Suggested Solutions to EC2102 Macroeconomic Analysis I
Tutorial 3, Week 5 (February 8-12, 2010)

Question 1

(i) First period budget constraint is:

$$C_1 + S_1 = w_1 (h - l_1).$$

Second period budget constraint is:

$$C_2 = w_2 (h - l_2) + (1 + r) S_1$$

Combining the two period budget constraints, we get the lifetime budget constraint (LBC):

$$C_1 + \frac{C_2}{1+r} = w_1 (h - l_1) + \frac{w_2 (h - l_2)}{1+r}.$$

(**Note**: This question is a partial equilibrium question, as it only says that the consumer takes w_t and r_t as given. If you were to think of this question as having a representative firm which is wholly owned by the representative consumer, who thus gets dividends, and if you wrote the above budget constraints with dividends, that is correct also.)

(ii) Expressing C_1 as a function of C_2, l_1, l_2 from LBC:

$$C_1 = w_1 (h - l_1) + \frac{w_2 (h - l_2)}{1+r} - \frac{C_2}{1+r} \tag{1}$$

Hence, his utility maximization problem is:

$$\max_{C_2, l_1, l_2} u (C_1 (C_2, l_1, l_2), l_1) + \beta u (C_2, l_2)$$
$$where\ C_1 = w_1 (h - l_1) + \frac{w_2 (h - l_2)}{1+r} - \frac{C_2}{1+r}$$

(iii)

$$\max_{l_1,l_2,C_2} u\left(C_1\left(C_2,l_1,l_2\right),l_1\right)+\beta u\left(C_2,l_2\right)$$

$$C_1 = w_1\left(h-l_1\right)+\frac{w_2\left(h-l_2\right)}{1+r}-\frac{C_2}{1+r}$$

$$\begin{aligned} F.O.C.\left(l_1\right) &: u_1(C_1^*,l_1^*)\frac{\partial C_1}{\partial l_1}+u_2(C_1^*,l_1^*)=0 \qquad (2)\\ &\Leftrightarrow u_1(C_1^*,l_1^*)\left(-w_1\right)+u_2(C_1^*,l_1^*)=0\\ F.O.C.\left(l_2\right) &: u_1(C_1^*,l_1^*)\frac{\partial Cc_1}{\partial l_2}+\beta u_2(C_2^*,l_2^*)=0 \qquad (3)\\ &\Leftrightarrow u_1(C_1^*,l_1^*)\left(-\frac{w_2}{1+r}\right)+\beta u_2(C_2^*,l_2^*)=0\\ F.O.C.\left(c_2\right) &: u_1(C_1^*,l_1^*)\frac{\partial C_1}{\partial c_2}+\beta u_1(C_2^*,l_2^*)=0 \qquad (4)\\ &\Leftrightarrow u_1(C_1^*,l_1^*)\left(-\frac{1}{1+r}\right)+\beta u_1(C_2^*,l_2^*)=0 \end{aligned}$$

(iv) Equation (2) :

$$\begin{aligned} u_1(C_1^*,l_1^*)\left(w_1\right) &= u_2(C_1^*,l_1^*)\\ w_1 &= \frac{u_2(C_1^*,l_1^*)}{u_1(C_1^*,l_1^*)}\\ -w_1 &= -\frac{u_2(C_1^*,l_1^*)}{u_1(C_1^*,l_1^*)}, \end{aligned}$$

which is just equation (II).

Equation (4) :

$$\begin{aligned} u_1(C_1^*,l_1^*)\left(\frac{1}{1+r}\right) &= \beta u_1(C_2^*,l_2^*)\\ \frac{u_1(C_1^*,l_1^*)}{\beta u_1(C_2^*,l_2^*)} &= 1+r \qquad (5)\\ -\frac{u_1(C_1^*,l_1^*)}{\beta u_1(C_2^*,l_2^*)} &= -\left(1+r\right) \end{aligned}$$

which is just equation (I)

Equation (3) :

$$\begin{aligned} u_1(C_1^*,l_1^*)\left(\frac{w_2}{1+r}\right) &= \beta u_2(C_2^*,l_2^*)\\ w_2 &= \frac{\beta u_2(C_2^*,l_2^*)}{u_1(C_1^*,l_1^*)}\left(1+r\right), \end{aligned}$$

but we can use equation (5), and hence,

$$
\begin{aligned}
w_2 &= \frac{\beta u_2(C_2^*, l_2^*)}{u_1(C_1^*, l_1^*)} \frac{u_1(C_1^*, l_1^*)}{\beta u_1(C_2^*, l_2^*)} = \frac{u_2(C_2^*, l_2^*)}{u_1(C_2^*, l_2^*)}, or \\
-w_2 &= -\frac{u_2(C_2^*, l_2^*)}{u_1(C_2^*, l_2^*)},
\end{aligned}
$$

which is just equation (III).

(v) $LHS\,(I)$ is the MRS between C_1 and C_2, evaluated at the optimum, and $RHS\,(I)$ is the relative cost of consuming today versus tomorrow (with a minus sign). Equation (I) thus says that in equilibrium, the agent's rate of tradeoff between (C_1, C_2) has to equal the relative cost of (C_1, C_2).

$LHS\,(II)$ is the MRS between l_1 and C_1, evaluated at the optimum, and $RHS\,(II)$ is the relative cost of consuming leisure today versus consumption today (with a minus sign). Equation (II) thus says that in equilibrium, the agent's rate of tradeoff between (l_1, C_1) has to equal the relative cost of (l_1, C_1).

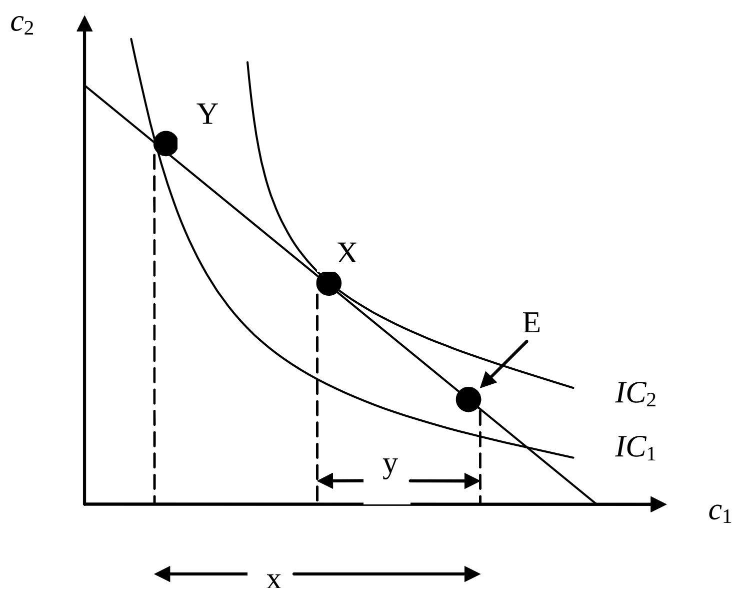
$LHS\,(III)$ is the MRS between l_2 and C_2, evaluated at the optimum, and $RHS\,(III)$ is the relative cost of consuming leisure tomorrow versus consumption tomorrow (with a minus sign). Equation (III) thus says that in equilibrium, the agent's rate of trade-off between (l_2, C_2) has to equal the relative cost of (l_2, C_2).

Question 2

A fully funded (FF) social security system basically involves asking each generation to save for its own retirement, unlike pay-as-you-go systems. An example of this is the CPF (Central Provident Fund) where each agent in the economy saves for his retirement in his own private account without any added contribution by other agents. I will use CPF and FF interchageably. The FF system is can be thought of as a "forced savings" programme.

If this FF system does not bind for an agent, what it means is that this agent was already planning on saving an amount in excess of what the FF system mandates, and thus, his savings-consumption decisions are unaffected. For instance, if the FF system requires that he saves x, but the individual was already planning on saving $y > x$, then what this individual would do is to save x with the FF system, then save the rest, $(y - x) > 0$ by himself. It is in this sense that the FF system does not bind.

If this FF system binds, what this means is that this agent was planning on saving an amount y which was less than what is mandated by the FF system, x, so this would distort the agent's decisions on consumptions and savings. In this case, since $y < x$, but since the system mandates that the agent saves x, this means that the agent has to scale back his consumption from what he would have chosen otherwise. This can be clearly seen from this diagram, which illustrates his intertemporal consumption choices over time periods 1 and 2, but is enough to illustrate the point at hand,:



Ths agent's optimal consumption bundle is at point X, where his budget line is tangent to the highest possible Indifference Curve. But he is now forced, because he has to save x, to consume at point Y. This makes him worse off as he is now on a lower Indifference Curve.

The intuition why the FF system makes people worse off is that people choose their savings optimally in any case, so by creating a FF system which "forces" people to save, either they were saving beyond the mandated amount, or they were not, in which case, they must be worse off since they were choosing savings optimally in the first place.