{**} = ½$ and $b^{**} = ½$; $a^{**} = 0$
  - The manager receives no fixed pay
    - But does receive incentive pay equal to 50 cents for every dollar of gross profit
  - Owner's expected net profit = ¼
- Second best, assume $A = 2$
  - Then $e^{**} = 1/3$ and $b^{**} = 1/3$; $a^{**} = 1/18$
  - Owner's expected net profit = 1/6

Insurance

- Insurance reduces incentive for precautions
- Risk-averse individual
  - Faces the possibility of a loss (l)
  - That will reduce his initial wealth (W_0)
  - The probability of loss is π
  - reduce π by spending on preventive measures (e)

Insurance

- An insurance company (principal)
  - premium is p for payment of x if loss occurs
  - Maximize: E(profit) = $p - \pi x$
- Expected utility: $(1-\pi)U(W_1) + (\pi)U(W_2)$
  - Wealth in state 1 (no loss): $W_1 = W_0 - e - p$
  - Wealth in state 2 (loss): $W_2 = W_0 - e - p - l + x$

First Best

- Firm perfectly monitors *e*
  - Set the terms to maximize its expected profit - subject to the participation constraint

    $(1-\pi)U(W_1) + (\pi)U(W_2) \geq \bar{U}$
  - Optimality: Full insurance with $x = l$
  - Socially efficient level of precaution

Second-best

- Insurance company cannot monitor *e*
  - incentive compatibility constraint must be added
  - second-best contract:
    - typically no full insurance
    - exposing the individual to some risk induces him to take some precaution

EXAMPLE 18.2 Insurance and Precaution against Car Theft

- Driver endowed with $100,000 of wealth
  - Purchase insurance against the theft of a $20,000 car
  - If installs a car alarm that costs $1,750
    - Probability of theft drops from 0.25 to 0.15
- No insurance
  - No car alarm: Expected utility = 11.45714
  - Install car alarm: Expected utility = 11.46113

- First best
  - Maximizes the insurance company's profit
    - Given that it requires the individual to install an alarm
    - Can costlessly verify whether the individual has complied
  - Full insurance
  - Highest premium p = 3,298
  - Company's profit = $298

EXAMPLE 18.2 Insurance and Precaution against Car Theft

- Second best
  - Company cannot monitor whether the individual has installed an alarm

1. Induce him to install the alarm by offering only partial insurance
   - Payment after theft = $3,374
   - p = $602 and company's profit = $96
2. Disregard the alarm and provide him with full insurance
   - p = $5,048 and company's profit = $48

EXAMPLE 18.3 Competitive Theft Insurance

- Car theft insurance
  - Is sold by perfectly competitive companies
- First best
  - Require him to install the alarm
  - Fully insure him for p = \$3,000
    - Fair insurance premium = the expected payout for a loss
  - Firm earns zero profit
- Second best
  - Equilibrium premium, p = \$506
  - Payment for loss is = \$3,374

Hidden Information

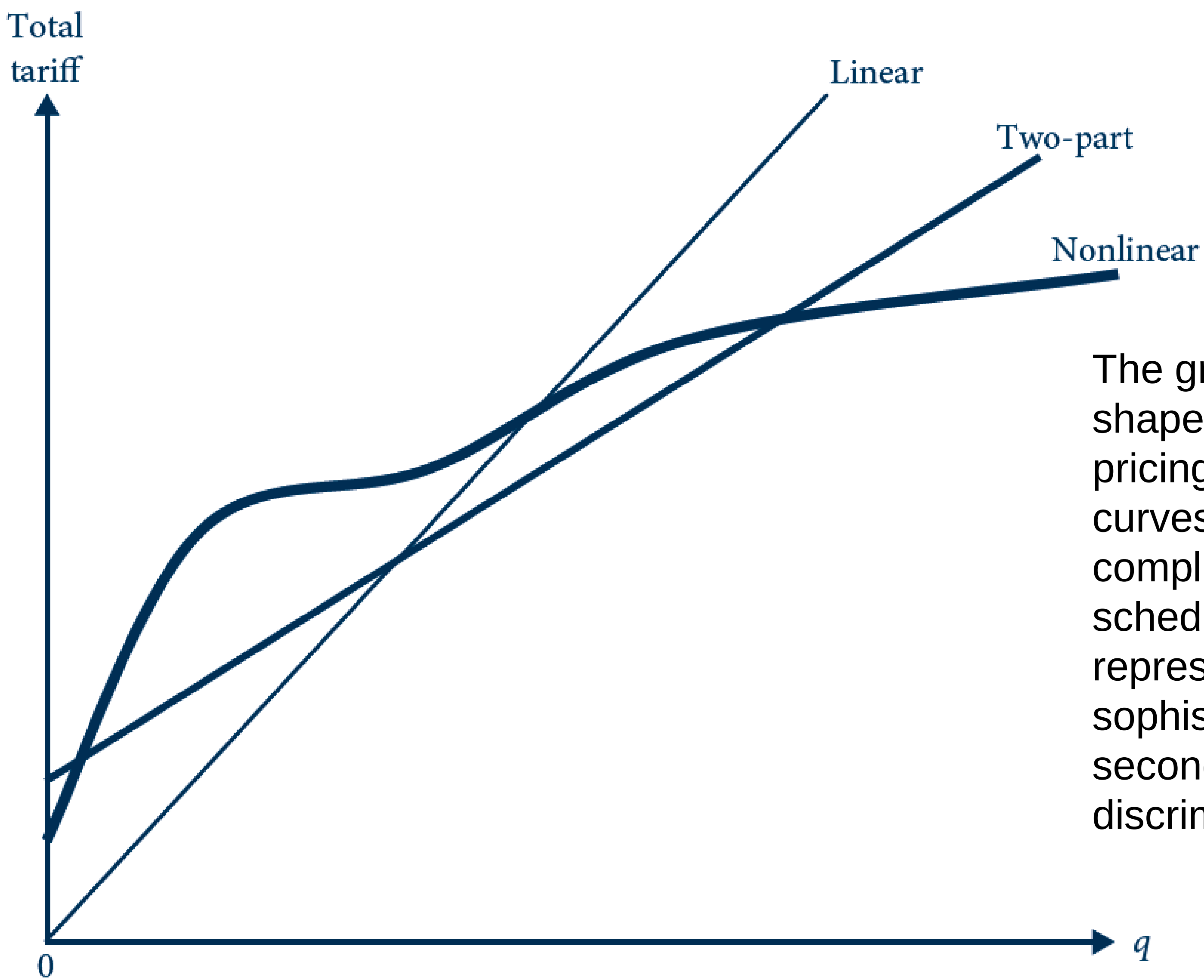
- Agent has private information:
  – Innate characteristics: cannot be chosen
- Principal: extract surplus through contract

Non-linear Pricing

- A monopolist (the principal): offers a nonlinear price schedule
  - Menu of different-sized bundles at different prices
  - Larger bundles sell for lower per-unit price

FIGURE 18.3

Shapes of Various Pricing Schedules



The graph shows the shape of three different pricing schedules. Darker curves are more complicated pricing schedules and so represent more sophisticated forms of second-degree price discrimination.

Model

- Consumer: consumes q and pay T has utility

$$U = \theta v(q) - T$$

  - Assume that $v'(q) > 0$ and $v''(q) < 0$
- Consumer's type is $\theta \in \{\theta_L < \theta_H\}$, $0 < \theta_L < \theta_H$
  - θ_H is the "high" type (with probability of β)
  - θ_L is the "low" type (with probability of $1-\beta$)
- monopolist
  - Has a constant average and marginal cost of c
  - Profit from selling q units is

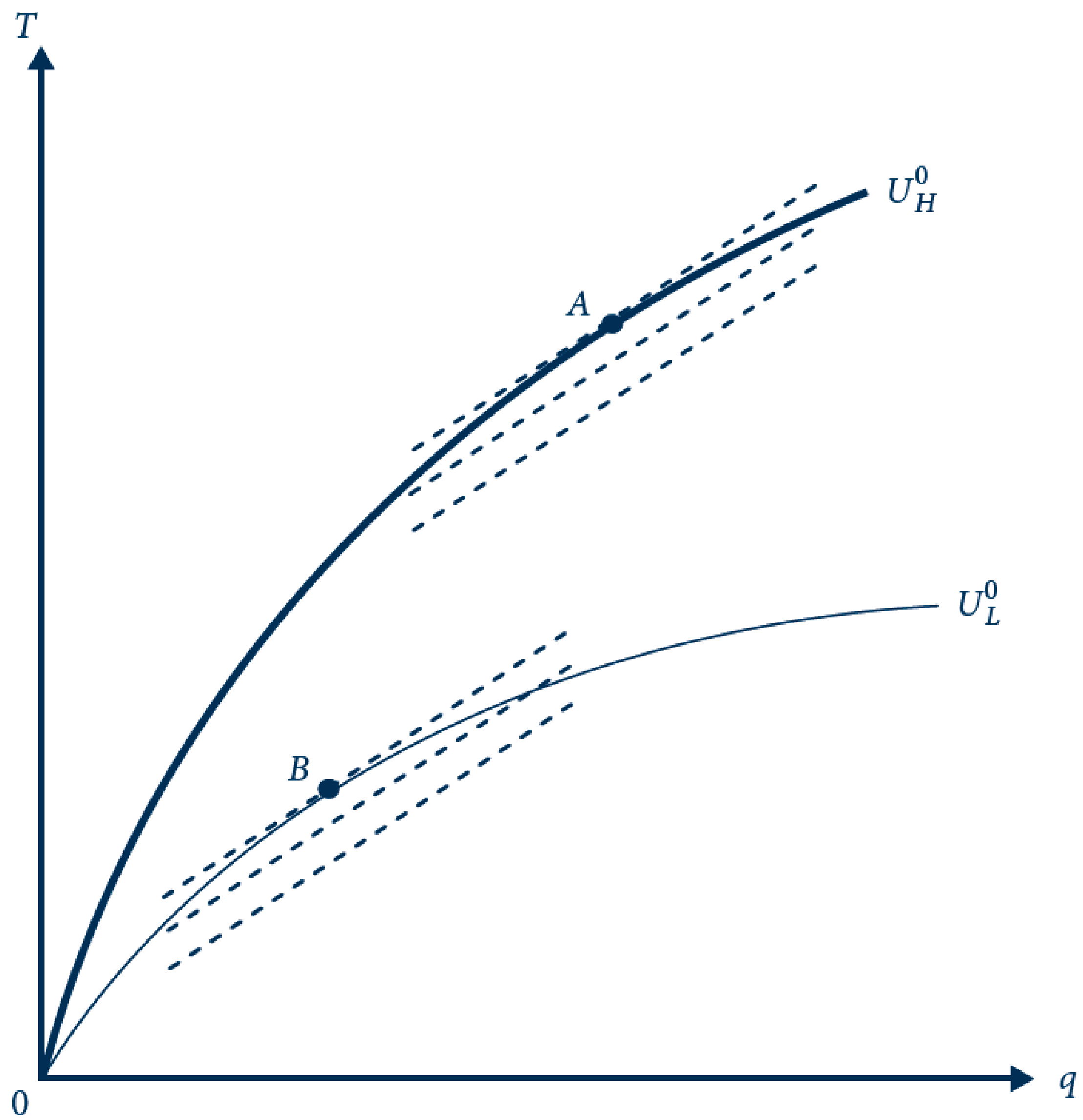
$$\Pi = T - cq$$

First-Best

- The monopolist observes θ
- Participation constraint: $\theta v(q) - T \geq 0$
- At the optimum: $\theta v'(q) = c$

FIGURE 18.4

First-Best Nonlinear Pricing



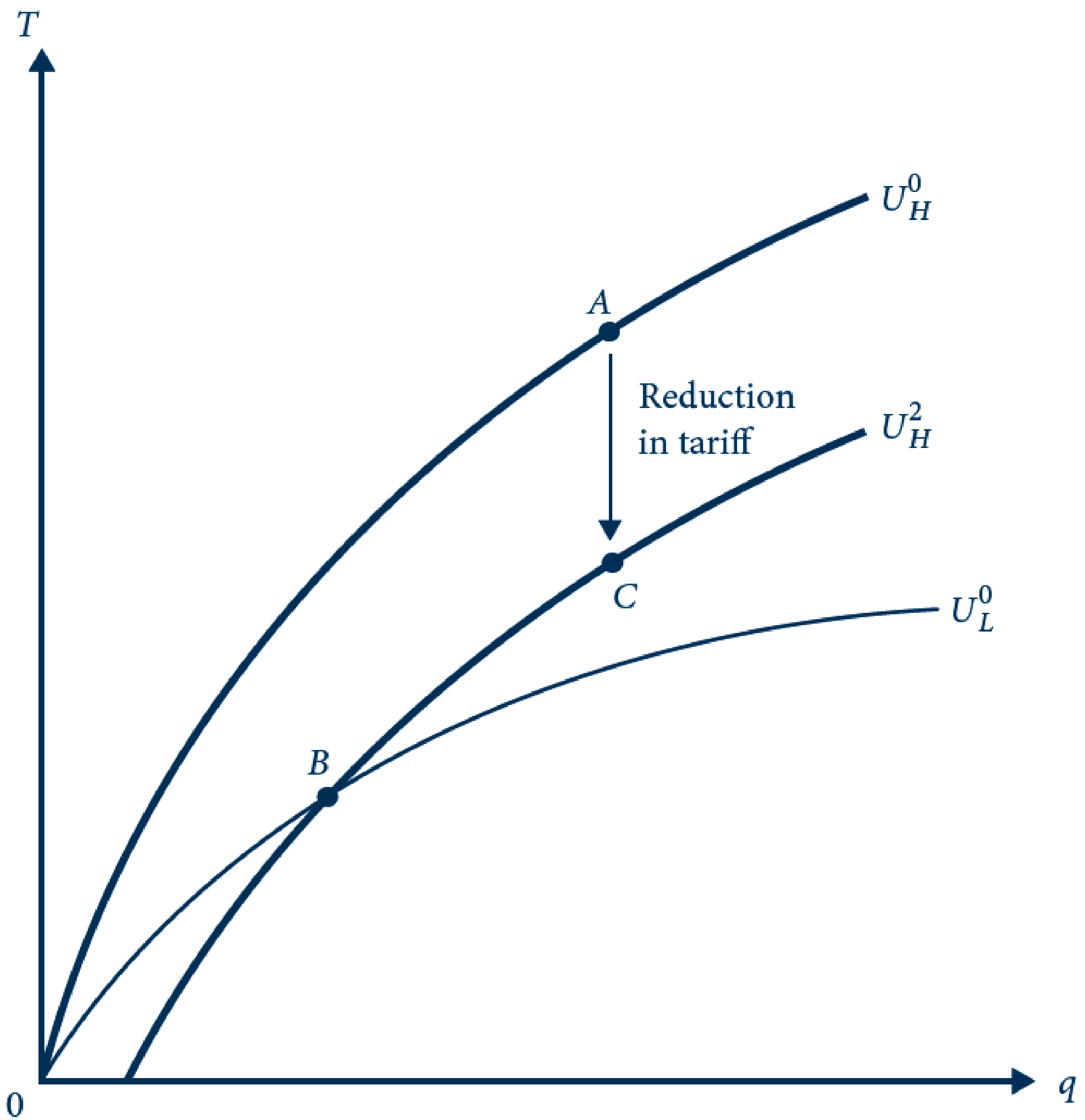
The consumer's indifference curves over the bundle of contractual terms are drawn as solid lines (the darker one for the high type and lighter for the low type); the monopolist's isoprofits are drawn as dashed lines. Point A is the first-best contract option offered to the high type, and point B is that offered to the low type.

Second Best

- The monopolist cannot observe the consumer's type
  - Knows the distribution
  - Choosing *A* is no longer incentive compatible for the high type
  - The monopolist must reduce the high-type's tariff, offering *C* instead of *A*

FIGURE 18.5

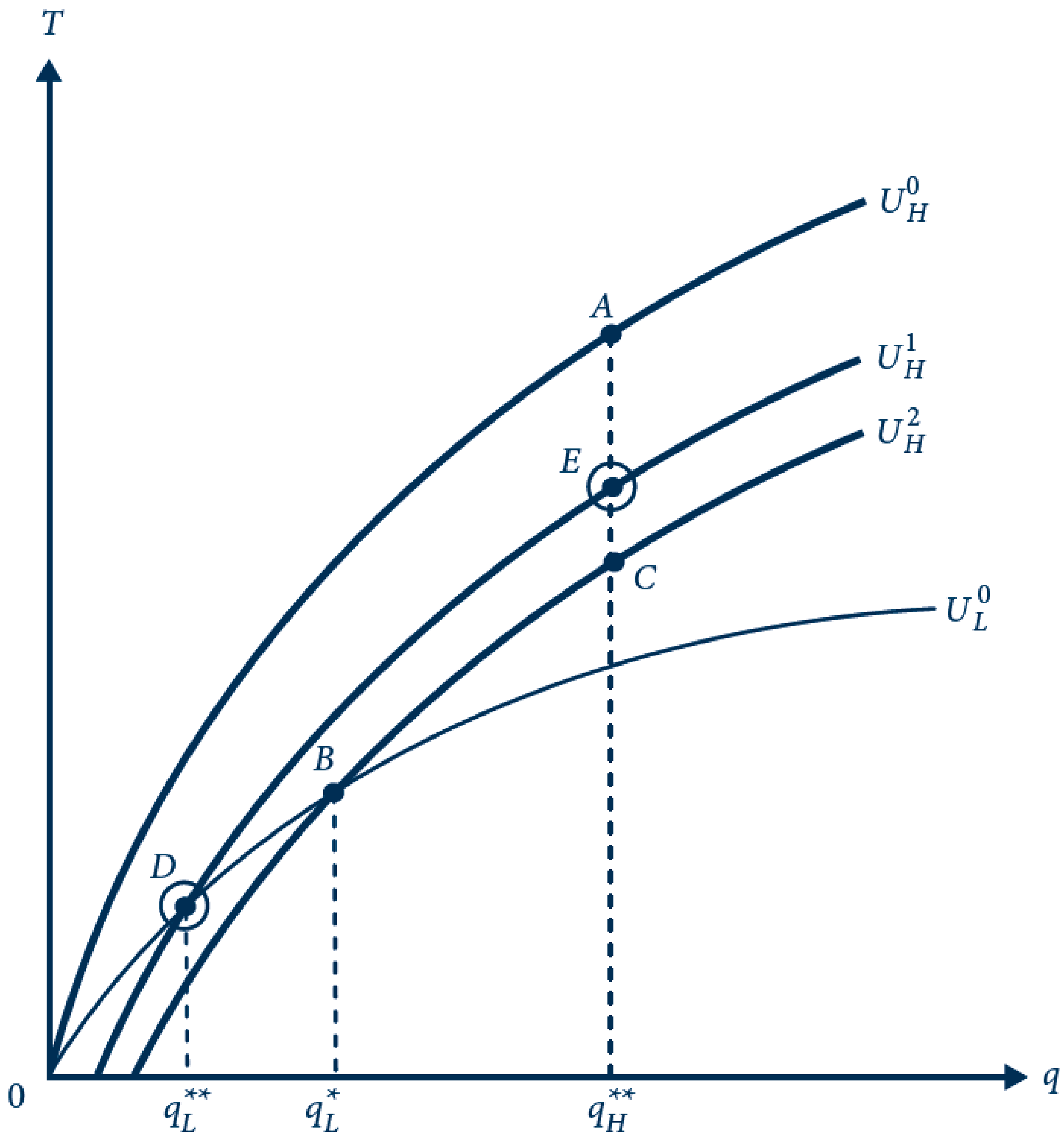
First Best Not Incentive Compatible



The first-best contract, involving points A and B, is not incentive compatible if the consumer has private information about his type. The high type can reach a higher indifference curve by choosing the bundle (B) that is targeted at the low type. To keep him from choosing B, the monopolist must reduce the high type's tariff by replacing bundle A with C.

FIGURE 18.6

Second-Best Nonlinear Pricing



The second-best contract is indicated by the circled points D and E. Relative to the incentive-compatible contract found in Figure 18.5 (points B and C), the second-best contract distorts the low type's quantity (indicated by the move from B to D) in order to make the low type's bundle less attractive to the high type. This allows the principal to charge tariff to the high type (indicated by the move from C to E).

EXAMPLE 18.4 Monopoly Coffee Shop

- The college has a single coffee shop
  - Faces a marginal cost of 5 cents per ounce
- The representative customer faces an equal probability of being one of two types
  - A coffee hound ($\theta_H = 20$)
  - A regular Joe ($\theta_L = 15$)
- Assume $v(q) = 2q^{0.5}$

EXAMPLE 18.4 Monopoly Coffee Shop

- Substituting such that marginal cost = marginal benefit, we get

$$q = (\theta/c)^2$$

$$q^*_L = 9 \qquad q^*_H = 16$$

$$T^*_L = 90 \qquad T^*_H = 160$$

$$E(\pi) = 62.5$$

EXAMPLE 18.4 Monopoly Coffee Shop

- Incentive compatibility when types are hidden
  - The first-best pricing scheme is not incentive compatible if the monopolist cannot observe type
    - Keeping the cup sizes the same, the price for the large cup would have to be reduced by 30 cents
    - The shop’s expected profit falls to 47.5

EXAMPLE 18.4 Monopoly Coffee Shop

- Second best
  - The shop can do better by reducing the size of the small cup
  - The size that is second best would be

$$\theta_L q_L^{-0.5} = c + (\theta_H - \theta_L) q_L^{-0.5}$$

$$q^{**}_L = 4$$

$$T^{**}_{\mathrm{L}} = 60$$

$$E(\pi) = 50$$

Insurance

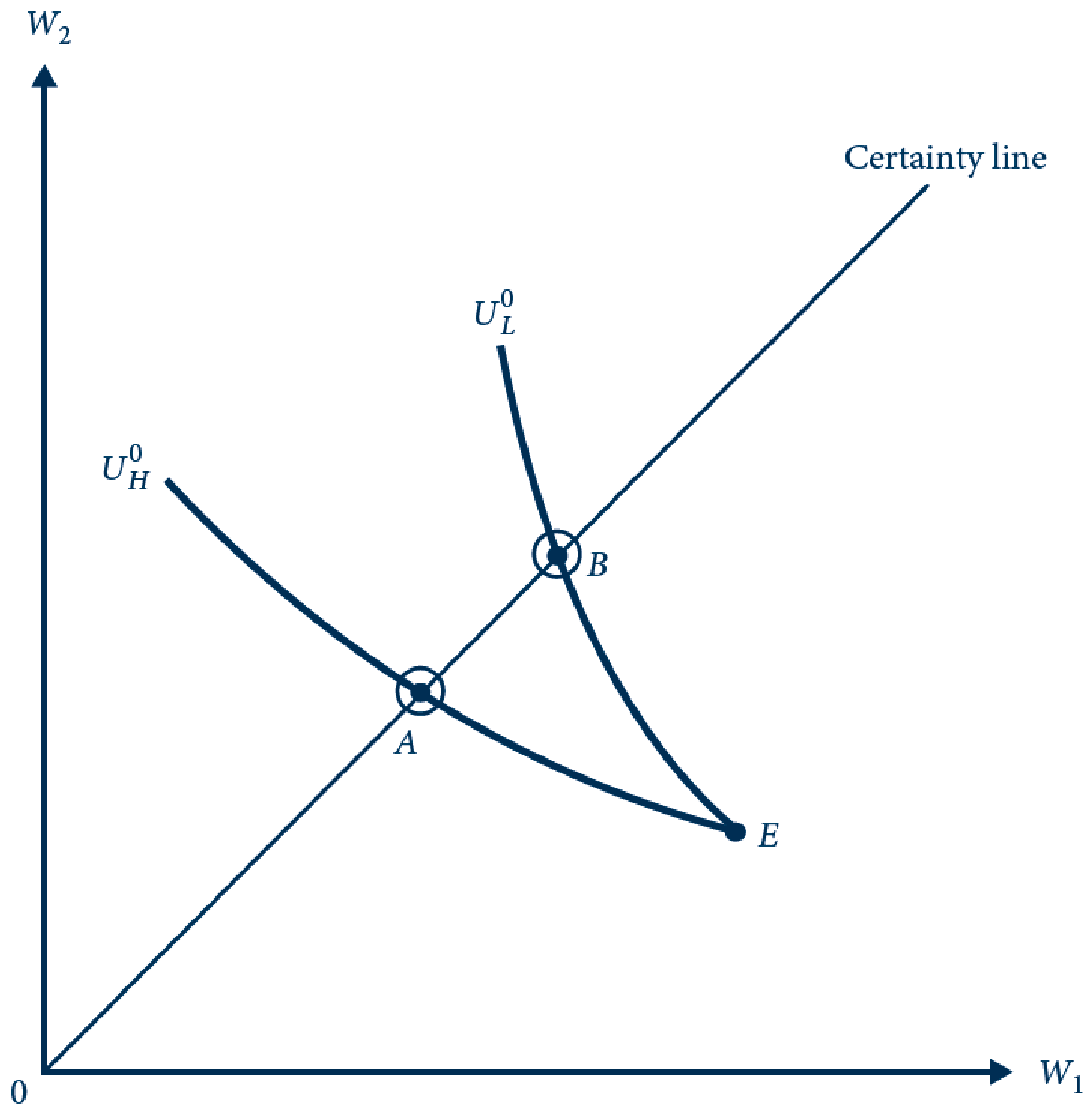
- risky types more likely to accept an insurance policy
- Two types of policy holders
  - θ_H = high risk; θ_L = low risk

First Best

- can observe the individual's risk type
- First best involves full insurance
  - Different premiums are charged to each type to extract all surplus

FIGURE 18.7

First Best for a Monopoly Insurer



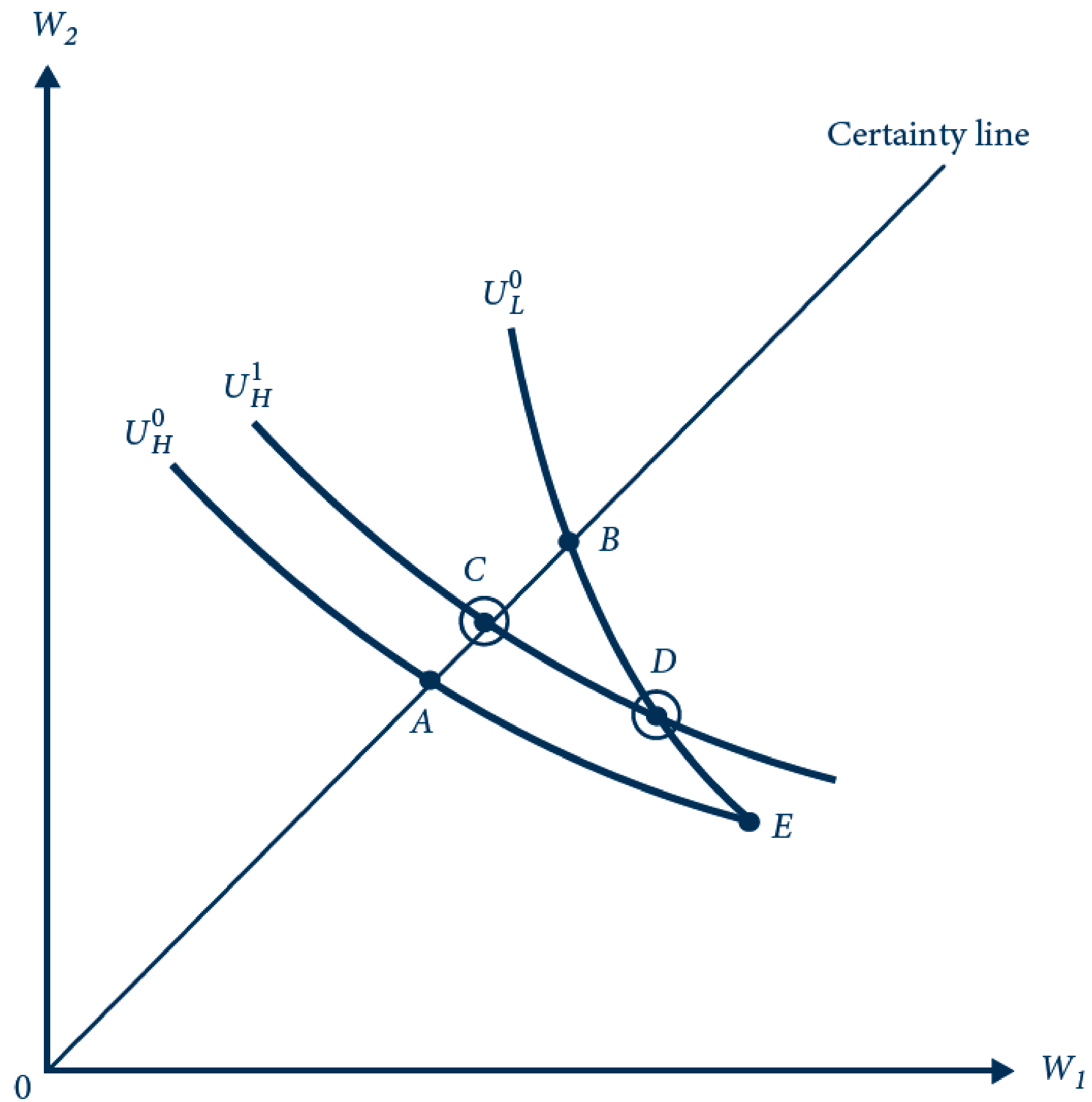
In the first best, the monopoly insurer offers policy A to the high-risk type and B to the low-risk type. Both types are fully insured. The premiums are sufficiently high to keep each type on his indifference curve through the no-insurance point (E).

Second-best

- The insurer cannot observe type
  - First-best contracts: not incentive compatible
  - If the insurer offered *A* and *B*, the high-risk type would choose *B*
- Make coverage to low-risk unattractive to high-risk

FIGURE 18.8

Second Best for a Monopoly Insurer



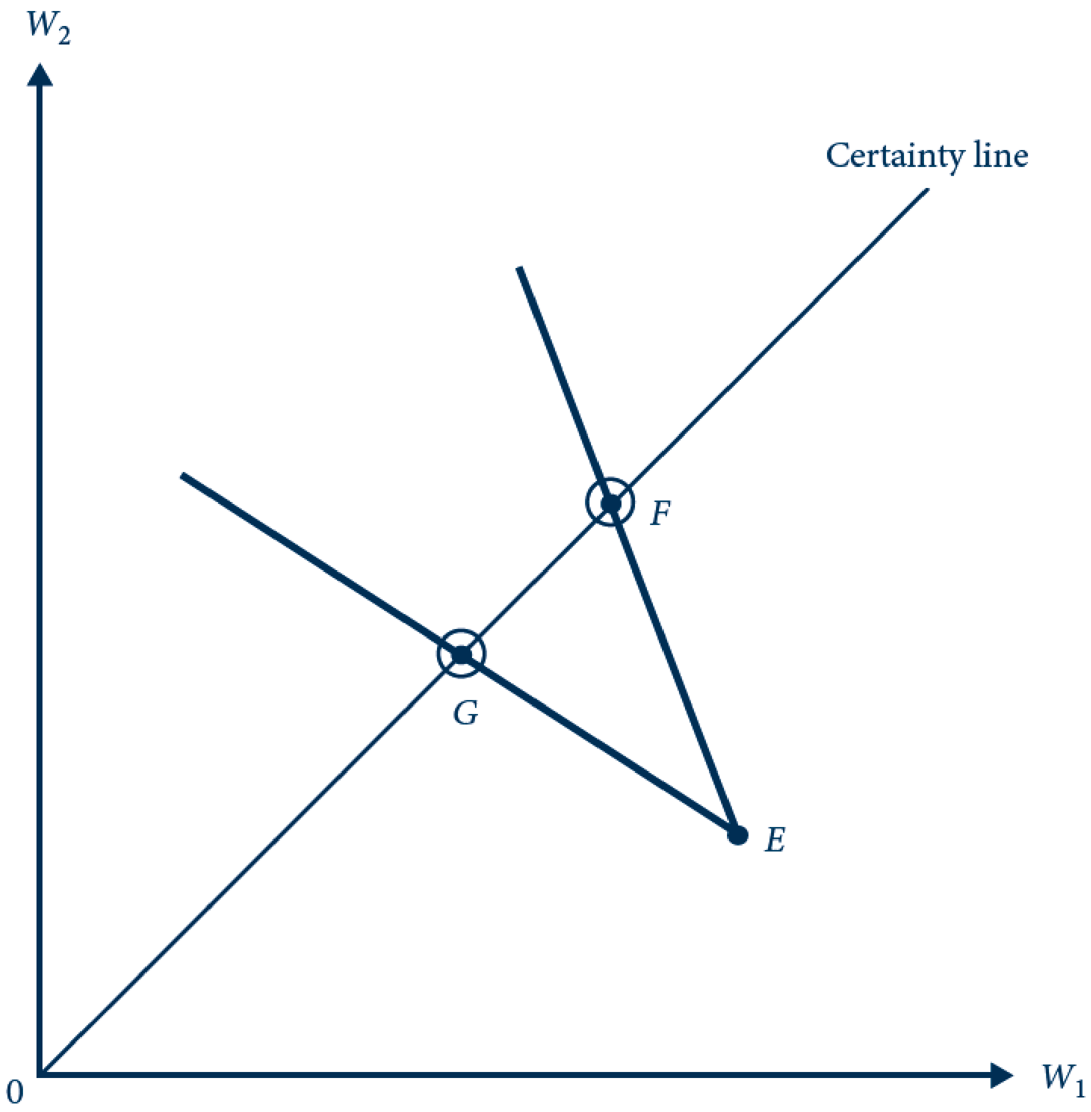
Second-best insurance policies are represented by the circled points: C for the high-risk type and D for the low-risk type.

Competitive Insurance Market

- perfectly competitive market
  - Fair insurance
  - Each type receives full insurance at a fair premium

FIGURE 18.9

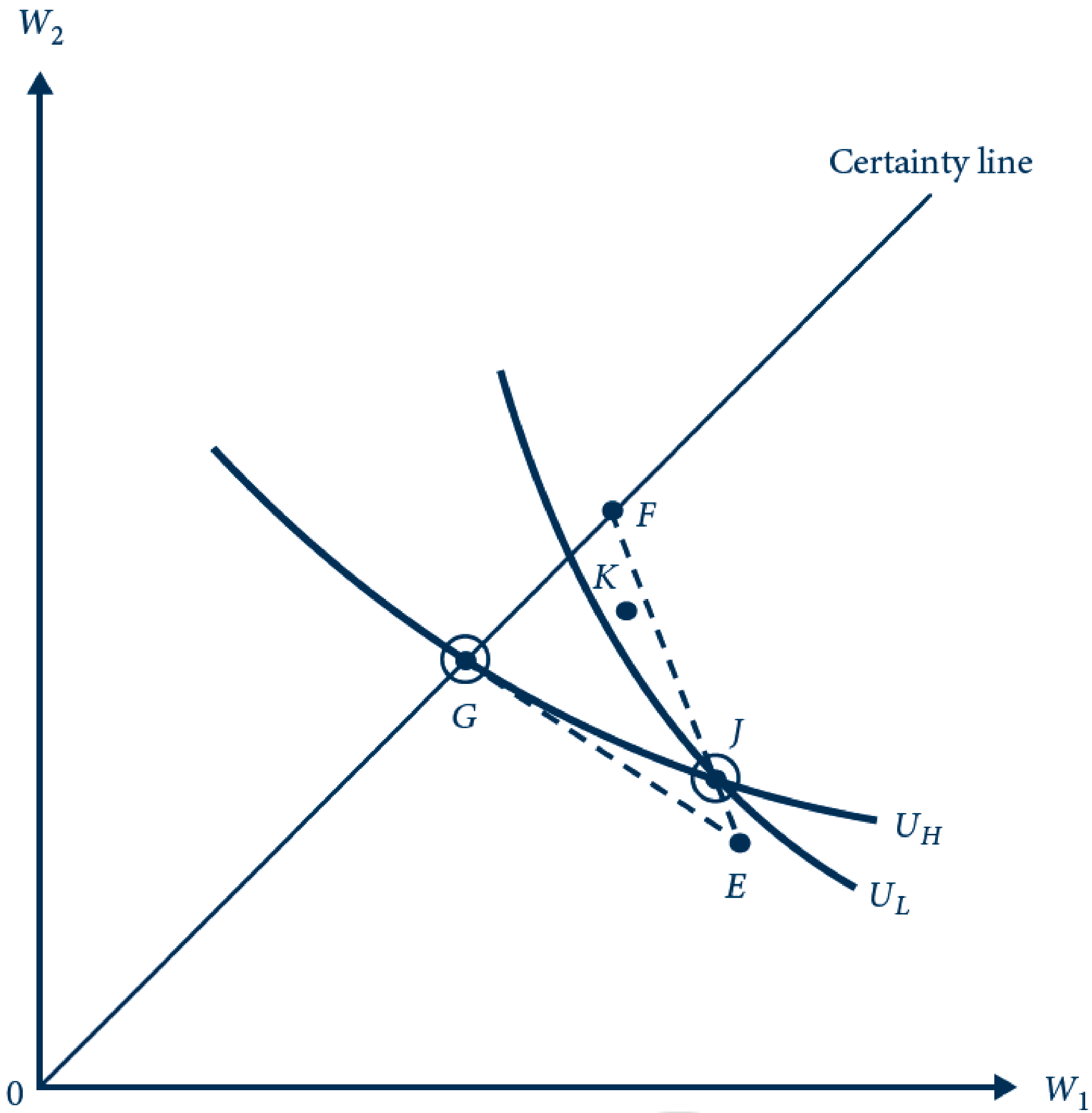
Competitive Insurance Equilibrium with Perfect Information



With perfect information, the competitive insurance market results in full insurance at fair premiums for each type. The high type is offered policy G; the low type, policy F.

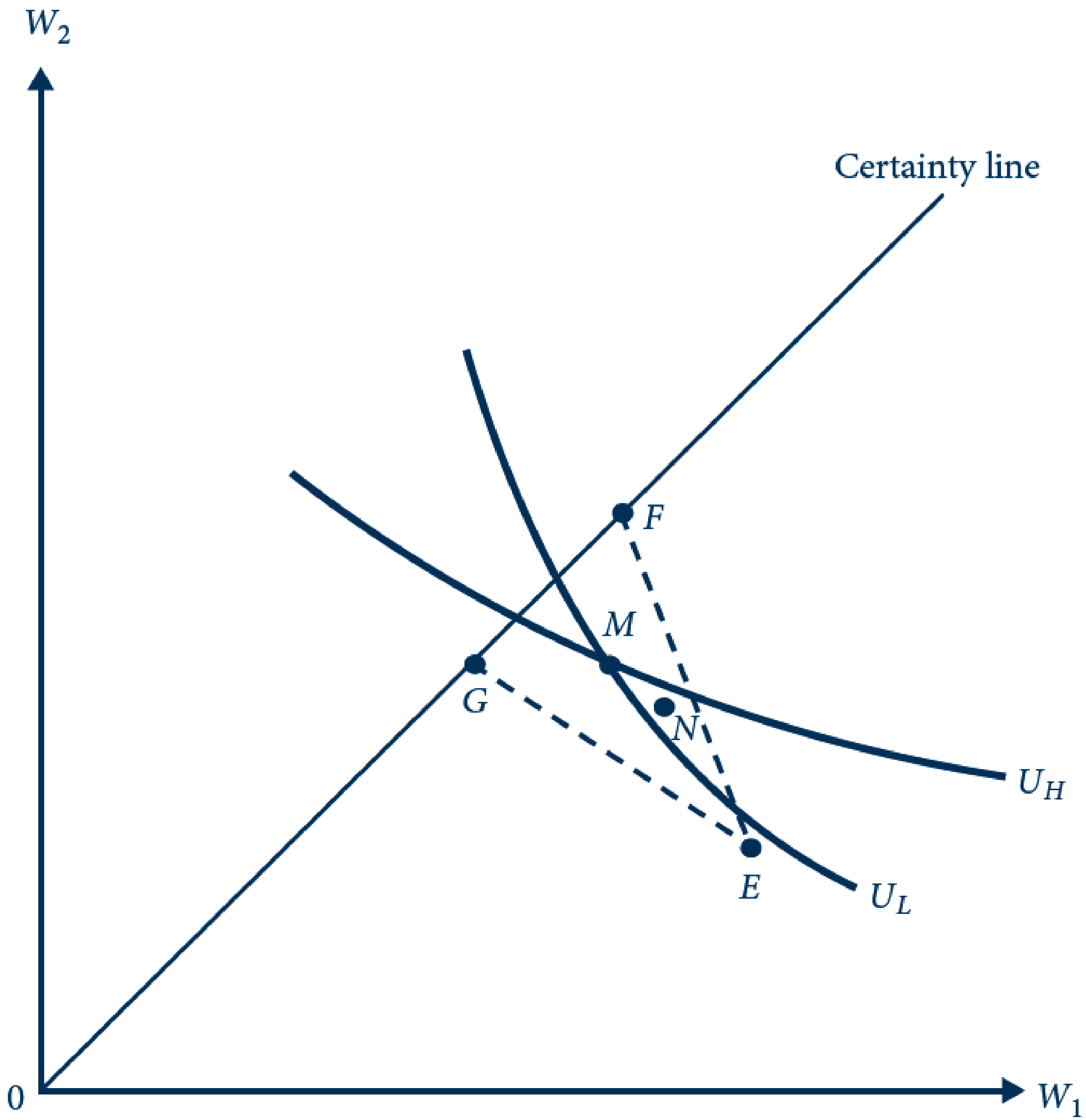
FIGURE 18.10

Competitive Insurance Equilibrium with Hidden Types



With hidden types, the high-risk type continues to be offered first-best policy G but the low-risk type is rationed, receiving only partial insurance at J in order to keep the high-risk type from pooling.

Impossibility of a Competitive Pooling Equilibrium



Pooling contract M cannot be an equilibrium because there exist insurance policies such as N that are profitable to insurers and are attractive to low-risk types but not to high-risk types.

EXAMPLE 18.5 Insuring the Little Red Corvette

- Probability of theft
  - Depends on the color of the car
  - Probability of theft is higher for red cars (π_H = 0.25) than for gray cars (π_L = 0.15)
- First best
  - The monopoly insurer
    - Can observe the car color
    - Offer different policies for different colors
  - Both colors are fully insured for the $20,000 loss of the car

- First best
  - Premium = maximum amount that each type would be willing to pay in lieu of going without insurance
    - = $5,426 for the high type (red cars)
    - = $3,287 for the gray cars
  - Expected profit from a policy
    - Sold for a red car = $426
    - Sold for a gray car = $287

EXAMPLE 18.5 Insuring the Little Red Corvette

- Second best
  - Insurer – knows only that 10% of all cars are red and the rest are gray
- Premium/insurance coverage bundles
  - (p_H, x_H) for high-risk, red cars
  - (p_L, x_L) for low-risk, gray cars
  - Red cars are fully insured, x_H = \$20,000
- Second-best values that result are

x_H^{**}= \$20,000; p_H^{**}= \$4,154

x_L^{**}= \$11,556; p_L^{**}= \$1,971

EXAMPLE 18.6 Competitive Insurance for the Little Red Corvette

- Insurance - competitive market
- Full information
  - Competitive equilibrium - full insurance for both types
    - Fair premium of $5,000 for high-risk, red cars
    - Fair premium of $3,000 for low-risk, gray cars
- If insurers cannot observe car colors
  - Red cars – same as under full information
  - Gray cars - involves a fair premium $p_L = 0.15x_L$
    - p_L = 453 and x_L = 3,020

Signaling Model

- informed player moves first, he can “signal” his type to the other party
  - The low-risk individual would benefit from providing his type to insurers
    - He should be willing to pay the difference between his equilibrium and his first-best surplus to issue such a signal

EXAMPLE 18.7 Certifying Car Color

- Competitive market for automobile insurance
  - Owner of a gray car
    - R – the most that he would be willing to pay to have his car color certified and reported to the market
      - Fully insured at a fair premium of $3,000
      - Earns surplus: ln(100,000 - 3,000 - R)
    - Without the certified report
      - Expected surplus: 0.85 ln(100,000 – 453) + 0.15 ln(100,000 - 453 - 20,000 + 3,020) = 11.4803
    - R = 207

EXAMPLE 18.7 Certifying Car Color

- Competitive market for automobile insurance
  - Owner of a red car
    - Would pay a bribe as high as $2,000
      - The difference between his fair premium with full information ($5,000)
      - And the fair premium charged to an individual known to be of low risk ($3,000)

Market of "Lemon"

- Asymmetric information hinders functioning of market
- Classic Paper by Akerlof (1970)
  - The Market for "Lemons": Quality Uncertainty and the Market Mechanism, *The Quarterly Journal of Economics*, Vol. 84, No. 3. (Aug., 1970), pp. 488-500.
  - Nobel Prize (2001)

What is a “Lemon”?

- Lemon: Bad second-hand car (American slangs)

- Sellers of used cars
  - Have more information on the condition of the car
  - But offering for sale can be bad signal of car quality
    - below some threshold for the owner to keep it

Second-hand Car market

- Valuation

	To Seller	To Buyer
Good Car	90	100
Bad car	10	20

- Equal proportion
  - Good cars: Bad cars = 1: 1
- Since 100>90; 20>10
  - Both cars should be traded.

Symmetric Information

- Suppose:
  - Supply of car finite
  - Demand for car is infinite
- Both sides have complete information
  - Good cars are sold at price: 100
  - Bad cars are sold at price: 20
- Both sides no idea on the quality
  - Cars are sold at average price: 60(=[100+20]/2)

Asymmetric information

- Sellers knows more about their cars
- But buyer does not. Hence, only one price
- Case 1: price ≥ 90
  – both cars are for sold.
  – Yet, buyers will pay no more than 60.
  – Hence, not an equilibrium.
- Case 2: price < 90
  – good cars are not sold.
  – Lemon cars are bought only if p ≤ 20
- Only equilibrium: only lemons are traded

- Sellers have private information about quality
  - And buyers know only the distribution
  - Market price, *p*
  - Sellers offer their cars for sale if and only if $q \le p$
  - Quality of a car offered for sale
    - Uniformly distributed between 0 and *p*
  - Expected quality:

$$\int_0^p q\left(\frac{1}{p}\right)dq = \frac{p}{2}$$

- Buyer's expected net surplus = $b - p/2$
- One equilibrium: $p^*=2b$

Auction

- Seller has an object to sell
- Seller does not know buyers value
- Need to design a way to extract
- First-price sealed auction bid
  - All bidders simultaneously submit secret bids
  - The auctioneer unseals the bids and awards the object to the highest bidder
  - The highest bidder pays his own bid

First-price sealed bid auction

- Weakly dominated strategy
  - If there is another strategy that does at least as well against all rivals’ strategies and strictly better against at least one
- First-price sealed auction bid
  - A buyer receives no surplus if he bids $b=v$
    - No matter what his rivals bid
  - By bidding $b < v$, there is a chance for some positive surplus

Second-price auction

- Second-price sealed auction bid
  - The highest bidder pays the next highest bid nduce bidders to reveal their valuations
- All bidding strategies
  - Are weakly dominated by the strategy of bidding exactly one's valuation
    - The winner's bid does not affect the amount he has to pay
      - That depends on someone else's bid

EXAMPLE 18.9 Art Auction

- Two buyers - bid for a painting in a first-price sealed-bid auction
  - Buyer *i*'s valuation, v_i
    - Random variable, uniformly distributed between 0 and 1
    - Independent of the other buyer's valuation
  - Buyers' valuations are private information
  - Symmetric equilibrium
    - Buyers bid a constant fraction of their valuations, $b_i = kv_i$
  - Solve for the equilibrium value of k

EXAMPLE 18.9 Art Auction

- Symmetric equilibrium
  - Buyer 1
    - Knows his own type v_1
    - Knows buyer 2's equilibrium strategy $b_2 = kv_2$
    - Best responds by choosing the bid b_1 maximizing his expected surplus = $(b_1/k)(v_1-b_1)$
  - So, $b_1 = v_1/2$, $k^* = ½$

- Order statistics
- *k*th-order statistic, $X_{(k)}$
  - The *k*th lowest draw from
  - *n* independent draws made from the same distribution, arranged from smallest to largest
- Expected value of the *k*th-order statistic
  - *n* draws taken from a uniform distribution between 0 and 1
  - $E(X_{(k)}) = k/(n+1)$

EXAMPLE 18.9 Art Auction

- Expected revenue
  - From the first-price auction = $E(max(b_1,b_2))=(1/2) E(max(v_1,v_2)) = 1/3$
    - Because $\max(v_1,v_2)$ is the largest-order statistic from two draws of a uniform random variable between 0 and 1
- Second-price auction
  - Buyers bid their true valuations: $b_i = v_i$
  - Seller's expected revenue = $E(min(b_1, b_2)) = 1/3$
    - Because $min(b_1, b_2) = min(v_1, v_2)$

Common values auctions

- The good has the same value to all bidders
- Do not know exactly what that value is
- The winner's curse
  - The winning bidder realizes that every other bidder though the object was worth less
    - He probably overestimated the value when bidding

Nonlinear pricing with a continuum of types

- Nonlinear pricing model for a monopolist
  - The monopolist offers a menu of bundles
    - One for each type θ
    - A bundle is a specification of a quantity $q(\theta)$ and a total tariff for this quantity $T(\theta)$
  - The consumer has private information about his type
  - The monopolist knows only the distribution from which θ is drawn

Nonlinear pricing with a continuum of types

- Nonlinear pricing model for a monopolist
  - $\varphi(\theta)$ - associated probability density function
  - $\Phi(\theta)$ - cumulative distribution function
  - All types fall in the interval between θ_L at the low end and θ_H at the high end
  - Consumer's utility function, $U(\theta) = \theta\, v(q(\theta)) - T(\theta)$
  - Monopolist's profit from serving type θ is $\Pi(\theta) = T(\theta) - cq(\theta)$
    - Where c is the constant marginal and average cost of production

Nonlinear pricing with a continuum of types

- First best
  - Each type is offered the socially optimal quantity: $\theta v'(q) = c$
  - Each type is charged the tariff that extracts all of his surplus: $T(\theta) = \theta v(q(\theta))$
  - The monopolist earns profit: $\theta v(q(\theta)) - cq(\theta)$

Nonlinear pricing with a continuum of types

- Second best
  - The menu of bundles q(θ) and T(θ) that maximizes its expected profit

$$\int_{\theta_L}^{\theta_H} \Pi(\theta)\varphi(\theta)d\theta = \int_{\theta_L}^{\theta_H} [T(\theta) - cq(\theta)]\varphi(\theta)d\theta$$

  - Subject to participation and incentive compatibility constraints for the consumer

$$\theta_L v\left(q\left(\theta_L\right)\right) - T\left(\theta_L\right) \geq 0$$

$$\theta v'\left(q\left(\theta\right)\right)q'(\theta) - T'\left(\theta\right) = 0$$

Nonlinear pricing with a continuum of types

- Rewriting the problem
  - Maximize

$$\int_{\theta_L}^{\theta_H} [\theta v(q(\theta)) - U(\theta) - cq(\theta)]\varphi(\theta)d\theta$$

    - Subject to the participation constraint and the incentive compatibility constraint

$$\theta_L v\left(q\left(\theta_L\right)\right) - T\left(\theta_L\right) \geq 0 \quad \text{and} \quad U'(\theta) = v\left(q\left(\theta\right)\right)$$

  - The Hamiltonian:

$$H = [\theta v(q(\theta)) - U(\theta) - cq(\theta)]\varphi(\theta) +$$
$$+ \lambda(\theta)v(q(\theta)) + U(\theta)\lambda'(\theta)$$

Nonlinear pricing with a continuum of types

- Optimal control solution

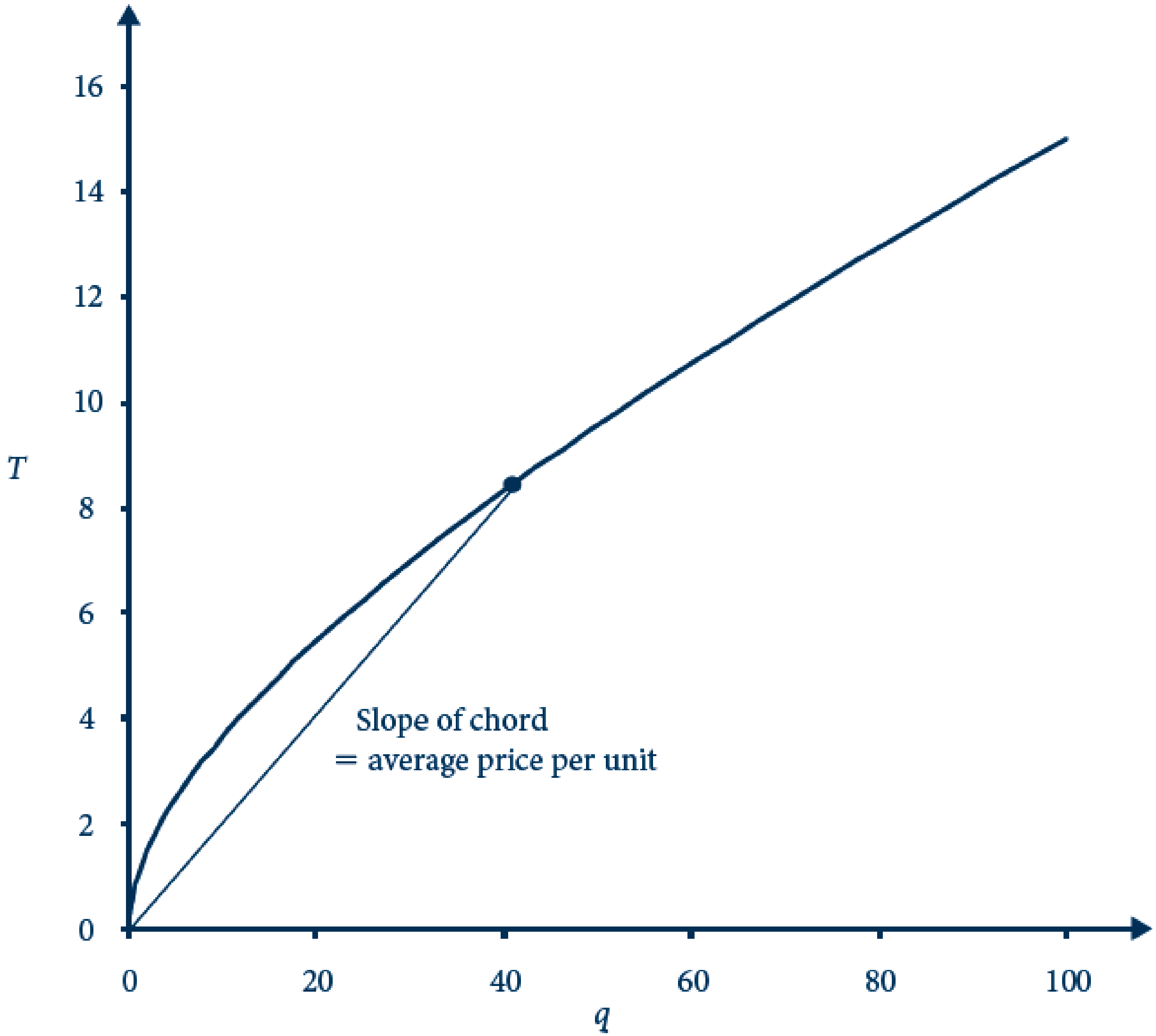
$$\frac{\partial H}{\partial q} = [\theta v'(q(\theta)) - c]\varphi(\theta) + \lambda(\theta)v'(\theta) = 0$$

$$\frac{\partial H}{\partial U} = -\varphi(\theta) + \lambda'(\theta) = 0$$

$$\theta v'(q(\theta)) = c + \frac{1-\Phi(\theta)}{\varphi(\theta)} v'(q(\theta))$$

FIGURE E 18.1

Nonlinear Pricing Schedule for Continuum of Types



The graph is based on calculations for uniformly distributed types. Larger bundles receive per-unit price discount.