EC2102 Macroeconomic Analysis I

Semester 2, 2008/9

Midterm Exam

4 March 2009

Please read the following INSTRUCTIONS carefully:

1. Do NOT turn over this page until you are told to do so, i.e., Do NOT start reading the questions until you are told to do so.
2. There are 3 pages in your question booklet, including this front page.
3. The duration of this exam is 1 hour 10 minutes.
4. This exam consists of 2 questions; all of them are compulsory questions, worth a total of 100 points. Please allocate your time wisely.
5. Write ALL your answers in the exam booklets provided only, including any figures you may have drawn.
6. Please write your tutorial time slot (and/or tutorial group number) and matriculation number on all the exam booklets used. Do NOT write your name on the exam booklets. Please indicate on the front page of your exam booklet the number of exam booklets handed in.
7. You are not allowed to bring the question paper out of the examination venue. Hand up all unused exam booklets as well.
8. Points will be taken off if instructions are not followed.

Question 1 (60 points)

Suppose an agent lives for 3 periods. Let y_t be his endowment in time period t, $t = 1, 2, 3$. His per period utility function is $u(c_t)$, where c_t denotes quantity of consumption goods, whereby $u'(c_t) > 0$ and $u''(c_t) < 0$ for $c_t > 0$. He has access to a credit market where he can lend and borrow freely at r_t, the interest rate between time period t and $t+1$, which he takes as given. Let $\beta \in (0, 1)$ be his discount factor.

PART A

(i) Write down each period's budget constraint for this agent. (6 points)

(ii) Combine the per period budget constraints in part (i) to write down his lifetime budget constraint (LBC). (Hint: Use the same trick as we did in class). (8 points)

(iii) Write down this agent's optimization problem where the LBC is the constraint. (5 points)

(iv) Suppose that $r_1 = r_2 = r$. From the LBC, express c_1 as a function of c_2 and c_3, and now write down the agent's optimization problem again, but without any constraint. (8 points)

(v) Derive the First Order Conditions (FOCs) to the optimization problem of part (iv). (8 points)

(vi) Rearrange the FOCs derived in part (v) to show that these conditions hold:

$$-\frac{u'(c_1^*)}{\beta u'(c_2^*)} = -(1+r); \qquad (I)$$

$$-\frac{u'(c_2^*)}{\beta u'(c_3^*)} = -(1+r). \qquad (II)$$

(8 points)

(vii) Explain what equations (I) and (II) mean. (7 points)

PART B

Now suppose that this individual does not have access to the credit market, and also assume that consumption goods cannot be stored. Without any computation, explain what his optimal consumption choices are each period. (10 points)

Question 2 (40 points)

Time goes on forever. An economy consists of an infinitely-lived individual who values consumption and leisure, and has h units of time each period; an infinitely lived representative firm which is owned by the consumer; and a government. Let us assume that the government plans to spend G every period, where $G > 0$ (i.e., $G_t = G > 0$ for all t), and its tax revenues of T_t levied on the representative consumer is such that its lifetime budget constraint holds. Production at the representative firm is $Y_t = z_t F(K_t, N_t)$.

Suppose also that the economy is initially in an equilibrium. The labour and goods market equilibria in time period 1 are illustrated in figure 1.

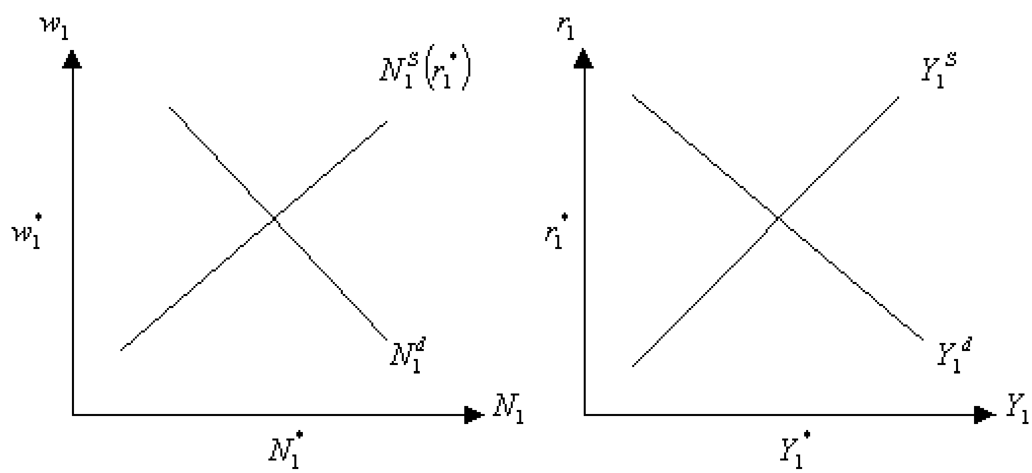

Figure 1

At the beginning of time period 1 you expect there to be an decrease in z_2 from z_2 to $\widehat{z}_2$. Explain clearly the impact of this on equilibria in the labour and goods market in time period 1 as illustrated in figure 1, taking care to explain clearly the impact on the decisions made in time period 1 by the representative consumer and the representative firm.

__________________ End of Midterm __________________

Suggested Solutions to
EC2102 Macroeconomic Analysis I
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Question 1 (60 points)

(*i*)

$$
\begin{aligned}
c_1 + s_1 &= y_1 && (1)\\
c_2 + s_2 &= y_2 + (1 + r_1)\, s_1 && (2)\\
c_3 + s_3 &= y_3 + (1 + r_2)\, s_2 && (3)
\end{aligned}
$$

but since the agent dies at the end of period 3, $s_3 = 0$. (You can write $s_3 = 0$ either in part (*i*) or (*ii*)) (6 points)

(*ii*) From (1),

$$s_1 = y_1 - c_1,$$

and substituting the above into (2),

$$
\begin{aligned}
c_2 + s_2 &= y_2 + (1 + r_1)(y_1 - c_1)\,, \text{ or}\\
c_1 + \frac{c_2}{1 + r_1} + \frac{s_2}{1 + r_1} &= y_1 + \frac{y_2}{1 + r_1} && (4)
\end{aligned}
$$

From (3),

$$s_2 = \frac{c_3}{1 + r_2} - \frac{y_3}{1 + r_2},$$

so substituting the above into (4), we get his LBC:

$$c_1 + \frac{c_2}{1 + r_1} + \frac{c_3}{(1 + r_1)(1 + r_2)} = y_1 + \frac{y_2}{1 + r_1} + \frac{y_3}{(1 + r_1)(1 + r_2)}. \qquad (5)$$

(8 points)

(*iii*)

$$
\begin{aligned}
&\max_{c_1, c_2, c_3} u(c_1) + \beta u(c_2) + \beta^2 u(c_3)\\
&s.t.\ (5).
\end{aligned}
$$

(5 points)

(iv) If $r_1 = r_2 = r$, the LBC becomes:

$$c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} = y_1 + \frac{y_2}{1+r} + \frac{y_3}{(1+r)^2} \equiv \varpi.$$

Therefore,

$$c_1 = c_1(c_2, c_3) = \varpi - \frac{c_2}{1+r} - \frac{c_3}{(1+r)^2}. \tag{6}$$

Hence, the agent's optimization problem now is

$$\begin{array}{l} \max_{c_2,c_3} u(c_1(c_2,c_3)) + \beta u(c_2) + \beta^2 u(c_3) \\ \textit{where } c_1(c_2,c_3) \text{ is given by (6).} \end{array}$$

(8 points)

(v)

$$F.O.C.(c_2) \quad : \quad u'(c_1^*)\left(\frac{-1}{1+r}\right) + \beta u'(c_2^*) = 0 \tag{7}$$

$$F.O.C.(c_3) \quad : \quad u'(c_1^*)\left(\frac{-1}{(1+r)^2}\right) + \beta^2 u'(c_3^*) = 0 \tag{8}$$

(8 points)

(vi) From (7),

$$\begin{aligned} \frac{u'(c_1^*)}{u'(c_2^*)} &= \beta(1+r) \qquad (9) \\ \frac{u'(c_1^*)}{\beta u'(c_2^*)} &= 1+r, \text{ or} \\ -\frac{u'(c_1^*)}{\beta u'(c_2^*)} &= -(1+r), \end{aligned}$$

which is just equation (I).

From (8),

$$\frac{u'(c_1^*)}{\beta(1+r)u'(c_3^*)} = \beta(1+r),$$

but using equation (9), the above becomes

$$\frac{u'(c_1^*)}{\beta(1+r)u'(c_3^*)} = \frac{u'(c_1^*)}{u'(c_2^*)}$$

which is just

$$\begin{aligned} \frac{u'(c_2^*)}{\beta u'(c_3^*)} &= (1+r), \text{ or} \\ -\frac{u'(c_2^*)}{\beta u'(c_3^*)} &= -(1+r), \end{aligned}$$

which is just equation (II).

Rearrange the FOCs derived in part (v) to show that these conditions hold:

$$-\frac{u'(c_1^*)}{\beta u'(c_2^*)} = -(1+r); \qquad (I)$$

$$-\frac{u'(c_2^*)}{\beta u'(c_3^*)} = -(1+r). \qquad (II)$$

(8 points)

(vii) $RHS\,(I)$ is the relative price of consumption between time periods 1 and 2 (with a minus). $LHS\,(I)$ is the marginal rate of substitution between consumption in time periods 1 and 2 evaluated at the optimum. Equation (I) just states that in equilibrium, the agent's rate of tradeoff between consumption in time periods 1 and 2 has to equal the relative price of consumption between time periods 1 and 2.

$RHS\,(II)$ is the relative price of consumption between time period 2 and 3 (with a minus). $LHS\,(II)$ is the marginal rate of substitution between consumption in time periods 2 and 3 evaluated at the optimum. Equation (II) states that in equilibrium, the agent's rate of tradeoff between consumption in time periods 2 and 3 has to equal the relative price of consumption between time periods 2 and 3. (7 points)

PART B

Since he has no access to the credit market, he cannot borrow or save each period by going to the credit market, and since he cannot store the good, he cannot save either. Hence, it must be that he consumes his endowment each period, that is, $c_t^* = y_t$ (10 points)