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Fully Funded v. Pay As You Go Social Security

In these notes we consider the effect on macroeconomic outcomes of different Social Security systems. Social Security Act was first introduced in 1935 mostly for income distribution reasons, its main purpose was to ensure a minimum level of income in retirement. The idea was that individuals might suffer from myopia and not save adequately for their retirement.

To study Social Security programs, we modify the two period model used to study the Ricardian Equivalence result to include overlapping generations of consumers. Each consumer still lives for 2 periods (young and old), but in each period a new generation of consumers is born.

		time period of birth					
		-1	0	1	2	3	4...
	0	o	y				
	1		o	y			
t, time	2			o	y		
period	3				o	y	
	4					o	y
	...						

Assume $N' = (1+n)\,N$, where N is the number of current old consumers and N' is the number of current young consumers. So, for each old there are $1+n$ young consumers. The government expenditure is 0 in each period. The role of the government is to conduct a social security program.

In what follows we will compare outcomes in the model economy without SS, with PAYG SS, and with FF SS. We will then describe when FF SS is a prefered system to PAYG and discuss whether SS reform can be paid for without making anyone worse off and making some people better off under the new system.

1 No Social Security Program

In this economy (without any SS program and government expenditure) each consumer solves

$$\begin{aligned} & \max_{c,c',s} U(c,c') && \text{(CP: No SS)} \\ & \text{subject to} \\ c+s &= y \\ c' &= y'+s(1+r) \end{aligned}$$

taking y, y', r as given. Each consumer's lifetime budget constraint is

$$c+\frac{c'}{1+r}=y+\frac{y'}{1+r}. \qquad \text{(BC with no SS)}$$

We denote $we^{No\ SS}=y+\frac{y'}{1+r}$.

Definition 1 *We say that the economy is dynamically inefficient whenever $n>r$, i.e. population growth exceeds the rate of return on savings.*

We will show below that if the economy is dynamically inefficent, there exists a transfer scheme that can improve everyone's utility.

2 Pay As You Go Social Security System

This is the system currently in place in the US. People currently working pay for social security benefits of the currently retired. We model this system by assuming that each young pays $t=b$ in social security taxes; each old then receives $b(1+n)$ in benefits since for each old, there are $1+n$ young consumers. So, we have that olds' taxes are negative, $t'=-b(1+n)$. Each person in a model with PAYG SS, solves

$$\begin{aligned} & \max_{c,c',s} U(c,c') && \text{(CP: PAYG SS)} \\ & \text{subject to} \\ c+s &= y-b \\ c' &= y'+s(1+r)+b(1+n) \end{aligned}$$

taking y, y', r as given. Each consumer's lifetime budget constraint is

$$c+\frac{c'}{1+r}=y-b+\frac{y'}{1+r}+\frac{b(1+n)}{1+r}. \qquad \text{(BC with PAYG SS)}$$

We denote $we^{PAYG} = y - b + \frac{y'}{1+r} + \frac{b(1+n)}{1+r}$.

Compare this BC to the BC in the model with no SS program. Clearly, consumers are better off with PAYG SS system as long as it increases their lifetime wealth, that is, as long as $we^{PAYG} > we^{NoSS}$, which simplifies to $\frac{b(1+n)}{1+r} > b$, and further to $n > r$. In other words, if the population growth rate is higher than the rate of return on savings (i.e. the economy is dynamically inefficient), the government can improve everyone's welfare by redistributing wealth across groups of consumers (by taxing individuals when young and redistributing the proceeds among the old).

Figure 1 below illustrates that in a situation with $n > r$, as a result of PAYG SS in place, consumers' lifetime budget constraint shifts out and enables him to achieve a higher level of welfare. Point A represents the optimal consumption bundle of consumers in the economy without SS, while Point B represents the optimal consumption bundle of consumers in the economy with PAYG SS and $n > r$.

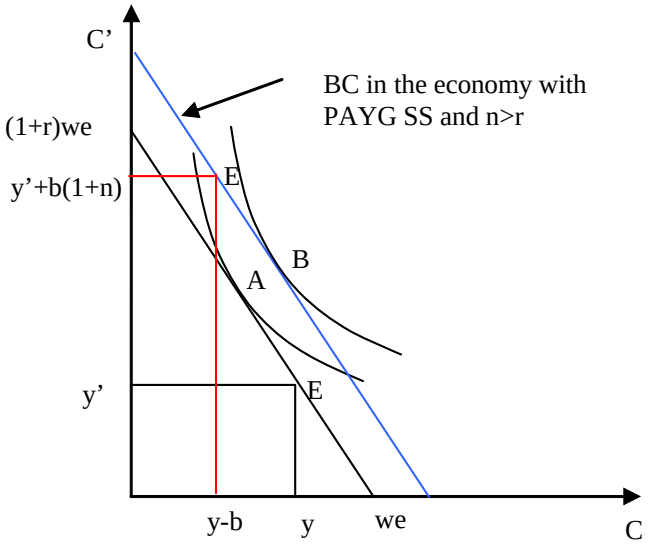

Figure 1. No SS v. PAYG SS, n>r

If, however, $n < r$, the PAYG SS system would lower consumers' lifetime wealth and decrease their welfare.

3 Fully Funded Social Security System

This type of system is usually referred to as privatized Social Security. The government requires individuals to save a certain amount and invests these savings into

these individuals' private accounts. Essentially, everyone gets the full gross return on their own savings. We model the FF SS system by assuming that each consumer is required to hand over b units of consumption good to the government when young. This b represents mandatory savings and is referred to as the social security tax. The government then invests the proceeds of the SS tax at rate r. Hence, the return on private savings is the same as the return on forced savings. Each person in a model with FF SS, solves

$$\begin{aligned} & \max_{c,c',s} U(c,c') && \text{(CP: FF SS)} \\ & \text{subject to} \\ c+s &= y-b \\ c' &= y'+s(1+r)+b(1+r) \end{aligned}$$

taking y, y', r as given. Compare this consumers' problem (CP: FF SS) with the consumers' problem under PAYG SS described by (CP: PAYG SS). The only difference is the return on b. We can think of the social security tax b as a part of total savings in both cases. Under PAYG system, the return on these savings is $1+n$ while under the FF system, the return is the same as the return on private savings $1+r$.

Each consumer's lifetime budget constraint is

$$\begin{aligned} c+\frac{c'}{1+r} &= y-b+\frac{y'}{1+r}+\frac{b(1+r)}{1+r}, \text{ i.e.} \\ c+\frac{c'}{1+r} &= y+\frac{y'}{1+r} && \text{(BC with FF SS)} \end{aligned}$$

Compare this constraint with the BC in the model with no SS system at all. They are the same. So, if negative private savings are allowed, then FF SS makes no difference at all. If a consumer in the world with no SS optimally saves 10 apples, then in the world with FF SS and the government demanding him to save 7 apples, he will save 3 apples privately and enjoy the same consumption in each period. If the government requires him to save 12 apples he will borrow 2 apples privately and still enjoy the same consumption in each period. If, however, borrowing is not allowed, the latter scenario would make the consumer worse off. Figure 2 below illustrates the situation in which the FF SS adversely affects consumers. In the world without SS, consumers choose A as the optimal consumption bundle and save s. When the government demands that consumers save b which is above their desired level of total savings, and if they are not allowed to borrow privately, then they are forced to a lower indifference curve (point B).

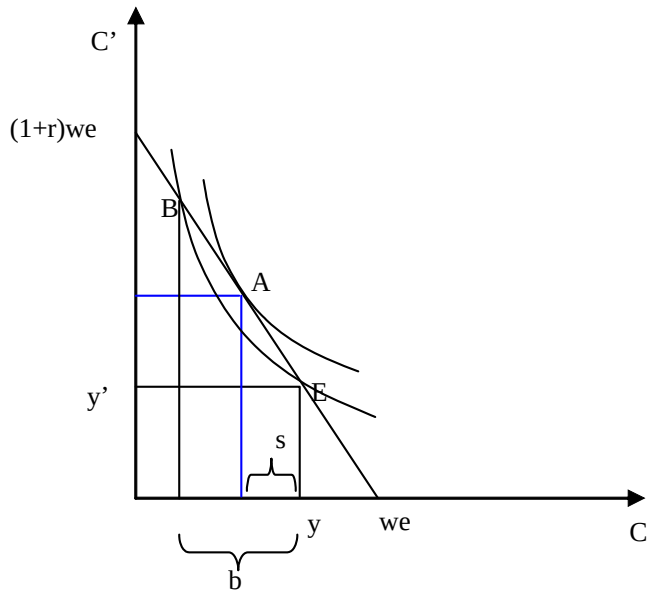

Figure 2. No SS v. FF SS with higher b than the total optimal level of savings (and no borrowing allowed).

The conclusion we can make is that the FF system is at best an ineffective program (relative to the outcomes in the model with no social security) while the PAYG SS system could increase consumers' welfare (when $n > r$) or decrease consumers' welfare (when $n < r$).

Most developed countries today have the PAYG SS system in place. In the view of very low population growth rates in these countries, the PAYG system represents a problem. Since $n < r$, a FF system would yield higher returns on workers' savings, a higher lifetime wealth and hence, a higher welfare. This, however, is not enough to make an argument for a switch to a FF system. As we read in Paul Krugman's article, the transition to the FF system is very costly, for the U.S., it is on the order of $3 trillion. The cost arises from the fact that the government would now have to pay for the retirement benefits of those who made no or too little contribution to their personal accounts. In the example below we will use a hypothetical economy with 0 population growth rate to compute how many generations it will take before the first benefits will be seen from privatizing SS.

Example: Social Security Reform

- Consider a hypothetical economy where $n = 0$ and $r = 0.05$. The government

oversees a PAYG SS system. It taxes each young in the amount of $b = 20$ and redistributes the proceeds equally among the old so that each old gets $b(1+n) = 20$. Suppose $y = 50$, $y' = 0$, $U(c, c') = \ln c + \ln c'$. Each person in this economy solves

$$\begin{aligned} & \max_{c,c'} \ln c + \ln c' \\ & \text{subject to} \\ c + s &= y - b \\ c' &= y' + s(1+r) + b(1+n) \end{aligned}$$

The two conditions that determine the optimal consumption choice are

$$\begin{aligned} & \begin{cases} \frac{c'}{c} = 1 + r \\ c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - b + \frac{(1+n)}{1+r} b \end{cases}, \text{ or} \\ & \begin{cases} \frac{c'}{c} = 1.05 \\ c + \frac{c'}{1.05} = 50 + 0 - 20 + 20/1.05 = 49.0476 \end{cases} \\ \Rightarrow \quad & \underline{c^* = 24.5238,\ c'^* = 25.75} \end{aligned}$$

With this PAYG system, the level of utility enjoyed by consumer is given by

$$U(c^*, c'^*) = \ln 24.5238 + \ln 25.75 = 6.4481.$$

- Next consider the same hypothetical economy but with FF SS system in place. The government taxes each young in the amount of $b = 20$ and invests the proceeds at the rate of r, the gross return is paid out to the same person when he is old. Each person in this economy solves

$$\begin{aligned} & \max_{c,c',s} U(c, c') \\ & \text{subject to} \\ c + s &= y - b \\ c' &= y' + (s+b)(1+r) \end{aligned}$$

The two conditions that determine the optimal consumption choice are given by

$$\begin{aligned} & \begin{cases} \frac{c'}{c} = 1 + r \\ c + \frac{c'}{1+r} = y + \frac{y'}{1+r} \end{cases}, \text{ or} \\ & \begin{cases} \frac{c'}{c} = 1.05 \\ c + \frac{c'}{1.05} = 50 \end{cases} \\ \Rightarrow \quad & \underline{c^* = 25,\ c'^* = 26.25} \end{aligned}$$

We have to make sure this is an interior solution (that b is smaller than the total savings). Here, the total savings $s + b = y - c = 50 - 25 = 25$. So, the person pays 20 to the government and invests 5 privately. The optimal consumption bundle is the same as in the case of no social security at all. The utility level enjoyed by consumers is given by

$$U(c^*, c'^*) = \ln 25 + \ln 26.25 = 6.4865.$$

- We just verified that this hypothetical economy is better off under a FF system (since $n < r$). If the status quo is the PAYG SS, then switching to the FF system would make all future generations better off except those people who are old at the time of the switch. These are the guys who paid their dues but receive no benefit. In what follows we ask the following question: *Is it possible to privatize social security in our hypothetical economy without making anyone worse off and making some people better off?*

- The idea is to allow the government to run a deficit (issue bonds) to pay for the benefits of those who are old at the time of the switch and pay it off by taxing future generations that benefit from privatizing SS. We require that the government balances its present value budget constraint (the present value of government expenditures must equal the present value of the tax revenues).

- We will also show that even if the government makes it its goal to pay off the debt as soon as possible, which means taxing its citizens after the reform at the level that extracts all of their benefit associated with the reform, it would still take infinitely many generations to pay for the reform. In other words, the reform cannot pay for itself. Let's show it now.

- Suppose the switch occurs at time $\bar{T}$. Suppose there are $\bar{N}$ old people at time $\bar{T}$. Then to make the old at the time of the switch be as well off as they would be with no reform, the government must pay their promised benefits of $b(1+n)$ per each old, so the present value of the transfers equal $\bar{N}b(1+n)$. This is the left hand side of the equation below. Let's assume that to pay off this debt, the government levies a tax t on future generations when they are young. The present value of tax revenues are given by the right hand side of the equation below. The equation is the government's balanced PV budget constraint:

$$\bar{N}b(1+n) = t\overbrace{\bar{N}(1+n)}^{\#\text{ of young in }\bar{T}} + t\overbrace{\bar{N}(1+n)^2}^{\#\text{ of young in }\bar{T}+1}/(1+r) + \frac{t\bar{N}(1+n)^3}{(1+r)^2} + ... + \frac{t\bar{N}(1+n)^x}{(1+r)^{x-1}},$$

where x denotes the last generations which is taxed.

- Let's find the tax which extracts all of the benefit associated with the reform. What is that t? We know the level of utility attained under the status quo system is 6.4481. A person born during or after the reform solves

$$\begin{aligned} & \max_{c,c'} \ln c + \ln c' \\ & \text{subject to} \\ c + \frac{c'}{1+r} &= y + \frac{y'}{1+r} - t \end{aligned}$$

The two conditions that determine the optimal consumption choice are given by

$$\begin{cases} \frac{c'}{c} = 1 + r \\ c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - t \end{cases}, \text{ or}$$

$$\begin{cases} \frac{c'}{c} = 1.05 \\ c + \frac{c'}{1.05} = 50 - t \end{cases}$$

$$\Rightarrow \quad c^* = 25 - \frac{t}{2}, \; c'^* = \left(25 - \frac{t}{2}\right) 1.05$$

The level of utility is

$$U\left(c^*, c'^*\right) = \ln\left(25 - \frac{t}{2}\right) + \ln\left(\left(25 - \frac{t}{2}\right) 1.05\right)$$

Setting this level equal to the utility level achieved without the reform we find out the max t that can be charged.

$$\begin{aligned} \ln\left(25 - \frac{t}{2}\right) + \ln\left(\left(25 - \frac{t}{2}\right) 1.05\right) &= 6.4481 \\ \ln\left[\left(25 - \frac{t}{2}\right)^2 1.05\right] &= 6.4481 \\ \left[\left(25 - \frac{t}{2}\right)^2 1.05\right] &= e^{6.4481} \\ t &= .95 \end{aligned}$$

Hence, if the government makes it a priority to pay off the debt as soon as possible, they will charge this tax.

- This tax could also be found by noting that by taxing future generations, we simply shift their BC back towards their pre-reform BC. In order to find the tax that yields the pre-reform utility level, we simply need to shift the BC all the way back, i.e. impose the tax that equates the new lifetime wealth to the pre-reform level. So we solve

$$\begin{aligned} y + \frac{y'}{1+r} - t &= y - b + \frac{y'}{1+r} + \frac{b(1+n)}{1+r}, \text{ i.e.} \\ -t &= -b + \frac{b(1+n)}{1+r} \\ t &= b - \frac{b(1+n)}{1+r} = \frac{b(r-n)}{1+r} = 20(0.05)/1.05 = .95 \end{aligned}$$

  Indeed, we get the same answer!

- We can then use the government's BC to compute how many generations it would take to pay off the current transfer $\bar{N}b(1+n)$.

$$\begin{aligned} &\bar{N}b(1+n) = \\ &= \bar{N}[t(1+n) + t(1+n)^2/(1+r) + t(1+n)^3/(1+r) + ... + t(1+n)^x/(1+r)^{x-1}] \end{aligned}$$

$$20 \approx .95 + .95\left(\frac{1}{1.05}\right) + .95\left(\frac{1}{1.05}\right)^2 + ... + .95\left(\frac{1}{1.05}\right)^{\infty}$$

  It takes infinitely many generations to pay off the transition cost of the SS reform. The main point of this exercise is to show that the transition is costly and when proposals are made in favor of privatizing social security they should focus a lot more on ways transition costs can be paid off.

- We could prove this point more generally by applying the formula for the infinite sum of a geometric time series. For your reference, the infinite sum of a geometric time series is $\sum_{t=0}^{\infty}\beta^t = \frac{1}{1-\beta}$. We know the tax is given by $\frac{b(r-n)}{1+r}$. Then the present value of tax revenues is

$$\bar{N}t(1+n) + \frac{\bar{N}t(1+n)^2}{1+r} + \frac{\bar{N}t(1+n)^3}{(1+r)^2} + ... =$$

$$\bar{N}t(1+n)\left[1 + \frac{1+n}{1+r} + \left(\frac{1+n}{1+r}\right)^2 + ...\right] =$$

$$\bar{N}t(1+n)\frac{1}{1-\frac{1+n}{1+r}} = \bar{N}\frac{b(r-n)}{1+r}(1+n)\frac{1}{\frac{r-n}{1+r}} = \bar{N}b(1+n),$$

i.e. exactly equal to the transfer required to pay for the current old.

The European Union is facing a slowdown of population growth, in fact, $n < 0$ in many of the European Union countries. So, a simple argument in favor of reforming the PAYG system to the FF system that we are now familiar with (rate of return on private savings being higher than the rate of return on a PAYG system) compels us of a need for a SS reform. One of the conditions of being a part of the European Union, however, is keeping the debt to GDP ratio below a certain level. This policy is now a major obstacle for implementing the SS reform because such a reform would require a large government debt.

Keep in mind that our model has nothing about redistributive aspects of the current PAYG system. The current PAYG SS system redistributes some wealth from the rich to the poor.