

## Examples Williamson Goes Through:

1. Equilibrium effects of an increase in government expenditures today  $G_1$
2. Equilibrium effects of a persistent increase in government expenditures
3. Equilibrium effects of a decrease in current capital stock  $K_1$
4. Equilibrium effects of an increase in current TFP  $z_1$
5. Equilibrium effects of an increase in future TFP, say  $z_2$
6. Sectoral shock

## Example 1: Effect of an increase in $G_1$

Examples: any temporary changes like fighting a quick war, increased spending due to natural disasters

$\Delta G_1 > 0 \implies \nearrow$  in  $PV$  of govt expenditures  $\implies \nearrow$  in  $PV$  of taxes as well.

Why?

Government's LBC:

$$\underbrace{G_1 + \frac{G_2}{1+r_1} + \dots}_{PV \text{ of expenditures}} = \underbrace{T_1 + \frac{T_2}{1+r_1} + \dots}_{PV \text{ of taxes, } \tau}$$

Hence,

$$\Delta G_1 = \Delta \tau$$

$\implies \Delta \omega^d = -\Delta \tau < 0$ . This implies:

Labour Market:

$N_1^s(r_1^*)$  curve shifts to the right to  $\tilde{N}_1^s(r_1^*)$  ( $\omega^d$  falls and consume less leisure which is a normal good, that is, work more), and

Goods Market:

Supply:  $Y_1^s$  curve shifts to the right to  $\tilde{Y}_1^s$  because  $N_1^s(r_1^*)$  shifted to the right

Demand: Two effects:

1. Government spends more:

$$\Delta G_1 > 0$$

2. Because consumer's lifetime disposable wealth has decreased,  $Y_1^d$  is affected by a change in consumption:

- direct decrease in  $C_1$ ,
- magnitude of decrease in  $C_1$ ?  $\Delta \omega^d = -\Delta \tau = -\Delta G_1 < 0$ ,

$$\Delta C_1 = MPC \Delta \omega^d = -MPC \Delta G_1 < 0.$$

Overall change in  $Y_1^d$ ?

Recall that we saw in the last lecture for some exogenous  $\Delta E$ ,

$$\Delta Y_1^d = \left( \frac{1}{1 - MPC} \right) \Delta E,$$

so here,

$$\begin{aligned}
 \Delta Y_1^d &= \left( \frac{1}{1 - MPC} \right) (\Delta G_1 + \Delta C_1) \\
 &= \left( \frac{1}{1 - MPC} \right) (\Delta G_1 - MPC \Delta G_1) \\
 &= \left( \frac{1}{1 - MPC} \right) \Delta G_1 (1 - MPC) \\
 &= \Delta G_1.
 \end{aligned}$$

This means that  $Y_1^d$  changes by  $\Delta G_1$ , i.e.,  $Y_1^d$  shifts out to the right to  $\tilde{Y}_1^d$ .

Unambiguously, equilibrium  $Y_1$  increases. But it looks like equilibrium  $r_1$  could increase or decrease.

As temporary change in  $G_1$ , effect on lifetime wealth small, so  $N_1^S$  and hence  $Y_1^S$  don't change by much. And  $Y_1^d$  changes by  $\Delta G_1$ . Since change in lifetime wealth small, so effect on  $Y_1^d$  relatively large.

So  $Y_1^d$  shifts to the right by more than  $Y_1^S$ .

At the original equilibrium interest rate of  $r_1^*$ , output demand (point A on  $\tilde{Y}_1^d$ ) exceeds output supply (point B on  $\tilde{Y}_1^S$ ). Hence, goods market cannot be in equilibrium. To restore equilibrium, interest rate needs to rise because a rise in interest rate means that consumers want to save more, so they consume less today,  $C_1$  falls (substitution effect of interest rate change dominates), and a rise in interest rate means that the rate of return on alternative asset to firms (bonds) increases, so  $I_1$  falls. Hence, there is a movement along the  $\tilde{Y}_1^d$  curve upwards from point A.

At the same time, as interest rate rises, representative consumer works more, since the price of leisure today is more expensive relative to the price of leisure tomorrow and this substitution effect dominates, so  $\tilde{N}_1^S(r_1^*)$  starts to shift to the right, and thus causes a movement up along the  $\tilde{Y}_1^S$  curve from point B.

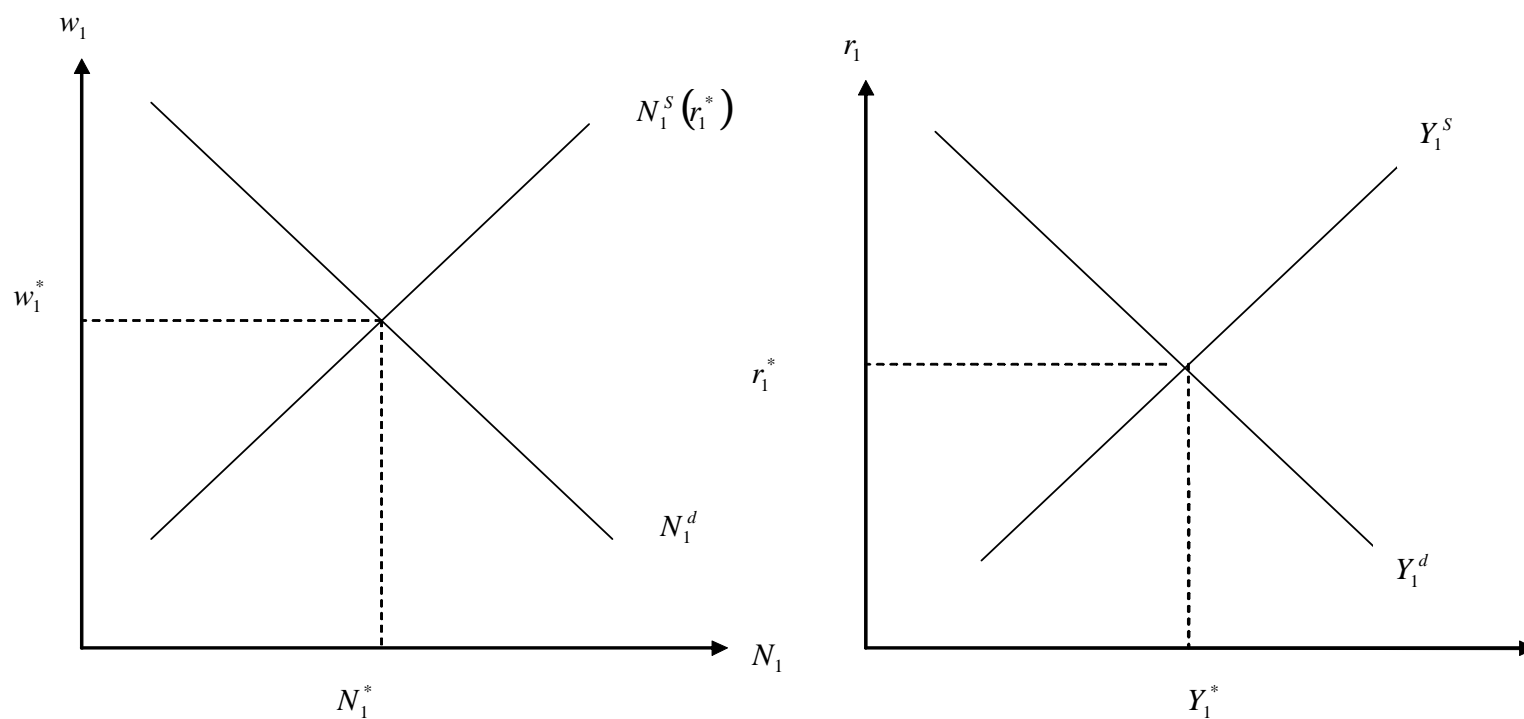
Equilibrium in the goods market is restored when equilibrium  $r_1$  has risen sufficiently so that goods demanded equals good supplied at point C, where the equilibrium  $r_1$  has risen from  $r_1^*$  to  $\tilde{r}_1^*$ , and equilibrium output increases from  $Y_1^*$  to  $\tilde{Y}_1^*$ ; and in the labour market,  $\tilde{N}_1^S(r_1^*)$  has shifted to the right to  $\tilde{N}_1^S(\tilde{r}_1^*)$ , where labour supplied equals labour demanded at the new wage rate which has fallen from  $w_1^*$  to  $\tilde{w}_1^*$ , and  $N_1$  has risen from  $N_1^*$  to  $\tilde{N}_1^*$ .

There is a “**crowding out**” effect of  $G_1$  increase on  $C_1$  and  $I_1$  :

- $C_1$  falls due to  $\searrow$  in  $\omega^d$ , falls due to  $\nearrow$  in equilibrium  $r_1$  from  $r_1^*$  to  $\tilde{r}_1^*$ , so overall:  $C_1 \searrow$
- $I_1$  falls due to  $\nearrow$  in  $r_1$ . Less investment now means lower production capacity in the future.

In equilibrium: a temporary  $\nearrow$  in government expenditures leads to a  $\nearrow$  in current output, an  $\nearrow$  in real interest rate, an  $\nearrow$  in current employment, a  $\searrow$  in real wages, a  $\searrow$  in current investment, and a  $\searrow$  in current consumption.

Figure "9.16" An Increase in  $G_1$  on labour and goods market equilibria





## Example 2: Effect of a permanent increase in $G$

Example: permanent terrorist threat, so funding for counter-terrorism measures increases permanently

$\Delta G_1 = \Delta G_2 = \Delta G_t = \Delta G > 0$  for all  $t$  means that PV of govt expenditures  $\nearrow \implies$  PV of taxes  $\nearrow$  as well since

$$\Delta G_1 + \frac{\Delta G_2}{1 + r_1} + \frac{\Delta G_3}{(1 + r_1)(1 + r_2)} \dots = \Delta \tau$$

$\implies \Delta \omega^d = -\Delta \tau < 0$ . This implies:

Labour Market:

$N_1^s$  curve shifts to the right from  $N_1^s(r_1^*)$  to  $\widetilde{N}_1^s(r_1^*)$  (because  $\omega^d \searrow$ )

Goods Market:

Supply:  $Y_1^S$  shifts to the right to  $\tilde{Y}_1^S$  because  $N_1^s(r_1^*)$  shifted to  $\tilde{N}_1^s(r_1^*)$

Demand: 3 effects

1: direct increase from government of  $\Delta G_1$

2: we know that the PV of taxes  $\nearrow$  by  $\Delta\tau$ . Because consumer's lifetime disposable wealth has decreased,  $Y_1^d$  is affected by a change in consumption:  
 $\Delta\omega^d = -\Delta\tau = -\left(\Delta G_1 + \frac{\Delta G_2}{1+r_1} + \dots\right) < 0,$

$$\Delta C_1 = MPC \Delta\omega^d = -MPC \left( \Delta G_1 + \frac{\Delta G_2}{1+r_1} + \dots \right) < 0.$$

3. Increase in future real income of

$$\Delta Y = \frac{\Delta Y_2}{1 + r_1} + \frac{\Delta Y_3}{(1 + r_1)(1 + r_2)} + \dots$$

Overall,

$$\begin{aligned} & \Delta Y_1^d \\ = & \left( \frac{1}{1 - MPC} \right) \Delta G_1 \\ & - \left( \frac{1}{1 - MPC} \right) \left[ MPC \left( \Delta G_1 + \frac{\Delta G_2}{1 + r_1} + \frac{\Delta G_3}{(1 + r_1)(1 + r_2)} + \dots \right) \right] \\ & + \left( \frac{1}{1 - MPC} \right) \left[ MPC \left( \frac{\Delta Y_2}{1 + r_1} + \frac{\Delta Y_3}{(1 + r_1)(1 + r_2)} + \dots \right) \right] \\ = & \Delta G_1 + \left( \frac{MPC}{1 - MPC} \right) \left[ \frac{\Delta Y_2 - \Delta G_2}{1 + r_1} + \frac{\Delta Y_3 - \Delta G_3}{(1 + r_1)(1 + r_2)} + \dots \right] \end{aligned}$$

Let us conjecture that  $\Delta Y_t = \Delta G_t$  every period from  $t \geq 2$ . That is, in all future periods, each of those periods' income would increase by the same amount as government spending. If this is so, then

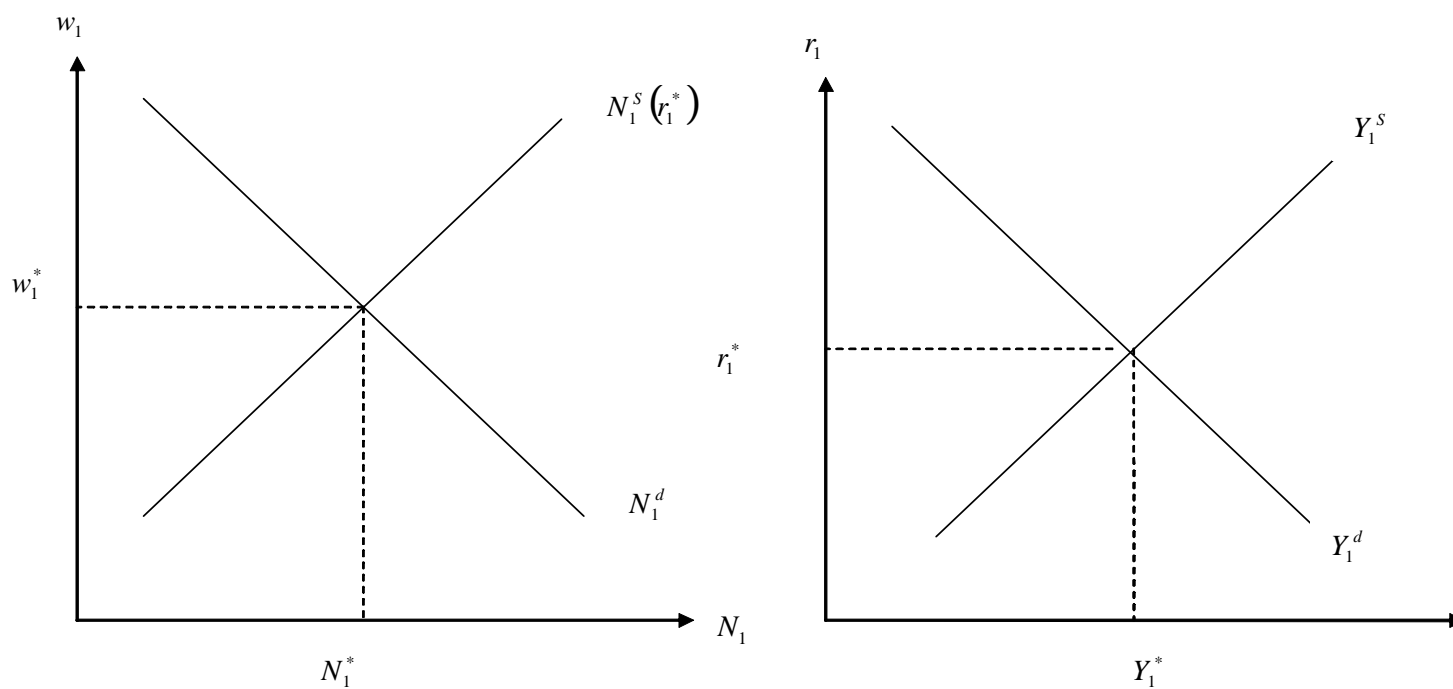
$$\Delta Y_1^d = \Delta G_1 = \Delta G.$$

Since  $Y_1^d$  shifted to the right to  $\tilde{Y}_1^d$ , and  $Y_1^S$  shifts to the right to  $\tilde{Y}_1^S$ , real output today must have increased, though it appears that  $r_1$  may rise or not. However,  $r_1$  does not change, because the change in government expenditures is permanent, so consumption every period is affected in the same way, and hence, there is no incentive to change savings to smooth consumption over time, implying no change in credit market. Hence,  $r_1$  remains unchanged, and  $\Delta Y_1 = \Delta Y_1^d = \Delta G$ . Since there is no change in investment, no change in future capital stock. Hence, nothing in the future has changed except that for all future periods only government expenditures change, so it must be that in all future periods  $\Delta Y_t = \Delta G_t$ , which proves the conjecture.

In equilibrium: a permanent  $\nearrow$  in government expenditures leads to:  
real current output  $\nearrow$ , real interest rate is unchanged, consumption is unchanged, investment is unchanged, real current wage rate has  $\downarrow$ , and current employment  $\nearrow$

So here, no crowding out effect.  $C_1$  and  $I_1$  remain unchanged. The entire cost of permanent change in government expenditures has resulted in a decrease in leisure, where the representative consumer worked harder to support higher government spending.

Figure "9.18" A Permanent Increase in Government Expenditures on equilibria in Goods and Labour markets



## Example 3: Effect of an increase in $z_1$

Examples: favourable weather, favourable change in government regulations; basically any factor that allows you to produce more with the same amount of inputs for a short period of time.

When  $\Delta z_1 > 0$

Labour Market:

$N_1^d$  shifts to the right from  $N_1^d$  to  $\tilde{N}_1^d$  because  $MPN_1 \nearrow$  and firm increases demand for labour

Goods Market:

Supply:  $Y_1^S$  shifts out to  $\tilde{Y}_1^S$  due to  $N_1^d$  shifting out from  $N_1^d$  to  $\tilde{N}_1^d$

Demand: no change in  $Y_1^d$

At the original equilibrium interest rate of  $r_1^*$ , output demand (point A on  $Y_1^d$ ) is less than output supply (point B on  $\tilde{Y}_1^s$ ). Hence, goods market cannot be in equilibrium. To restore equilibrium, interest rate needs to fall, because a fall in interest rate means that consumers want to save less, so they consume more today,  $C_1$  increases (substitution effect of interest rate change dominates), and a fall in interest rate means that the rate of return on alternative asset to firms (bonds) decreases, so  $I_1$  increases. Hence, there is a movement along the  $Y_1^d$  curve downwards from point A.

At the same time, as interest rate falls, representative consumer works less, since the price of leisure today is less expensive relative to the price of leisure tomorrow and this substitution effect dominates, so  $N_1^S(r_1^*)$  starts to shift to the left, and thus causes a movement down along the  $\tilde{Y}_1^s$  curve from point B.

Equilibrium in the goods market is restored when equilibrium  $r_1$  has fallen sufficiently so that goods demanded equals good supplied at point C, where



the equilibrium  $r_1$  has fallen from  $r_1^*$  to  $\tilde{r}_1^*$ , and equilibrium output increases from  $Y_1^*$  to  $\tilde{Y}_1^*$ .

But in the labour market, as interest rate rises, labour supplied falls from  $N_1^s(r_1^*)$  to  $\tilde{N}_1^s(\tilde{r}_1^*)$ . Equilibrium wage rate rises unambiguously from  $w_1^*$  to  $\tilde{w}_1^*$ , but the net effect on current employment is ambiguous

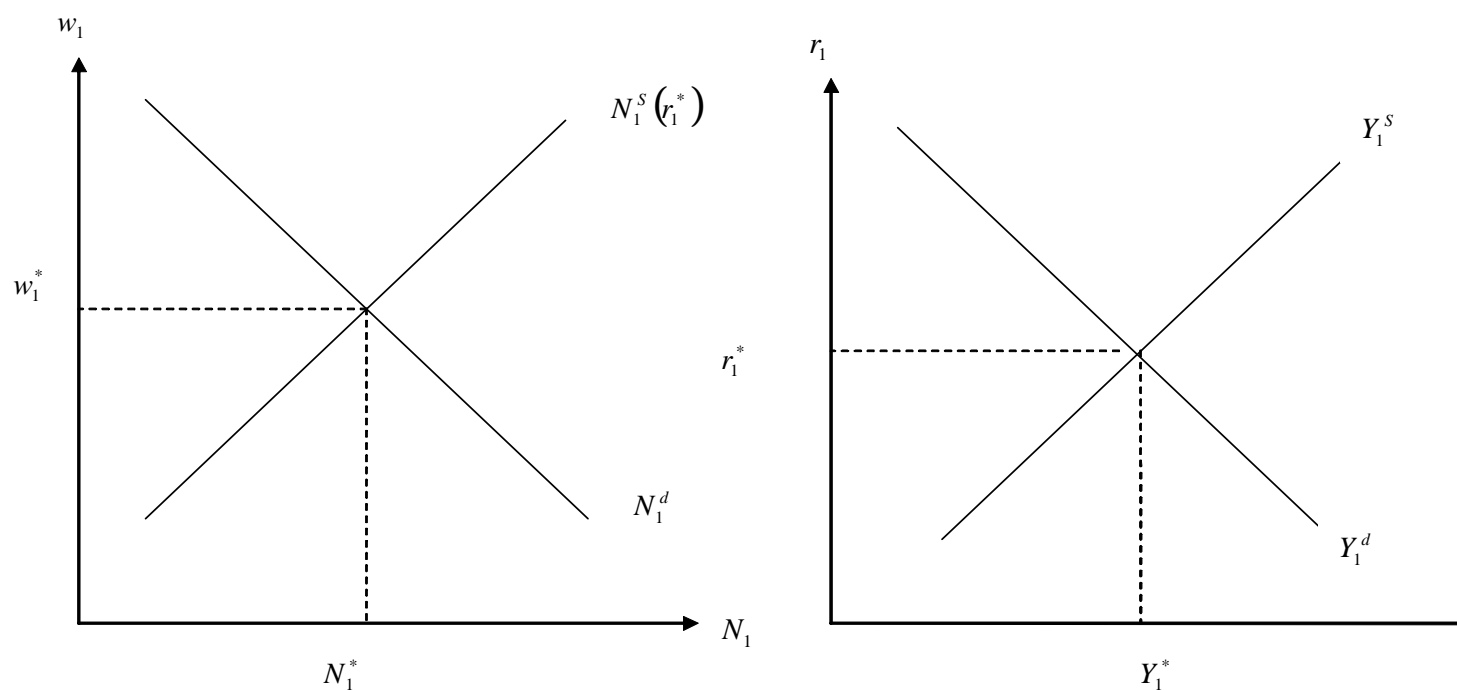
But data tells us that real interest rate effect on labour supply is small, so overall, employment will rise from  $N_1^*$  to  $\tilde{N}_1^*$ .

Story: increase in  $z_1$  increases labour demand from  $N_1^d$  to  $\tilde{N}_1^d$ , so movement along the labour supply curve  $N_1^s(r_1^*)$  means an increase in wages as well as higher current employment. This then increases the quantity of goods that can be supplied in the goods market, as labour supply increases from  $Y_1^s$  to  $\tilde{Y}_1^s$ . This causes a fall in real interest rate from  $r_1^*$  to  $\tilde{r}_1^*$ , resulting in a decrease in labour supply from  $N_1^s(r_1^*)$  to  $N_1^s(\tilde{r}_1^*)$ , which is smaller than the increase

in labour demand. Consequently, real wages and current employment rise in the labour market. In the goods market, consumption increases following the fall in interest rate and increase in current income, and investment rises due to the fall in interest rate. In equilibrium in the goods market, the increase in output demanded via increasing  $C_1$  and  $I_1$  equals the increase in output supplied (movement along the  $Y^d$  curve as  $Y^s$  rises).

Note: later on, we will see that temporary shocks to TFP is a good candidate in accounting for business cycle fluctuations.

"Figure 9.21" Change in  $z_1$  on equilibria in Goods and Labour markets



## Example 4: Effect of an expected increase in future *TFP*, say $z_2$

Let  $\Delta z_2 > 0$

Examples: anticipate new sources of fuel, anticipate new production processes that make production more efficient in the future.

Goods Market:

increase in  $z_2$  means higher  $MPK_2$ , so higher return to investment, so  $I_1$  increases

investment component of  $Y_1^d \nearrow$ , so  $Y_1^d \nearrow$  from  $Y_1^d$  to  $\hat{Y}_1^d$

Recall that  $w_1 = MPN_1 = z_1 F_2(K_1, N_1)$ ? Likewise, tomorrow,  $w_2 = MPN_2 = z_2 F_2(K_2, N_2)$

$\nearrow$  in tomorrow's wage means  $\nearrow$  in lifetime wealth  $\Rightarrow \nearrow$  in today's consumption, so consumption component of  $Y_1^d \nearrow$ , and multiplier effect, so let  $Y_1^d$  shift further out from  $\hat{Y}_1^d$  to  $\tilde{Y}_1^d$

### Labour Market:

- increase in  $\omega^d$  implies  $N_1^s$  shifts in to  $\tilde{N}_1^s(r_1^*)$ , but this causes a leftward shift of  $Y_1^S$  to  $\tilde{Y}_1^S$ .

As the increase in wealth is small, this leftward shift of  $Y_1^S$  to  $\tilde{Y}_1^S$  is small relative to the rightward shift of  $Y_1^d$  to  $\tilde{Y}_1^d$ .

At the original equilibrium interest rate of  $r_1^*$ , output demand (point A on  $\tilde{Y}_1^d$ ) exceeds output supply (point B on  $\tilde{Y}_1^S$ ). Hence, goods market cannot be in equilibrium. To restore equilibrium, interest rate needs to rise because a rise in interest rate means that consumers want to save more, so they consume less

today,  $C_1$  falls (substitution effect of interest rate change dominates), and a rise in interest rate means that the rate of return on alternative asset to firms (bonds) increases, so  $I_1$  falls. Hence, there is a movement along the  $\tilde{Y}_1^d$  curve upwards from point A.

At the same time, as interest rate rises, representative consumer works more, since the price of leisure today is more expensive relative to the price of leisure tomorrow and this substitution effect dominates, so  $\tilde{N}_1^S(r_1^*)$  starts to shift to the right, and thus causes a movement up along the  $\tilde{Y}_1^s$  curve from point B.

Equilibrium in the goods market is restored when equilibrium  $r_1$  has risen sufficiently so that goods demanded equals good supplied at point C, where the equilibrium  $r_1$  has risen from  $r_1^*$  to  $\tilde{r}_1^*$ , and equilibrium output increases from  $Y_1^*$  to  $\tilde{Y}_1^*$ .

In the labour market, the small wealth effect must be dominated by the rightward shift of labour supply through an interest rate change, so the rightward shift of  $\tilde{N}_1^S(r_1^*)$  to  $\tilde{N}_1^S(\tilde{r}_1^*)$ . This is because  $Y_1^*$  increases to  $\tilde{Y}_1^*$ , and since  $z_1$  and  $K_1$  have not changed, an increase in current output must be through an increase in current employment. Hence, labour supplied equals labour demanded at the new wage rate which has fallen from  $w_1^*$  to  $\tilde{w}_1^*$ , and  $N_1$  has risen from  $N_1^*$  to  $\tilde{N}_1^*$ .

In equilibrium: an expected increase in future  $TFP$  leads to  $\nearrow$  in real interest rate and  $\nearrow$  in current output, an  $\nearrow$  in current employment, a fall in equilibrium wages, and an  $\nearrow$  in current investment. Overall effect on current consumption is ambiguous.

- Current investment  $\nearrow$  because even though it falls due to  $\nearrow$  in real interest rate and rises due to an  $\nearrow$  in  $z_2$ , overall investment must have risen because that was the original shock to the economy.
- Consumption is ambiguous because it falls due to  $\nearrow$  in real interest rate, rises to  $\nearrow$  in current income, and rises due to  $\nearrow$  in future income

Figure "9.22" Equilibrium effects of an expected change in  $z_2$ 