Tutorial material 6

1. The difference between classical theory and Keynesian theory

In classical theory, supply determines output. Output is determined by capital stock, labor, and production function:

$$\bar{Y} = F(\bar{K}, \bar{L})$$

The equilibrium of goods and services market is to determine the allocation of resources:

$$Y = C + I(r) + G$$

The interest rate is flexible. The change of r guarantees the equilibrium in the goods and services market. Here r is the "invisible hand". The invisible hand, i.e. the price system which includes only one price, r, in our simple model here, determines the allocation of output. In this sense, the invisible hand is visible, since it is exactly the price system.

In Keynesian theory, demand determines output. Output level and allocation of output is simultaneously determined by the equilibrium of goods and services market:

$$Y = C(Y - T) + I + G$$

For simplification, we assume investment I is constant here. We say output level is determined by demand since Y is the solution of the above equation, which is also called the Keynesian cross.

2. The derivation of $f'(k) = MPK$

We know that

$$MPK = F_1(K, L)$$

And

$$f'(k) = F_1(\frac{K}{L}, 1)$$

since

$f(k) = F(k, 1) = F(\frac{K}{L}, 1)$.

So our aim is to show

$$F_1(K, L) = F_1(\frac{K}{L}, 1)$$

We show a general property of F(K,L), i.e. for any z>0, $F_1(zK, zL) = F_1(K, L)$.

This is shown by taking derivative of K on the both sides of $F(zK, zL) = zF(K, L)$.

3. The derivation of $\Delta k = i - (\delta + n)k$ when this is a positive population growth rate.

From the relationship

$$\Delta k = \Delta\left(\frac{K}{L}\right) = \frac{\Delta K}{L} - \frac{K}{L^2}\Delta L = \frac{\Delta K}{L} - \frac{K}{L}\frac{\Delta L}{L}$$

The result follows from plugging in

$$\Delta K = I - \delta K$$

and $\frac{\Delta L}{L} = n$, i.e.

$$\Delta k = \frac{\Delta K}{L} - \frac{K}{L}\frac{\Delta L}{L} = \frac{I - \delta K}{L} - nk = \frac{I}{L} - \delta\frac{K}{L} - nk$$
$$= i - \delta k - nk = i - (\delta + n)k.$$