National University of Singapore
Microeconomics Anaylsis III, EC4101
Tutorial on: Exchange Economy

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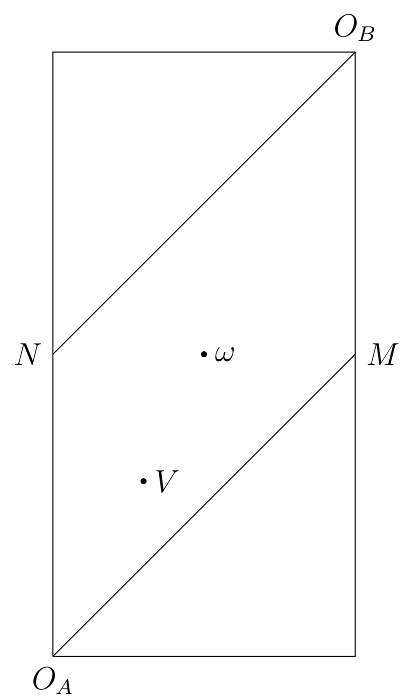
1. Imagine a two-consumer (A, B) and two-good (x_1, x_2) economy in which the total endowment of good 1 is 1 and the total endowment of good 2 is 2. Both consumers have utility functions given by

$$U(x_1, x_2) = \min\{x_1, x_2\}.$$

   (a) What is the set of Pareto efficient allocations in the economy?

   Answer:



The entire region $O_A M O_B N$ is the set of Pareto efficient allocations.

To see why, note that improving agent A's utility; say from point V would require giving agent A more of x_1 but this also lowers agent B's amount of x_1, lowering B's utility from the level $\hat{u}_B$.

Similarly, all other points in the region $O_A M O_B N$ can be checked to fulfil the requirement of Pareto efficiency, which is that one agent's utility can be improved only if other agent's utility is lowered.

(b) If the endowment vector of each consumer is given by

$$(\omega_1, \omega_2) = (\frac{1}{2}, 1),$$

what is the set of competitive equilibria? If the endowment vector of each consumer A is

$$(\omega_1^A, \omega_2^A) = (\frac{1}{2}, \frac{1}{2})$$

and that of consumer B is

$$(\omega_1^B, \omega_2^B) = (\frac{1}{2}, \frac{3}{2}),$$

what is the set of competitive equilibria?

Answer:

Competitive equilibrium is a pair of prices and allocations such that

i. each agent is maximizing utility at the given prices and choosing the particular allocations

ii. market clears; i.e. aggregate demand = aggregate endowment

To calculate the equilibrium first normalize the prices of good 1 by setting $p_1 = 1$and choose p_2 to be the price of good x_2 (i.e. good 2 price is measured in terms of good 1). Note that in this way, we have just fixed the relative prices not the absolute prices. Agent A's utility maximization involves choosing a bundle along the 45 degree line:

$$\begin{aligned} p_1 x_{1A}^* + p_2 x_{2A}^* &= p_1 \times \frac{1}{2} + p_2 \times 1 \\ (p_1 + p_2) x_{1A}^* &= \frac{1}{2} p_1 + p_2 \\ x_{1A}^* &= \frac{\frac{1}{2} p_1 + p_2}{p_1 + p_2} = \frac{\frac{1}{2} + p_2}{1 + p_2} \end{aligned}$$

By symmetry between the agents for the given utility and endowment specifications

$$x_{1A}^* = x_{1B}^*$$

so market clearing of good x_1 requires

$$\begin{aligned} x_{1A}^* + x_{1B}^* &= \omega_1^A + \omega_1^B \\ 2x_{1A}^* &= \frac{\frac{1}{2} + p_2}{1 + p_2} = 1 \\ 1 + 2p_2 &= 1 + p_2 \\ p_2 &= 0 \end{aligned}$$

So equilibrium prices are $(p_1, p_2) = (1, 0)$ and euqilibrium allocations are $(x^*_{1A}, x^*_{2A}) = (\frac{1}{2}, 1) = (x^*_{2A}, x^*_{2B})$ that clears the market. Explanation: Why is $p_2 = 0$? Good 2 is in excess supply: each consumer would like to trade good 2 for good 1, which means there will be no trade in equilibirum. Due to excess supply of good 2, its price is driven down to zero.

Your Tasks:
Try solving the competitive equilibrium when $(\omega^A_1, \omega^A_2) = (\frac{1}{2}, \frac{1}{2})$ and $(\omega^B_1, \omega^B_2) = (\frac{1}{2}, \frac{3}{2})$ following a similar procedure.