# Suggested Solutions to EC2102 Macroeconomic Analysis I
# Tutorial 1, Week 3 (January 25-29, 2010)

**Question 1**

($i$) Let us set up the maximization problem for Mr. $j$, dropping the superscript $j$ for now. Mr. $j$ wants to

$$\begin{aligned} & \max_{c_1, c_2} \; u(c_1) + \beta u(c_2) \\ s.t.\; c_1 + \frac{c_2}{1+r} &= y_1 + \frac{y_2}{1+r} \equiv \omega. \end{aligned}$$

Rewriting the lifetime budget constraint, we have that

$$c_2(c_1) = (\omega - c_1)(1+r).$$

Using the above and the fact that $u(c_t) = \sqrt{c_t}$ for $t = 1, 2$, Mr $j$'s maximization problem is just

$$\max_{c_1} \sqrt{c_1} + \beta\sqrt{(\omega - c_1)(1+r)}.$$

($ii$) The F.O.C. to Mr $j$'s maximization problem in ($i$), dropping the superscript $j$, is

$$\frac{1}{2\sqrt{c_1^*}} + \frac{-\beta(1+r)}{2\sqrt{(\omega - c_1^*)(1+r)}} = 0, \tag{1}$$

which yields

$$c_1^* = \frac{\omega}{\beta^2(1+r)+1} = \frac{y_1(1+r)+y_2}{\left[\beta^2(1+r)+1\right](1+r)}. \tag{2}$$

From the first period budget constraint, in equilibrium

$$s_1^* = y_1 - c_1^* = y_1 - \frac{y_1(1+r)+y_2}{\left[\beta^2(1+r)+1\right](1+r)}.$$

From the lifetime budget constraint, in equilibrium,

$$c_2^* = (\omega - c_1^*)(1+r) = \left[\frac{\beta^2(1+r)}{\beta^2(1+r)+1}\right]\left[y_1(1+r)+y_2\right].$$

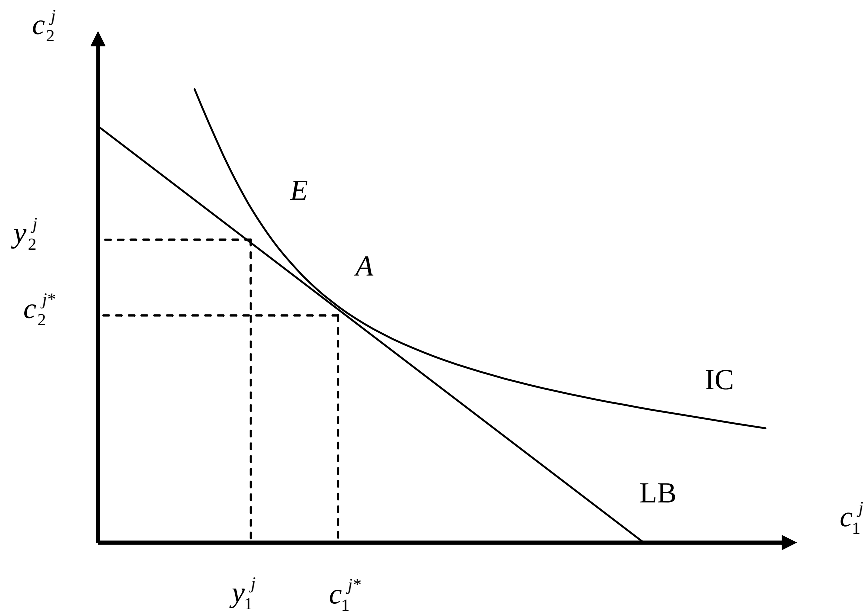
Putting the superscripts back and substituting $y_1^A = 10$, $y_2^A = 50$, and $y_1^B = 50$ and $y_2^B = 10$, we have that

$$
\begin{aligned}
c_1^{A*} &= \frac{10(1+r)+50}{\left[\beta^2(1+r)+1\right](1+r)}, \\
s_1^{A*} &= 10 - \frac{10(1+r)+50}{\left[\beta^2(1+r)+1\right](1+r)}, \\
c_2^{A*} &= \left[\frac{\beta^2(1+r)}{\beta^2(1+r)+1}\right][10(1+r)+50]; \text{ and}
\end{aligned}
$$

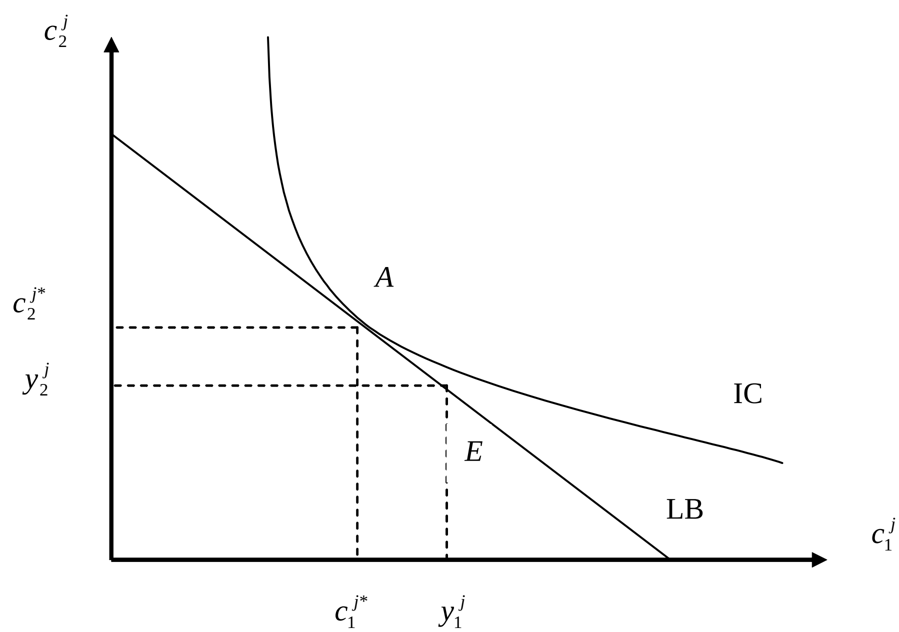
$$
\begin{aligned}
c_1^{B*} &= \frac{50(1+r)+10}{\left[\beta^2(1+r)+1\right](1+r)}, \\
s_1^{B*} &= 50 - \frac{50(1+r)+10}{\left[\beta^2(1+r)+1\right](1+r)}, \\
c_2^{B*} &= \left[\frac{\beta^2(1+r)}{\beta^2(1+r)+1}\right][50(1+r)+10].
\end{aligned}
$$

$(iii)$ From the above, we do not know if Mr $j$ is a borrower, lender, or is neither borrowing nor lending. Let $E$ denote his endowment point, and let $A$ denote his optimal consumption bundle. We have three cases: Case 1, where Mr $j$ is a borrower; Case 2, where Mr $j$ is a lender; and Case 3, where Mr $j$ is neither lending nor borrowing.

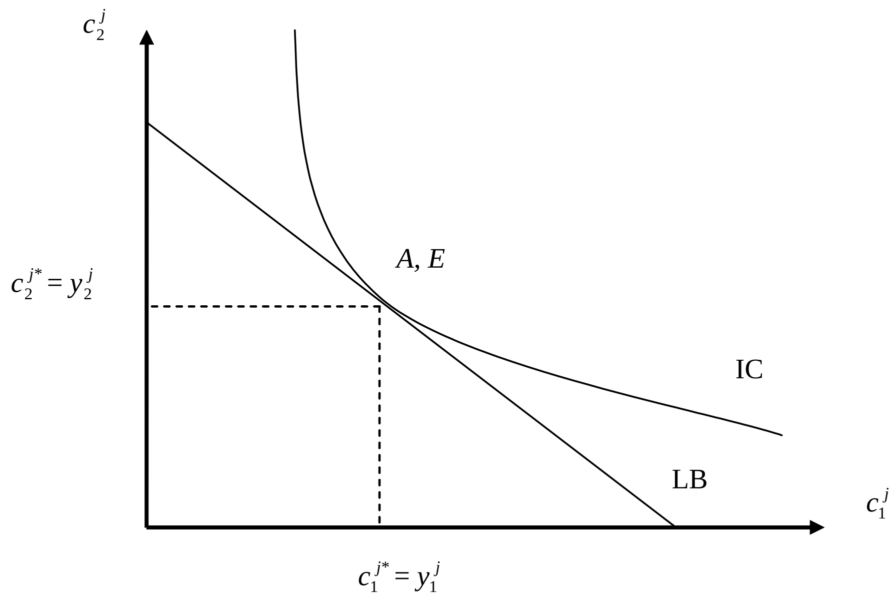
Case 1: Mr $j$ is a borrower




Case 2: Mr $j$ is a lender



Case 3: Mr $j$ is neither a borrower nor a lender




If an agent is maximizing his lifetime utility, the F.O.C., equation (1), must be satisfied, which means that $(c_1^*, c_2^*)$ is such that the highest possible indifference curve is tangent to the $LBC$.

$(iv)$ Using (2) with the appropriate superscripts,

$$\frac{\partial c_1^{j*}}{\partial y_1^j} = \frac{1}{\beta^2 (1+r) + 1} < 1 \text{ since } \beta \in (0,1) \text{ and } r > 0.$$