

## Solution to midterm exam

1. Derive  $NX=S-I$  from the national income accounts identity  $Y=C+I+G+NX$ . (10 points)

Answer: From  $Y=C+I+G+NX$ , we have  $NX=Y-C-I-G$ . This implies  $NX=S-I$ , since  $S=Y-C-G$  by the definition of  $S$ .

2. In a big city, at any day time, there are 40% of hired taxis whose passengers get off from them. And 80% of empty taxis can counter passengers who want to hire them. How much fraction of taxis are empty in the steady state? (10 points)

Answer: We denote the number of empty taxis  $U$  and the number of hired taxis  $E$ . We denote the total number of taxis  $L$ . Thus  $L=U+E$ . Let  $s$  denote the percent of hired taxis whose passengers get off from them. Let  $f$  denote the percent of empty taxis which can counter passengers hiring them.

By the equilibrium condition in the steady state,  $Uf=Es$ , we have  $\frac{U}{L} = \frac{s}{f+s} = \frac{0.4}{0.8+0.4} = \frac{1}{3}$ . Thus  $\frac{1}{3}$  of taxis are empty in the steady state.

3. John Taylor has proposed the following monetary policy for the nominal interest rate  $i_t$ :

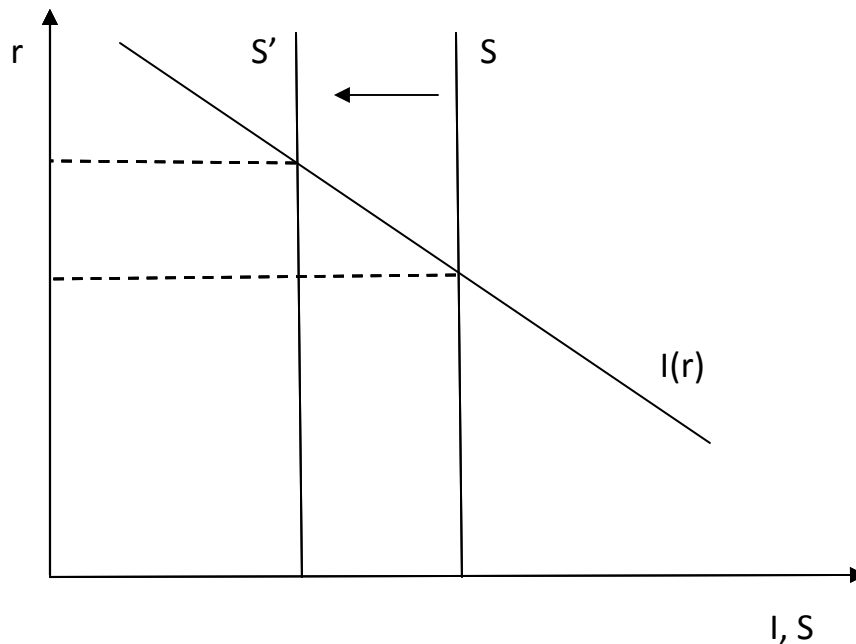
$$i_t = \rho + \pi_t + \theta_\pi(\pi_t - \pi_t^*) + \theta_Y(Y_t - \bar{Y}_t).$$

What do  $\rho$ ,  $\pi_t^*$ ,  $\theta_\pi$  and  $\theta_Y$  represent in this equation? (10 points)

Answer:  $\rho$  is the natural rate of interest.  $\pi_t^*$  is the central bank's target for the inflation rate.  $\theta_\pi$  measures how much the central bank adjusts the interest rate when inflation deviates from its target.  $\theta_Y$  measures how much the central bank adjusts the interest rate when output deviates from its natural level.

4. This question helps you to understand the term of “crowding effect”. In a closed economy, please use the classical theory to analyze the effects of increase in government purchases,  $G$ , on private investment,  $I$ , and real interest rate,  $r$ . Suppose that government increases purchases by  $\Delta G$ . Show that  $\Delta I = -\Delta G$ . Thus the increase in  $G$  implies the decrease in  $I$ . Government purchases are said to crowd out investment. (14 points)

Answer: Since  $S=Y-C-G$ , increase of  $G$  implies decrease of  $S$ . We use a graph to show the impacts of this event.



From the above graph we can see that increase of  $G$  causes  $I$  to decrease and  $r$  to increase.

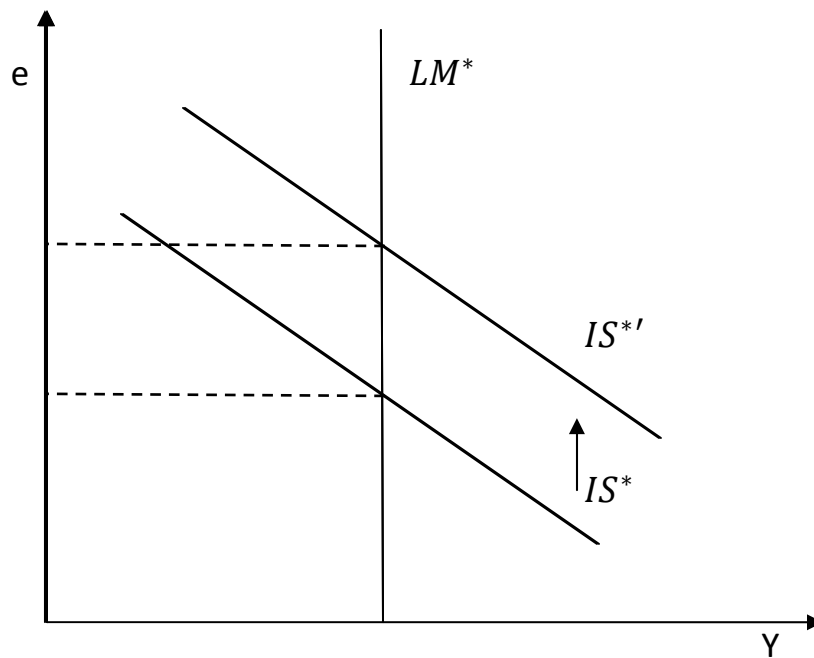
Suppose that government increases purchases by  $\Delta G$ . Since  $Y=C+I+G$ , we know that  $I=Y-C-G$ . Thus we have  $\Delta I = -\Delta G$ .

5. In a small open economy, the private housing investment increases because mortgage becomes easier to apply for in the financial market. Please use Mundell-Fleming model to predict what would happen to aggregate

income, the exchange rate, and the trade balance under both floating and fixed exchange rates. (16 points)

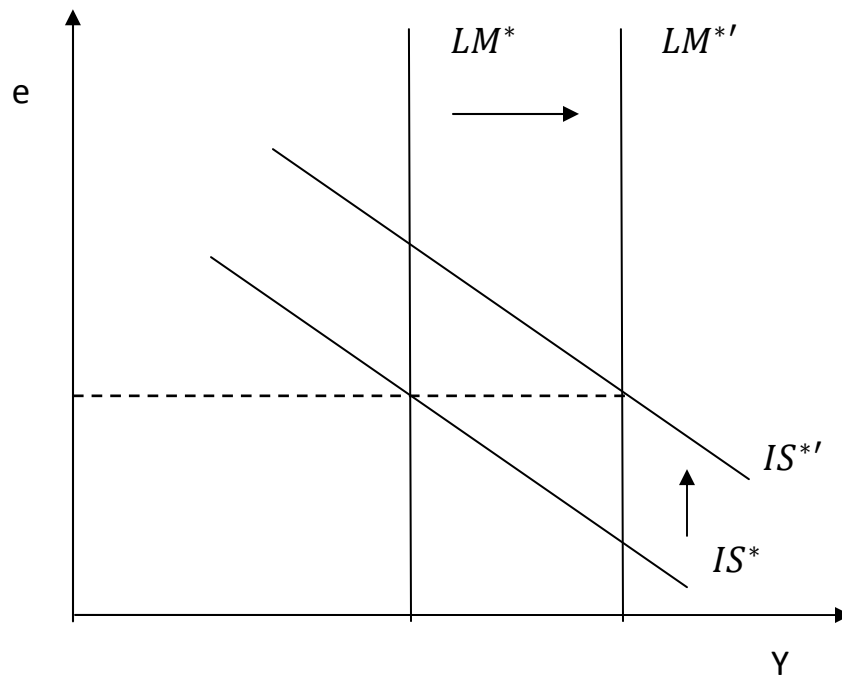
Answer: The increase of private housing investment causes the total investment to increase. This causes  $IS^*$  curve to shift upward.

Under floating exchange rate, we have the graph:



Thus  $Y$  does not change and  $e$  increases.  $NX$  decreases, since  $NX(e)$  has negative relationship with  $e$ .

Under fixed exchange rate, we have the graph:



Thus  $Y$  increases and  $e$  does not change.  $NX$  does not change since  $e$  does not change.

6. Consider a small open economy in the long run. The economy is described by the following equations:

$$Y = C + I + G + NX$$

$$Y = 100$$

$$T = 20$$

$$G = 20$$

$$C = 10 + 0.9(Y - T)$$

$$I = 80 - 30r$$

$$NX = 40 - 6\varepsilon$$

And the world interest rate  $r^* = 2$ .

- (1) Solve for national saving, investment, the trade balance and the equilibrium exchange rate. (8 points)

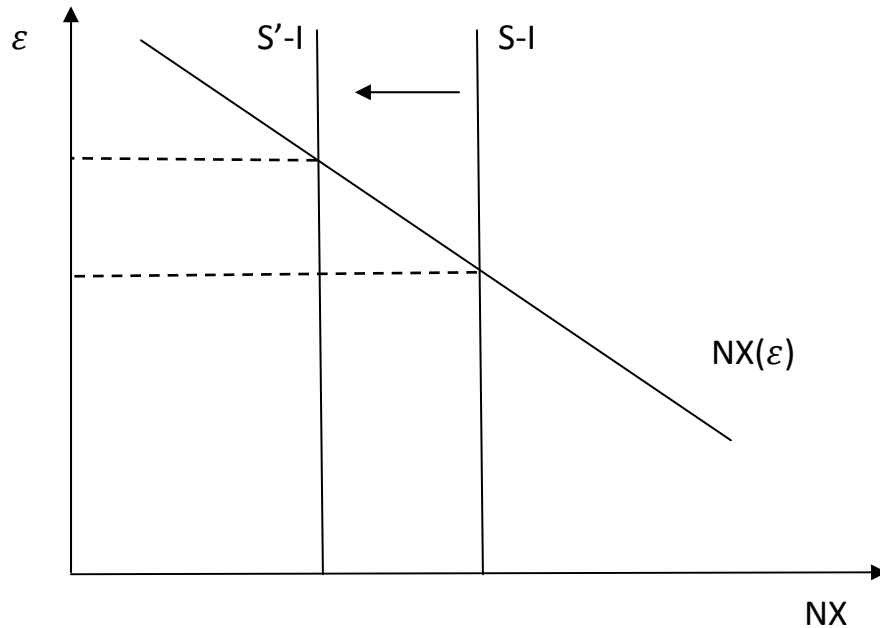
(2) Suppose that government cuts  $T$  from 20 to 10. Solve for national saving, investment, the trade balance and the equilibrium exchange rate. Use a graph to explain what you find. (12 points)

Answer:

(1) By the consumption function, we have  $C=10+0.9(Y-T)=82$ . Thus national saving  $S=Y-C-G=100-82-20=-2$ . By the investment function  $I=80-30r=80-30r^*=20$ . Thus net exports  $NX=S-I=-2-20=-22$ . Solving the equation  $NX=40-6\varepsilon=-22$ , we have the equilibrium exchange rate  $\varepsilon = \frac{31}{3}$ .

(2) By the consumption function, we have  $C=10+0.9(Y-T)=91$ . Thus national saving  $S=Y-C-G=100-91-20=-11$ . By the investment function  $I=80-30r=80-30r^*=20$ . Thus net exports  $NX=S-I=-11-20=-31$ . Solving the equation  $NX=40-6\varepsilon=-31$ , we have the equilibrium exchange rate  $\varepsilon = \frac{71}{6}$ .

Comparing with (1), we find that decrease of  $T$  causes  $NX$  to decrease and  $\varepsilon$  to increase. The reason is that decrease of  $T$  causes increase of consumption. The increase of consumption in turn causes decrease of saving since  $S=Y-C-G$ . The impacts on  $NX$  and  $\varepsilon$  follow from the following graph:



7. Consider a closed economy in the short run.

The consumption function is  $C=10+0.9(Y-T)$ . The investment function is  $I=80-30r$ . And the fiscal policies of this economy are  $G=20$ , and  $T=20$ .

The money demand function of this economy is  $\left(\frac{M}{P}\right)^d = Y - 60r$ . And the money supply  $M$  is 330, and the price level  $P$  is 3.

- (1) Derive the equations of IS curve and LM curve. Solve for the equilibrium interest rate  $r$  and the equilibrium income  $Y$ . (15 points)
- (2) Derive the equation of the aggregate demand curve. (5 points)



End of exam.

Answer:

- (1) By the equilibrium of goods and services market, we have

$Y=C+I+G=10+0.9(Y-20)+80-30r+20$ . Thus we have the equation of IS curve  $Y=920-300r$ .

By the equilibrium of money market, we have  $\frac{M}{P} = Y - 60r$ . We know that  $M=330$  and  $P=3$ . Thus we have the equation of LM curve  $Y=110+60r$ .

Combining the equations of IS curve and LM curve, we can solve the equilibrium  $r=2.25$  and the equilibrium  $Y=245$ .

(2) From the equilibrium of money market, we have  $\frac{330}{P} = Y - 60r$ . From the IS curve we have  $r = \frac{920}{300} - \frac{Y}{300}$ . Combining these two equations we have the aggregate demand equation  $\frac{330}{P} = 1.2Y - 184$ .