Quick recap of representative firm's problem (1/2)

The representative firm is owned by the representative consumer.
- as shareholder, consumer receives dividends π_t from the firm every period, in real terms. This is his capital income

The representative firm employs the representative consumer/worker, whose wage rate is w_t, in real terms
- as worker who puts in N_t units of time, his labor income is $N_t w_t$

The representative firm uses labor and capital to produce a (final) good y_t every period, in real terms.

This firm at time t has K_t units of capital, and has to decide how much labour to hire, and how much capital stock it wants in period $(t+1)$.

$$K_{t+1} = (1-\delta) K_t + I_t$$

Quick recap of representative firm's problem (2/2)

How is capital produced?
- the firm can produce one unit of capital using one unit of output

What is this final good?
- It is output supplied of consumption goods.

Who buys this output of consumption goods?
- the representative consumer demands consumption goods

In our model, representative firm hires worker, produces consumption goods, and is owned by representative consumer.
Rrepresentative consumer works, buys consumption goods, and receives dividends from owning 100% of representative firm

Representative Firm's Max. Problem (1/4)

$$\max_{N_1,K_2} \pi_1 + \frac{\pi_2}{1+r_1} + \ldots$$

$$\max_{N_1,K_2} z_1 F(K_1,N_1) - w_1 N_1 - K_2 + (1-\delta)K_1 + \frac{z_2 F(K_2,N_2)}{1+r_1} + \frac{(1-\delta)K_2}{1+r_1} \underbrace{-\frac{w_2 N_2}{1+r_1} - \frac{K_3}{1+r_1}}_{\text{we don't care about these terms today}} + \ldots$$

Taking the $F.O.C.s$:

$$F.O.C.(N_1) \quad : \quad z_1 F_2(K_1,N_1) - w_1 = 0$$

$$F.O.C.(K_2) \quad : \quad -1 + \frac{z_2 F_1(K_2,N_2) + 1 - \delta}{1+r_1} = 0,$$

which after rearranging gives

Representative Firm's Max. Problem (2/4)

$$F.O.C.(N_1) \quad : \quad \underbrace{z_1 F_2(K_1, N_1)}_{MPN_1} = w_1, \tag{24}$$

$$F.O.C.(K_2) \quad : \quad \underbrace{z_2 F_1(K_2, N_2) - \delta}_{\substack{MPK_2 - \delta, \text{ or net } MPK_2 \\ \text{return to inv., net of } \delta}} = \underbrace{r_1}_{\text{OC of inv.}} \tag{25}$$

opportunity cost of investment is the rate of return on alternative asset, i.e., what you get on your savings, r_1.

Two assets in the economy: bonds traded in credit market, and capital held by representative firm. Since consumer owns firm, it's as if consumer owns capital. Firm acts in consumer's interests, and will invest up to the point where marginal benefit of investment (net MPK_2) exactly equals the marginal cost of investment (r_1)

Representative Firm's Max. Problem (3/4)

Equation (25) : (from $F.O.C.(K_2)$)

$$MPK_2 - \delta = z_2 F_1(K_2, N_2) - \delta = r_1$$

If r_1 increases, for equation (25) to hold, $F_1(K_2, N_2)$ needs to increase, and since total output increases at a decreasing rate in capital, K_2 needs to fall, i.e., I_1 needs to fall, as illustrated in figure "9.6".

For equation (24) : (from $F.O.C.(N_1)$)

$$MPN_1 = z_1 F_2(K_1, N_1) = w_1$$

if w_1 rises, $z_1 F_2(K_1, N_1)$ needs to rise, and since total product is increasing at a decreasing rate in labour, N_1 needs to fall; demand less labour today

Representative Firm's Max. Problem (4/4)

But MPN_1 is N_1^d because firm's optimal decision for labour demand is where $MPN_1 = w_1$ from equation (24). Because representative consumer/worker, higher labour demand means higher demand for more hours worked in a period. Hence, figure "9.4".

Representative Firm: Comparative Statics

Change in current TFP, z_1:

Labour demand: $\nearrow z_1 \Longrightarrow MPN_1 \nearrow \Longrightarrow \nearrow$ return to labour $\Longrightarrow$ demand more labour today at every wage rate, so MPN_1 curve shifts out for any given wage rate. (figure "9.5")

Expected change in TFP in second period , z_2:

Investment: $\nearrow z_2 \Longrightarrow MPK_2 \nearrow \Longrightarrow \nearrow$return to investment $\Longrightarrow$invest more today, so net MPK_2 curve shifts out for any given r_1. (figure "9.7")

The Government

- Expenditures: G_t
- Levy lump-sum taxes: T_t
- Can issue debt at the end of each period: B_t

Budget constraints are:

- period t: $G_t + (1 + r_{t-1})B_{t-1} = T_t + B_t$
- lifetime BC:

$$G_1 + \frac{G_2}{1+r_1} + \frac{G_3}{(1+r_1)(1+r_2)} ... = T_1 + \frac{T_2}{1+r_1} + \frac{T_3}{(1+r_1)(1+r_2)} + ... \quad (26)$$

Competitive Equilibrium (1/2)

Given that the government is spending and taxing (G_t, T_t) for all t such that its LBC holds, a competitive equilibrium here is an allocation $\left(C_t^*, S_t^*, N_t^{S*}, N_t^{D*}, I_t^*\right)$ and prices (r_t^*, w_t^*), for all t, such that:

- the representative consumer chooses $\left(C_t^*, S_t^*, N_t^{S*}\right)$ each period in order to maximize lifetime utility, taking (r_t^*, w_t^*) as given;

- the representative firm maximizes its present value of profits by choosing $\left(N_t^{D*}, I_t^*\right)$ each period, taking (r_t^*, w_t^*) as given; and

Competitive Equilibrium (2/2)

- the labour market and the goods market clear at (r_t^*, w_t^*):
  - the labour market clears when labour supply equals labour demand each period, i.e., $N_t^* = N_t^{S*} = N_t^{D*}$, and
  - the goods market clears when goods supplied equals goods demanded each period, $Y_t^* = Y_t^s = Y_t^d = C_t^* + I_t^* + G_t$.

recall that there are 3 markets here: Labour Market, Goods Market and Credit Market, so by Walras's Law, if goods and labour markets clear each period, then credit market also clears in every period.

Constructing Output Supply Curve Y_1^S (1/2)

We already have labour demand N_1^d and labour supply $N_1^S(r_1)$: labour market equilibrium is (N_1^*, w_1^*) (figure "9.10")

If interest rate $r_1 \nearrow$ from $\widetilde{r}_1$ to $\widehat{r}_1$:
- given any wage rate, work more, so rightward shift of N_1^S, because price of current leisure is more expensive relative to price of future leisure, and substitution effect domintes
- optimal employment $\nearrow$ and optimal wage rate $\downarrow$
- movement up the production function,
  reflecting $Y_1^S \nearrow$ when optimal employment$\nearrow$
- Hence, goods supply is increasing in r_1, so Y_1^S is upward sloping
  (figure "9.11")

Constructing Output Supply Curve Y_1^S (2/2)

Output supply curve Y_1^S reflects combinations of $\left(Y_1^S, r_1\right)$ for which the labour market is in equilibrium.

Contrast this with "usual" AS-AD analysis when output supplied (AS) is plotted on (P_1, Y_1) diagram. Here, (r_1, Y_1) to reflect the concern with "real", as opposed to "nominal", variables.

Note: changes in r_1 are movements along the Y_1^S curve

Shifts of Output Supply Curve Y_1^S (1/2)

Question: When does Y_1^S curve shift?
Answer: When $N_1^S(r_1)$, N_1^d, and/or production function change.

From previous lecture, we dealt with:
- $N_1^S(r_1)$ changes if lifetime disposable wealth ω^d changes
- N_1^d changes if current TFP z_1 changes
- production function $Y_1 = z_1 F(K_1, N_1)$ changes if current TFP z_1 changes

Here, let us analyze an increase in current or future expenditures of government, i.e., G_1 or G_2 or $G_3, \ldots \nearrow$ (try an increase in z_1 as <u>homework</u>)

- immediate implication is: for government's lifetime BC to hold, taxes have to increase, either now or in the future

Shifts of Output Supply Curve Y_1^S (2/2)

- hence, lifetime disposable wealth ω^d of representative consumer falls since $\nearrow$ PV of taxes
- as $\omega^d \downarrow$, consume less leisure $\Longrightarrow$ work more $\left(dN_1^s(r_1)/d\omega^d < 0\right)$, so $N_1^s(r_1)$ shifts to the right and $N_1^* \nearrow$ and $w_1^* \downarrow$
- as $N_1^* \nearrow$, output supplied increases as we move up and along the production function.

Hence, for a given interest rate (which hasn't changed), output supplied increased, so Y_1^S curve shifts to the right. (figure "9.12")

(Implicit assumption here: can think of government expenditures in economy as worthless; do not increase productive capacity of representative firm, nor do they increase utility of representative consumer)

Construction of Output Demand Curve (1/5)

$$Y_1^d = C_1(r_1, Y_1^d) + I_1^d(r_1) + G_1$$

Y_1^d : total current demand for goods, or output demand
$C_1(r_1, Y_1^d)$: - demand for current consumption goods by representative consumer,
- decreasing function of the real interest rate because substitution effect dominates;

$I_1^d(r_1)$: - demand for current investment goods by representative firm
- decreasing function of the real interest rate because opp cost of investment increases as r_1 increases so invest less;

G_1 : - government's purchases/demand of current goods

Construction of Output Demand Curve (2/5)

Recall that C_1 decreases when r increases (substitution effect dominates).

But C_1 also depends on Y_1^d. Why?
- recall that when current income $Y_1 \nearrow$, other things being constant, disposable lifetime wealth $\omega^d \nearrow$, so demand for consumption goods today $C_1 \nearrow$. But we know from national income accounting that current income must equal current expenditure on goods, which is equal to total demand for current goods, i.e., Y_1^d

In other words, if $Y_1^d \nearrow$, $C_1 \nearrow$.

Which is why $C_1(r_1, Y_1^d)$.

Construction of Output Demand Curve (3/5)

But this is an added complication because now there is a **multiplier** process:
- suppose some exogenous factor changes C_1 or I_1 or G_1, leading to an increase in Y_1^d
- but this increase in demand for output creates an increase in income, some of which is spent on consumption
- and this increase in C_1 will again lead to an increase in Y_1^d, which in turn increases C_1, ...

Example:
- suppose some exogenous variable, like z_2 changes, which leads to an exogenous increase in spending ΔE
- let ΔY_1^d : change in current demand for output
- let $MPC\Delta Y_1^d$: change in demand for consumption goods

Construction of Output Demand Curve (4/5)

As we have seen from before, $0 < MPC < 1$ due to consumption-smoothing.

Hence,

$$\begin{aligned} \Delta Y_1^d &= MPC\Delta Y_1^d + \Delta E, \text{ so} \\ \Delta Y_1^d &= \underbrace{\left(\frac{1}{1 - MPC}\right)}_{multiplier} \Delta E, \end{aligned}$$

- notice that the higher then MPC, the larger the multiplier, so the larger the effect on Y_1^d, for some given exogenous factor affecting demand for goods.

Construction of Output Demand Curve (5/5)

Suppose r_1 changes

We know that $\frac{\Delta C_1}{\Delta r_1} < 0$, and $\frac{\Delta I_1^d}{\Delta r_1} < 0$.

Since

$$Y_1^d = C_1(r_1, Y_1^d) + I_1^d(r_1) + G_1,$$

we have that

$$\frac{\Delta Y_1^d}{\Delta r_1} = \left(\frac{1}{1 - MPC}\right)\left(\frac{\Delta C_1}{\Delta r_1} + \frac{\Delta I_1^d}{\Delta r_1}\right) < 0.$$

In other words, there is a negative relationship between Y_1^d and r_1 (figure "9.14")

Shifts of Output Demand Curve Y_1^d (1/2)

Question: What causes Y_1^d curve to shift?
Answer: Any factor that changes C_1 or I_1^d or G_1, except r_1.

Examples:
- decrease in the PV of taxes, which increases ω^d, and if the change is $\Delta\omega^d$, output demand curve shifts to the right by the amount $\frac{\Delta\omega^d}{1-MPC}$;
- increase in tomorrow's income by ΔY_2, then lifetime wealth increases by $\frac{\Delta Y_2}{1+r}$, so output demand curve shifts to the right by the amount $\frac{\Delta Y_2}{(1+r)(1-MPC)}$;
- increase in tomorrow's TFP, which causes investment today to rise. Suppose investment today changes by ΔI_1, then output demand curve shifts to the right by the amount $\frac{\Delta I_1}{1-MPC}$;

Shifts of Output Demand Curve Y_1^d (2/2)

- increase in G_1 by ΔG_1, which causes output demand curve shifts to the right by the amount $\frac{\Delta G_1}{1-MPC}$.

(figure "9.15")

Graphical Representation of Competitive Equilibrium

See figure “9.16”

Go back to definition of competitive equilibrium