Suggested Solutions to EC2102 Macroeconomic Analysis I

Semester 2, 2009/10

Midterm Exam

March 3, 2010

MCQs, Questions 1-9: worth 4 points each except for Question 2 which is worth 8 points.

Question 1 (i)

Question 2 (v)

Question 3 (B)

Question 4 (D)

Question 5 (B)

Question 6 (D)

Question 7 (E)

Question 8 (D)

Question 9 (C)

Question 10

PART A

(i) A competitive equilibrium for the economy considered here is an allocation $\left(c_{j1}^{*}, c_{j2}^{*}, s_{j1}^{*}\right)$, for each type of agent $j = A, B$, and an interest rate r^{*}, such that:

- for each individual j, taking the interest rate r^{*} as given, $\left(c_{j1}^{*}, c_{j2}^{*}, s_{j1}^{*}\right)$ is the solution to his lifetime utility maximization problem;
- the credit market clears at the interest rate r^{*}, i.e.,

$$200 s_1^{A*}\left(r^{*}\right) + 500 s_1^{B*}\left(r^{*}\right) = 0.$$

(**NOTE**: It is perfectly correct to include s_{j2}^{*} in your description of the environment also.)
(5 *points*)

(ii) Let us first define a type j individual's weath ω_j as

$$\omega_j \equiv y_{j1} + \frac{y_{j2}}{1+r}.$$

Dropping the subscript indicating type for ease of notation, an agent's utility maximization problem is

$$\begin{aligned} & \max_{c_1, c_2} c_1^{\alpha} + \beta c_2^{\alpha} \\ s.t.\ c_1 + \frac{c_2}{1+r} &= \omega. \end{aligned}$$

Since $c_2 = (\omega - c_1)(1+r)$, the above is just

$$\begin{aligned} & \max_{c_1} c_1^{\alpha} + \beta \left[(\omega - c_1)(1+r) \right]^{\alpha}, \text{ or} \\ & \max_{c_1} c_1^{\alpha} + \beta (1+r)^{\alpha} (\omega - c_1)^{\alpha} \end{aligned}$$

$\underline{(5\ points)}$

(iii) The F.O.C. to the problem set up in (i) is

$$F.O.C.: \alpha c_1^{\alpha-1} + \beta\alpha (1+r)^{\alpha} (\omega - c_1)^{\alpha-1} (-1) = 0,$$

where clearly c_1^* must satisfy the F.O.C. Manipulating the above F.O.C., we get

$$\begin{aligned} c_1^{\alpha-1} &= \beta (1+r)^{\alpha} (\omega - c_1)^{\alpha-1} \\ c_1 &= \left[\beta (1+r)^{\alpha} \right]^{\frac{1}{\alpha-1}} (\omega - c_1) \\ c_1 &= \beta^{\frac{1}{\alpha-1}} (1+r)^{\frac{\alpha}{\alpha-1}} \omega - \beta^{\frac{1}{\alpha-1}} (1+r)^{\frac{\alpha}{\alpha-1}} c_1 \\ c_1 \left[1 + \beta^{\frac{1}{\alpha-1}} (1+r)^{\frac{\alpha}{\alpha-1}} \right] &= \beta^{\frac{1}{\alpha-1}} (1+r)^{\frac{\alpha}{\alpha-1}} \omega. \end{aligned}$$

Hence we obtain that the solution to FOC, c_1^*, is

$$\begin{aligned} c_1^* &= \frac{\beta^{\frac{1}{\alpha-1}} (1+r)^{\frac{\alpha}{\alpha-1}} \omega}{1 + \beta^{\frac{1}{\alpha-1}} (1+r)^{\frac{\alpha}{\alpha-1}}} \text{ or} \\ c_1^* &= \frac{\omega}{\beta^{\frac{1}{1-\alpha}} (1+r)^{\frac{\alpha}{1-\alpha}} + 1}. \end{aligned}$$

Since in equilibrium the optimal choice of savings s_1^* must satisfy $s_1^* = y_1 - c_1^*$, the optimal savings choice today is

$$s_1^* = y_1 - \frac{\omega}{\beta^{\frac{1}{1-\alpha}} (1+r)^{\frac{\alpha}{1-\alpha}} + 1}.$$

Clearly

$$s_2^* = 0$$

since each agent lives for two periods. Lastly,

$$c_2^* = (\omega - c_1^*)(1+r) = \left(\omega - \frac{\omega}{\beta^{\frac{1}{1-\alpha}}(1+r)^{\frac{\alpha}{1-\alpha}}+1}\right)(1+r).$$

(17 *points*)

(*iv*) The credit market clears when aggregate savings equals zero, so it is equivalent to finding an interest rate r^* such that

$$\begin{aligned} 200s_{A1}^* + 500s_{B1}^* &= 0, \text{ or} \\ 2s_{A1}^* + 5s_{B1}^* &= 0, \end{aligned} \tag{1}$$

Substituting the expressions for savings from part (*iii*),

$$2y_{A1} - \frac{2\left(y_{A1} + \frac{y_{A2}}{1+r^*}\right)}{\beta^{\frac{1}{1-\alpha}}(1+r^*)^{\frac{\alpha}{1-\alpha}}+1} + 5y_{B1} - \frac{5\left(y_{B1} + \frac{y_{B2}}{1+r^*}\right)}{\beta^{\frac{1}{1-\alpha}}(1+r^*)^{\frac{\alpha}{1-\alpha}}+1} = 0$$

and upon simplifying,

$$r^* = \frac{1}{\beta} - 1.$$

Given $\beta = \frac{3}{4}$,

$$r^* = \frac{1}{3}.$$

(10 *points*)

(*v*) Let $\alpha = 0.5$.

$$\begin{aligned} c_{A1}^* &= c_{A2}^* = 10, \ s_{A1}^* = s_{A2}^* = 0; \text{ and} \\ c_{B1}^* &= c_{B2}^* = 4, \ s_{B1}^* = s_{B2}^* = 0. \end{aligned}$$

(8 *points*)

PART B

(*vi*) Let us now define a type j agent's disposable wealth as

$$\omega_j^d \equiv y_{j1} + \frac{y_{j2} - t}{1+r}$$

Dropping the subscript indicating type for ease of notation, an agent's utility maximization problem is

$$\begin{aligned} & \max_{c_1,c_2} c_1^\alpha + \beta c_2^\alpha \\ s.t.\ c_1 + \frac{c_2}{1+r} &= \omega^d. \end{aligned}$$

The difference between this setup and the setup in part (ii) is the inclusion of a tax in time period 2 in an individual's budget constraint per period, which implies that in his lifetime budget constraint we now have that the present value of consumption is equal to the present value of disposable wealth rather than the present value of wealth. In other words,

$$\omega^d = \omega - \underbrace{\frac{t}{1+r}}_{PV \text{ of taxes}},$$

which means that

$$\omega^d < \omega.$$

For the optimal consumption choice today, given r, it is now

$$\widehat{c}_1^* = \frac{\omega^d}{\beta^{\frac{1}{1-\alpha}}(1+r)^{\frac{\alpha}{1-\alpha}} + 1}$$

instead of, as we had shown in part (iii)

$$c_1^* = \frac{\omega}{\beta^{\frac{1}{1-\alpha}}(1+r)^{\frac{\alpha}{1-\alpha}} + 1}.$$

It is clear that $\widehat{c}_1^* < c_1^*$. Letting the new savings be $\widehat{s}_1^*$, since

$$\widehat{s}_1^* = y_1 - \widehat{c}_1^*,$$

it must be that $\widehat{s}_1^* > s_1^*$ since $\widehat{c}_1^* < c_1^*$.

For tomorrow's optimal consumption choice, $\widehat{c}_2^*$, it is

$$\widehat{c}_2^* = \left(\omega^d - \widehat{c}_1^*\right)(1+r).$$

Let $\gamma \equiv \beta^{\frac{1}{1-\alpha}}(1+r)^{\frac{\alpha}{1-\alpha}}$.

$$\begin{aligned} \widehat{c}_2^* &= \left(\omega^d - \widehat{c}_1^*\right)(1+r) \\ &= \left(\omega^d - \frac{\omega^d}{\gamma+1}\right)(1+r) = \omega^d\left(1 - \frac{1}{\gamma+1}\right)(1+r), \end{aligned}$$

whereas

$$\begin{aligned} c_2^* &= (\omega - c_1^*)(1+r) \\ &= \left(\omega - \frac{\omega}{\gamma+1}\right)(1+r) = \omega\left(1 - \frac{1}{\gamma+1}\right)(1+r) > \widehat{c}_2^* \end{aligned}$$

since $\omega^d < \omega$.

Clearly, savings in time period 2 have not changed and is still 0 since the agent lives for only two periods.

The intution as to why consumption dropped in both periods is this: since each agent is poorer because he has to pay more tax tomorrow, he will decrease consumption in both periods to smooth out this fall in disposable wealth, and since the tax is levied only tomorrow, the agent will save more today because consumption today has fallen.

(**NOTE**: You do not have show "so much calculations" in explaining what happened to consumption and savings choices for a given interest rate r. If you explained in words the intuition why consumption each period will drop and why savings in time period 1 will increase correctly, you still get full points for this part.)

$\big($15 *points*$\big)$