Work-Leisure Decision of Consumer (2-period economy) (1/6)

In our two-period economy, y_1, y_2 taken as exogenous.
How are they determined? By how much you choose to work.

Assume no government for now.

h : units of time available each period, same every period
l_t : leisure (decision/choice variable) in period t
$(h - l_t)$: amount of time spent working in period t

Work-Leisure Decision (2/6)

Individual's problem:

$$\begin{aligned} \max_{c_1,c_2,s_1,l_1,l_2} U(c_1,c_2,l_1,l_2) &= u(c_1,l_1)+\beta u(c_2,l_2) \\ & s.t. \\ c_1+s_1 &= w_1(h-l_1) \qquad (17) \\ c_2 &= w_2(h-l_2)+(1+r)s_1 \qquad (18) \end{aligned}$$

Combining equations (17) and (18) to get LBC:

$$c_1+\frac{c_2}{1+r}=w_1(h-l_1)+\frac{w_2(h-l_2)}{1+r}.$$

Looks very similar to our usual LBC with
$y_1 = w_1(h-l_1)$ and $y_2 = w_2(h-l_2)$.

Work-Leisure Decision (3/6)

Therefore, we can write consumer's problem as

$$\max_{c_1,c_2,l_1,l_2} u(c_1,l_1) + \beta u(c_2,l_2)$$
$$s.t.$$
$$c_1 + \frac{c_2}{1+r} = w_1(h-l_1) + \frac{w_2(h-l_2)}{1+r}$$

Again using the same trick of expressing c_2 as a function of c_1, l_1, l_2 from LBC:

$$c_2 = [w_1(h-l_1) - c_1](1+r) + w_2(h-l_2),$$

Work-Leisure Decision (4/6)

Consumer's problem is

$$\max_{c_1, l_1, l_2} u(c_1, l_1) + \beta u(c_2(c_1, l_1, l_2), l_2),$$
$$where\ c_2 = [w_1(h - l_1) - c_1](1 + r) + w_2(h - l_2).$$

Take 3 F.O.C.s since there are 3 decision variables now:

$$F.O.C.(c_1) : u_1(c_1^*, l_1^*) + \beta u_1(c_2^*, l_2^*)(-(1 + r)) = 0 \tag{19}$$
$$F.O.C.(l_1) : u_2(c_1^*, l_1^*) + \beta u_1(c_2^*, l_2^*)(-w_1)(1 + r) = 0 \tag{20}$$
$$F.O.C.(l_2) : u_1(c_2^*, l_2^*)(-w_2) + u_2(c_2^*, l_2^*) = 0 \tag{21}$$

Work-Leisure Decision (5/6)

which can be rearranged as:

$$(19) \quad : \quad \underbrace{-\frac{u_1(c_1^*, l_1^*)}{\beta u_1(c_2^*, l_2^*)}}_{MRS_{c_1,c_2}} = -(1+r)$$

$$(20) \quad : \quad \underbrace{-\frac{u_2\left(c_1^*, l_1^*\right)}{u_1(c_1^*, l_1^*)}}_{MRS_{l_1,c_1}} = -w_1$$

$$(21) \quad : \quad \underbrace{-\frac{u_2\left(c_2^*, l_2^*\right)}{u_1\left(c_2^*, l_2^*\right)}}_{MRS_{l_2,c_2}} = -w_2$$

Work-Leisure Decision (6/6)

Just as $(1 + r)$ is the price of today's consumption relative to tomorrow's consumption, w_1 is the price of consuming more leisure today relative to consuming consumption goods: If choose to consume more leisure, forego chance to earn wages (denoted in terms of consumption goods), so you can consume less consumption goods today. Likewise, w_2 is the price of consuming more leisure tomorrow relative to consuming consumption goods tomorrow

Homework: Verify that the above F.O.C.s are correct.

Stocktaking...

Finished consumer's intertemporal problem.

Readings: ch. 4: pp. 99-120, ch. 8

Now we're introducing the firm and closing the model.

Readings: ch. 4: pp.121-133, ch. 9

Complete the Model (1/5)

Consumer: representative consumer
Mr Average

Firm: representative firm

Add back government.

Who owns the firm? the consumer; consumer gets dividends π_t every period from the firm.

Complete the Model (2/5)

Consumer's first period budget constraint:

$$C_1 + S_1^P = w_1 (h - l_1) + \pi_1 - T_1,$$

where $w_1 (h - l_1) + \pi_1 - T_1 = y_1^d$, disposable income in period 1.

Consumer's second period budget constraint:

$$C_2 + S_2^P = w_2 (h - l_2) + (1 + r) S_1^P + \pi_2 - T_2,$$

where $w_2 (h - l_2) + (1 + r) S_1^P + \pi_2 - T_2 = y_2^d$, disposable income in period 2.

But $S_2^P = 0$ because two-period economy

Complete the Model (3/5)

Hence, consumer's LBC:

$$C_1 + \frac{C_2}{1+r} = w_1 (h - l_1) + \frac{w_2 (h - l_2)}{1+r} + \pi_1 + \frac{\pi_2}{1+r} - \left(T_1 + \frac{T_2}{1+r}\right)$$

But is it realistic to assume people die after 2 periods?
- If each period is one month, then obviously not.

Modern macroeconomic models usually model representative consumer, representative firm as infinitely lived. To capture idea that people make decisions regularly over many time periods. Plus, convenient assumption.

But how do you solve an infinitely lived representative consumer's problem?

Complete the Model (4/5)

You can guess from how we have modelled consumer's problem so far that we need to solve for a consumer's optimal $C_1, C_2, C_3, C_4, ..., S_1^P, S_2^P, S_3^P, S_4^P, ...,$ and $l_1, l_2, l_3, l_4, ...,$ which means...
a lot of variables to solve for.

Why not just solve it period by period, or "sequentially"? More intuitive, and easier. Captures the idea that at time period 1, you decide C_1, S_1^P, l_1, and then tomorrow, at time period 2, you decide C_2, S_2^P, l_2, etc.

Turns out that you can do it. When choose C_1, S_1^P, l_1 today, tomorrow is affected because S_1^P affects how much you can consume tomorrow. But this is very similar to looking at a two-period economy, which was what we have been doing all along.

Complete the Model (5/5)

Instead of choosing $\left(C_1, C_2, S_1^P, l_1, l_2\right)$ in a two-period lived consumer, in this sequential formulation the consumer chooses $\left(C_1, S_1^P, l_1\right)$ today, $\left(C_2, S_2^P, l_2\right)$ tomorrow, and so on.

The Representative Consumer; Mr Average (1/5)

Infinitely lived: can think of a dynasty with altruistic members; each member of the dynasty behaves as if he were infinitely lived

Endowed each period with h units of time he can devote to work
- labour income: $w_t(h - l_t) = w_t N_t$
- capital income: π_t (from ownership of representative firm)

Period t budget constraint:

$$C_t + S_t^P = w_t(h - l_t) + (1 + r_{t-1})S_{t-1}^P + \pi_t - T_t$$

consumption decisions are still made based on disposable lifetime wealth.

The Representative Consumer; Mr Average (2/5)

Time-separable utility function:

$$U(C_1, l_1, C_2, l_2, C_3, l_3...) = u(C_1, l_1) + \beta u(C_2, l_2) + \beta^2 u(C_3, l_3) + ...$$

In period t, given S_{t-1}^P his savings/debt from period $(t-1)$, the representative consumer chooses consumption C_t, savings S_t^P and leisure time l_t to maximize lifetime utility.

Period t decisions affect only period $t+1$ directly. In fact, S_t^P enters period $t+1$ budget constraint but not $t+2$ and beyond. Hence, when deciding his consumption and labor for period t the agent needs only to take into account the impact of his consumption/savings decision on the following period and hence it is similar to our two period economy problem. All the comparative statics still hold in this more complicated set-up.

The Representative Consumer; Mr Average (3/5)

Let $t = 1$. How do you write the representative consumer's problem?

$$\max_{C_1, l_1, S_1} u(C_1, l_1) + \beta u(C_2, l_2) + \ldots$$

$$\begin{aligned} s.t.\ C_1 + S_1^P &= w_1(h - l_1) + \pi_1 + (1 + r_0) S_0^P - T_1 \quad (22) \\ C_2 + S_2^P &= w_2(h - l_2) + \pi_2 + (1 + r_1) S_1^P - T_2 \quad (23) \end{aligned}$$

What is S_0^P? Assume it is zero here.

From (22) and (23),
$C_2 = w_2(h - l_2) + \pi_2 - T_2 +$
$\quad [w_1(h - l_1) + \pi_1 - T_1 - C_1](1 + r_1) - S_2^P,$

so C_2 is a function of C_1, l_1

The Representative Consumer; Mr Average (4/5)

Hence, representative consumer's problem in period 1 is:

$$\max_{C_1, l_1} u\left(C_1, l_1\right) + \beta u\left(C_2\left(C_1, l_1\right), l_2\right) + ...$$

$$where$$

$$C_2 = w_2(h - l_2) + \pi_2 - T_2 + \\ \left[w_1(h - l_1) + \pi_1 - T_1 - C_1\right]\left(1 + r_1\right) - S_2^P,$$

The Representative Consumer; Mr Average (5/5)

The F.O.C.s are:

$$\begin{array}{rl} F.O.C.(C_1): & u_1(C_1,l_1)-\beta(1+r_1)u_1(C_2,l_2)=0, \\ F.O.C.(l_1): & u_2(C_1,l_1)-w_1\beta(1+r_1)u_1(C_2,l_2)=0, \end{array}$$

which after rearranging are

$$\begin{aligned} \underbrace{-\frac{u_1(C_1,l_1)}{\beta u_1(C_2,l_2)}}_{MRS_{C_1,C_2}} &= -\left(1+r_1\right), \\ \underbrace{-\frac{u_2(C_1,l_1)}{u_1(C_1,l_1)}}_{MRS_{l_1,C_1}} &= -w_1. \end{aligned}$$

As usual, w_1 and $(1+r_1)$ are prices.

Representative consumer: Comparative Statics (1/6)

Current labour supply, N_1^S

1. If current wage rate w_1 increases, do you work more or less?

Substitution effect: since w_1 is the price of l_1 relative to c_1, the price of leisure has increased. Hence, want to consume less leisure, that is, you want to work more. Labour supply N_1^S increases.
Income effect: as leisure is a normal good, increasing w_1 means income has increased, so want to consume more leisure. Hence, labour supply N_1^S decreases.

Net effect? Ambigous. But data tells us that substitution effect dominates: empirically, observe that as wage rises, people work more (supply more labour). That is, labour supply N_1^S is upward sloping
Hence, $dN_1^S/dw_1 > 0$. Upward sloping labour supply curve. (figure "9.1")

Representative consumer: Comparative Statics (2/6)

2. Do you work more or less when real interest rate today r_1 increases?

$\frac{w_1(1+r_1)}{w_2}$: price of period one leisure relative to the second period leisure.

Substitution effect: $\nearrow r_1$ means $\nearrow$ in price of current leisure to future leisure, so consume less current leisure, that is, you work more, so $N_1^S \nearrow$.

Income effect: $\nearrow r_1$ means same amount of savings imply more disposable income in second period, feel richer, so $N_1^S \downarrow$
(OR positive income effect means consume more leisure)

Net effect ambiguous, but data tells us that when r_1 increases, $N_1^S \nearrow$, i.e., substitution effect dominates

Representative consumer: Comparative Statics (3/6)

Hence, $dN_1^S/dr_1 > 0$.

For the same wage rate, an increase in r_1 means that $N_1^S \nearrow$, so curve shifts to the right. (figure "9.2")

Representative consumer: Comparative Statics (4/6)

3. Do you work more or less when disposable wealth changes?

$\nearrow$ disposable wealth $\Longrightarrow$ consume more of normal goods $\Longrightarrow$ consume more leisure, i.e., work less

Hence, $dN_1^S/d\omega^d < 0$.

For the same wage, an increase in ω^d means that $N_1^S \downarrow$, so curve shifts to the left. (figure "9.3")

Representative consumer: Comparative Statics (5/6)

Current demand for Consumption Goods

1. Do you consume more or less when r_1 increases?

Like we did last time, assume substitution effect dominates, prefer to consume less today and more tomorrow since relative price of consuming today has increased.

Hence, consume less today when r_1 increases

Representative consumer: Comparative Statics (6/6)

2. Do you consume more or less when lifetime wealth increases?

If lifetime wealth increases, demand for current consumption goods increases

Demand for current consumption goods by representative consumer is only part of the total demand for current goods. We still have to consider firms and the government's demand for current consumption goods. Total demand for current consumption goods will later be summarized by the output demand curve.

The Representative Firm (1/4)

Production function: for each period output is

$$Y_t = z_t F(K_t, N_t),$$

where z_t is Total Factor Productivity (TFP) and $F(K_t, N_t)$ is assumed to have increasing in each of its arguments, but at a decreasing rate:

$$F_1(K_t, N_t) > 0,\ F_2(K_t, N_t) > 0,\ F_{11}(K_t, N_t) < 0 \text{ and } F_{22}(K_t, N_t) < 0.$$

Note that

$$\begin{aligned} MPK_t &= z_t F_1(K_t, N_t) \\ MPN_t &= z_t F_2(K_t, N_t). \end{aligned}$$

The Representative Firm (2/4)

Law of motion for capital:

$$K_{t+1} = (1 - \delta)K_t + I_t$$

where δ is the depreciation rate of capital, and I_t is investment in period t

Maximization problem of the firm:

The firm maximizes the PV of stream of profits (in the interest of its shareholders) which is:

$$\pi_1 + \frac{\pi_2}{1 + r_1} + \frac{\pi_3}{(1 + r_1)(1 + r_2)} + \ldots$$

where period t profits are:

$$\pi_t = z_t F(K_t, N_t) - w_t N_t - I_t$$

The Representative Firm (3/4)

Like in the representative consumer's problem, we just look at what the firm does each period.

Concretely, let us look at what representative firm is doing in period 1. Firm wants to pick (N_1, I_1) to maximize stream of profits. But equivalent to choosing (N_1, K_2) to maximize stream of profits.

$$\max_{N_1,K_2} \pi_1 + \frac{\pi_2}{1+r_1} + \ldots$$

$$\max_{N_1,K_2} z_1F(K_1,N_1) - w_1N_1 - K_2 + (1-\delta)K_1 + \frac{z_2F(K_2,N_2)}{1+r_1}$$

$$+ \frac{(1-\delta)K_2}{1+r_1} \qquad \underbrace{-\frac{w_2N_2}{1+r_1} - \frac{K_3}{1+r_1}}_{\text{we don't care about these terms today}} \qquad + \ldots$$

The Representative Firm (4/4)

HOMEWORK:

(i) What are the F.O.C.s of the representative firm's maximization problem on previous slide?

(ii) Solve the representative firm's maximization problem on previous page, but now with respect to (N_1, I_1). Do you get the same F.O.C.s? (Hint: you should.)