Suggested Solutions to EC2102 Macroeconomic Analysis I Tutorial 4, Week 6 (February 22-26, 2010)

(i) Since the production technology is $Y_t = z_t F(K_t, N_t) = z_t K_t^a N_t^{1-a} (0 < a < 1)$, the marginal product of capital in time period t, MPK_t:

$$MPK_t = \partial Y_t / \partial K_t = z_t F_1(K_t, N_t) = a z_t K_t^{a-1} N_t^{1-a} > 0; \tag{1}$$

and the marginal product of labor in time period t, MPN_t:

$$MPN_t = \partial Y_t / \partial N_t = z_t F_2(K_t, N_t) = (1-a) z_t K_t^a N_t^{-a} > 0. \tag{2}$$

(ii) The firm's maximization problem in period 1 is:

$$\max_{\{N_1, K_2\}} \pi_1 + \frac{\pi_2}{1+r_1} + \frac{\pi_3}{(1+r_1)(1+r_2)} + \cdots$$

Since

$$\pi_t = z_t F(K_t, N_t) - w_t N_t - I_t = z_t F(K_t, N_t) - w_t N_t - [K_{t+1} - (1-\delta) K_t]$$

Therefore, we can rewrite the firm's maximization problem in period 1 as:

$$\max_{\{N_1, K_2\}} z_1 F(K_1, N_1) - w_1 N_1 - [K_2 - (1-\delta) K_1] + \frac{z_2 F(K_2, N_2) - w_2 N_2 - [K_3 - (1-\delta) K_2]}{1+r_1} + \cdots$$

$$F.O.C.(N_1): z_1 F_2(K_1, N_1^*) - w_1 = 0 \tag{3}$$

$$F.O.C.(K_2): -1 + \frac{1}{1+r_1}[z_2 F_1(K_2^*, N_2) + (1-\delta)] = 0 \tag{4}$$

Substituting equation (1)and (2) into equation (3) and (4), we have

$$z_1 F_2(K_1, N_1^*) = (1-a) z_1 K_1^a \left(N_1^*\right)^{-a} = w_1$$
$$z_2 F_1(K_2^*, N_2) = a z_2 \left(K_2^*\right)^{a-1} N_2^{1-a} - \delta = r_1.$$

The economic meaning of F.O.C.s: equation (3) tells firm how much labor to demand in period 1, i.e. demand labor till MPN_1 exactly equals to marginal cost of labor, which is w_1; equation (4) tells firm how much capital to demand in period 1, i.e. net benefit of investing (MPK_2,net of depreciation) till MPK_2 exactly equals to marginal(opp.) cost of investment, which is r_1.

(iii) The representative consumer's maximization problem in period 1 is:

$$\max_{\{C_1, l_1, S_1\}} u(C_1, l_1) + \beta u(C_2, l_2) + \beta^2 u(C_3, l_3) + \cdots \tag{5}$$
s.t.
$$C_1 + S_1^P = w_1(h - l_1) + (1 + r_0) S_0^P + \pi_1 \tag{6}$$
$$C_2 + S_2^P = w_2(h - l_2) + (1 + r_1) S_1^P + \pi_2. \tag{7}$$
+...

By assuming $S_0^P = 0$, and combining equations (6) and (7), we can rewrite the maximization problem above as:

$$\max_{\{C_1, l_1\}} u(C_1, l_1) + \beta u(C_2, l_2) + \beta^2 u(C_3, l_3) + \cdots$$
where
$$C_2 = w_2(h - l_2) + (1 + r_1)[w_1(h - l_1) + \pi_1 - C_1] + \pi_2 - S_2^P.$$

$$F.O.C.(C_1) : u_1(C_1^*, l_1^*) + \beta u_1(C_2(C_1^*, l_1^*, l_2), l_2)[-(1 + r_1)] = 0 \tag{8}$$
$$F.O.C.(l_1) : u_2(C_1^*, l_1^*) + \beta u_1(C_2(C_1^*, l_1^*, l_2), l_2)[-w_1(1 + r_1)] = 0 \tag{9}$$

Equation (8):

$$\frac{u_1(C_1^*, l_1^*)}{\beta u_1(C_2(C_1^*, l_1^*, l_2), l_2)} = 1 + r_1$$
$$\text{since} \quad u(C_t, l_t) = \ln C_t + \ln l_t \quad u_1(C_t, l_t) = \frac{1}{C_t}$$
$$\text{so} \quad \frac{C_2(C_1^*, l_1^*, l_2)}{\beta C_1^*} = 1 + r_1;$$

Equation (9):

$$\frac{u_2(C_1^*, l_1^*)}{\beta u_1(C_2(C_1^*, l_1^*, l_2), l_2)} = w_1(1 + r_1)$$

$$\text{since} \quad u(C_t, l_t) = \ln C_t + \ln l_t \quad u_2(C_t, l_t) = \frac{1}{l_t}$$

$$\text{so} \quad \frac{C_2(C_1^*, l_1^*, l_2)}{\beta l_1^*} = w_1(1 + r_1)$$

$$\text{moreover} \quad \frac{C_2(C_1^*, l_1^*, l_2)}{\beta C_1^*} = 1 + r_1$$

$$\text{therefore} \quad \frac{C_1^*}{l_1^*} = w_1.$$

(iv) A competitive equilibrium is an allocation $(C_t^*, S_t^*, N_t^{S*}, N_t^{D*}, I_t^*)$ and a set of prices (r_t^*, w_t^*), such that,

- the representative consumer chooses (C_t^*, S_t^*, N_t^{S*}) each period in order to maximize lifetime utility, taking (r_t^*, w_t^*) as given;
- the representative firm maximizes its present value of profits by choosing (N_t^{D*}, I_t^*), taking (r_t^*, w_t^*) as given;
- the labor market and goods market clear at (r_t^*, w_t^*).

In time period 1, equilibria in the labor and goods markets are illustrated as:
Figure 1: labor market
Figure 2: goods market

(v) If the minimum wage $w_1^m < w_1^*$, how would the equilibrium in the labor and goods market change?

As minimum wage w_1^m is below market wage w_1^* level, nothing changes: (w_1^*, N_1^*) is still labor market equilibrium and (r_1^*, Y_1^*) is still goods market equilibrium. This is because minimum wage is now non-binding, since firm already pays a wage rate above the minimum wage.

Figure 3: labor market
Figure 4: goods market

(*vi*) Derivation of new output supply curve:

Output supply curve shows the relationship between interest rate and output. In the labor market diagram (Figure 6), initial supply curve is shown by $N_1^S(r_1^*)$. Now consider two different cases:

1. Suppose labor supply curve is $N_1^S(\tilde{r}_1^*)$, which is located to the right hand side of point T, where $r_1^c < \tilde{r}_1^* < r_1^*$. In this case, there will be no changes in labor demand. Thus, the output supply curve will be vertical for any interest rate higher than r_1^c, which is shown by the vertical line extending from KL in Figure 6.

2. Suppose labor supply curve is $N_1^S(\hat{r}_1^*)$, which is located to the left hand side of point T, where $\hat{r}_1^* < r_1^c < \tilde{r}_1^* < r_1^*$. In this case, as interest rate increases, labor supply increases, equilibrium level of labor increases, implying a positive relationship between interest rate and output supply. Thus the output supply curve is upward sloping for any interest rate less than r_1^c, which is shown by the JK segment of $\widehat{Y}_1^S$ in Figure 6.

Therefore, goods market equilibrium point is at L. Current employment falls from N_1^* to N_1^m; aggregate output falls from Y_1^* to Y_1^m; real interest rate goes up from r_1^* to r_1^m; real wage rate goes up from w_1^* to w_1^m.
Figure 5: labor market
Figure 6: goods market
Figure 7: firm's production function

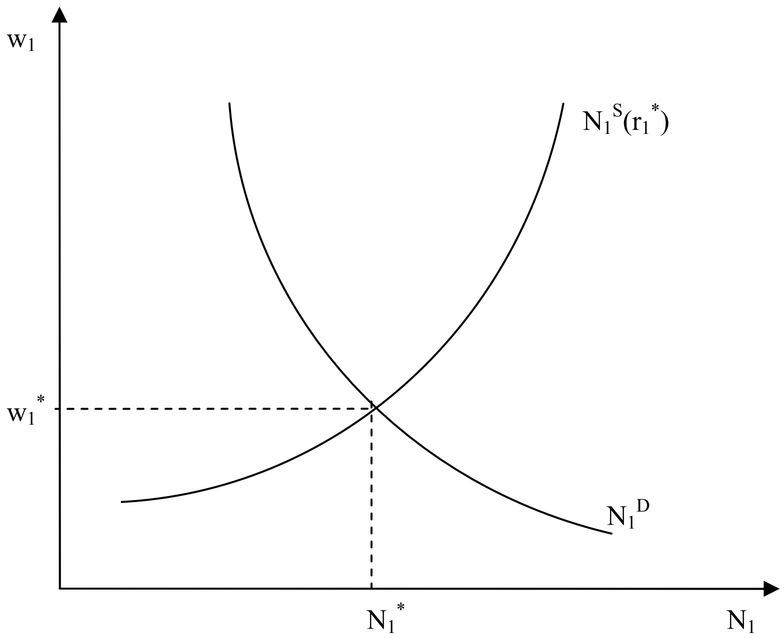

Figure 1

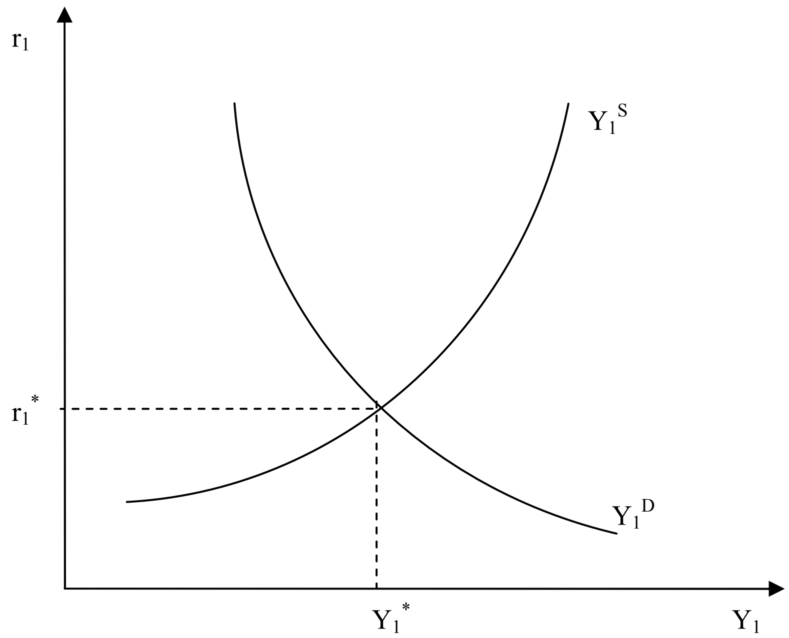

Figure 2

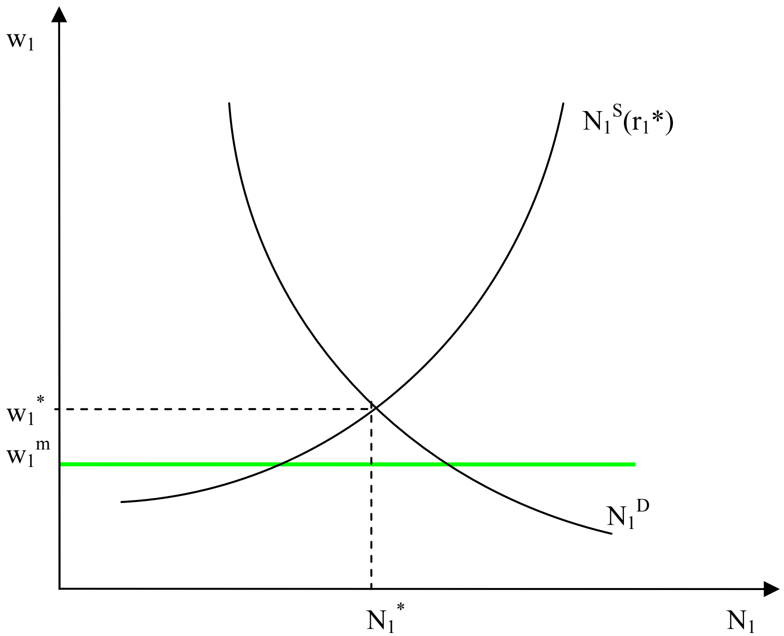

Figure 3

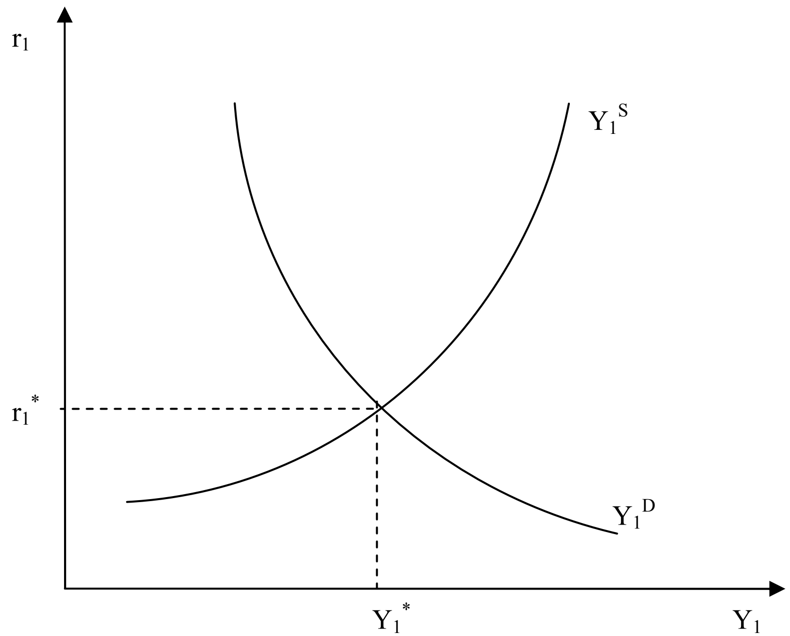

Figure 4

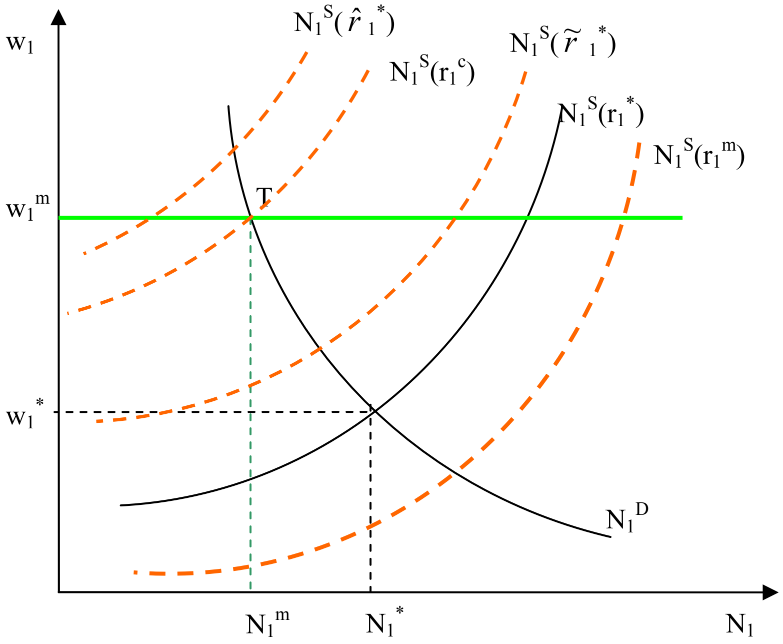

Figure 5

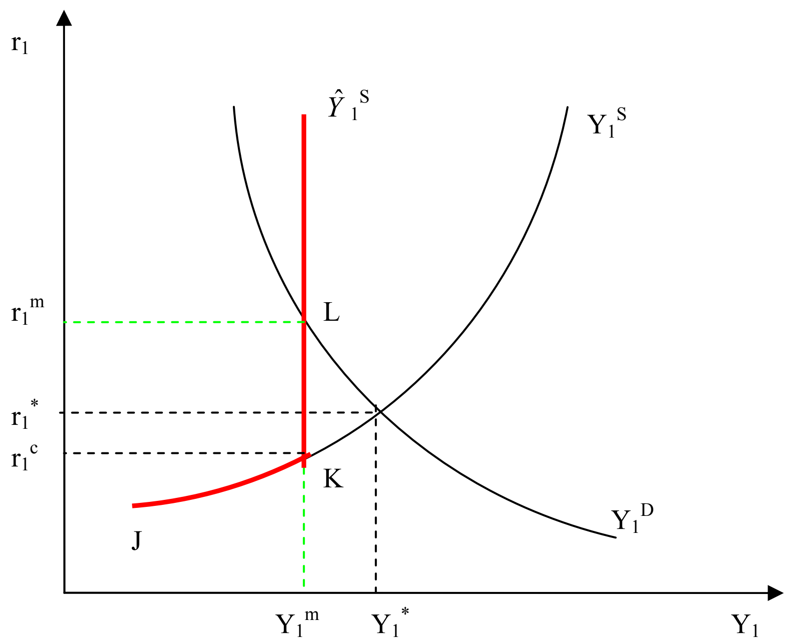

Figure 6

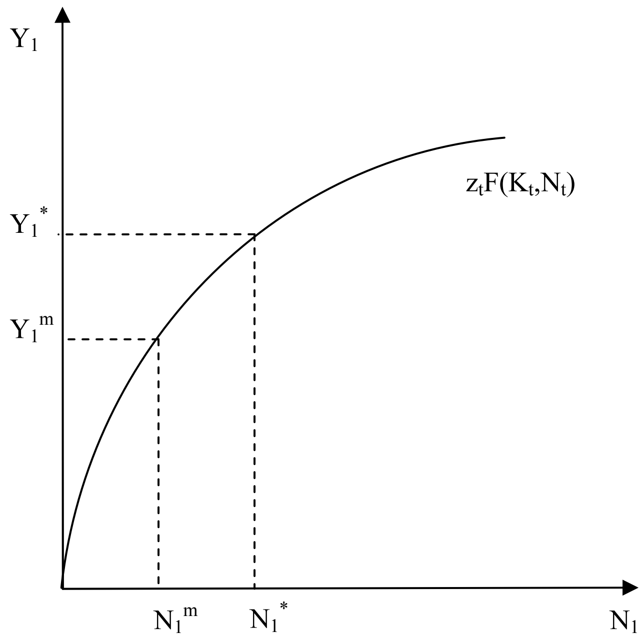

Figure 7