EC2102 Macroeconomic Analysis I

Semester 2, 2008/9

Midterm Exam

6 March 2009

Please read the following INSTRUCTIONS carefully:

1. Do NOT turn over this page until you are told to do so, i.e., Do NOT start reading the questions until you are told to do so.
2. There are 3 pages in your question booklet, including this front page.
3. The duration of this exam is 1 hour 10 minutes.
4. This exam consists of 2 questions; all of them are compulsory questions, worth a total of 100 points. Please allocate your time wisely.
5. Write ALL your answers in the exam booklets provided only, including any figures you may have drawn.
6. Please write your tutorial time slot (and/or tutorial group number) and matriculation number on all the exam booklets used. Do NOT write your name on the exam booklets. Please indicate on the front page of your exam booklet the number of exam booklets handed in.
7. You are not allowed to bring the question paper out of the examination venue. Hand up all unused exam booklets as well.
8. Points will be taken off if instructions are not followed.

Question 1 (60 points)

Let us consider an individual's problem. Suppose this individual lives for two periods, periods 1 and 2. There are h units of time each period. His per period utility function is $u(c_t) + v(l_t)$, where c_t denotes quantity of consumption goods and l_t denotes the amount of leisure consumed in time period t. Let r_t be the interest rate between time periods t and $t+1$, and w_t be the wage rate at time t. The agent takes all r_t and w_t as given. Let $\beta \in (0,1)$ be his discount factor. This individual pays a lump sum tax of τ_t at time t.

PART A

(i) Write down each period's budget constraint for this agent. (4 points)

(ii) Combine the per period budget constraints in part (i) to write down his lifetime budget constraint (LBC). (3 points)

(iii) Write down this agent's optimization problem where the LBC is the constraint. (5 points)

(iv) From the LBC, express l_1 as a function of c_1, c_2, and l_2, and now write down the agent's optimization problem again, but without any constraint. (8 points)

(v) Derive the First Order Conditions (FOC) to the optimization problem of part (iv). (12 points)

(vi) Rearrange the FOCs derived in part (v) to show that these conditions hold:

$$-\frac{u'(c_1^*)}{\beta u'(c_2^*)} = -(1+r_1); \tag{I}$$

$$-\frac{v'(l_1^*)}{u'(c_1^*)} = -w_1; \text{ and} \tag{II}$$

$$-\frac{v'(l_2^*)}{u'(c_2^*)} = -w_2. \tag{III}$$

(12 points)

(vii) Explain what equations (I), (II), and (III) mean. (6 points)

PART B

Suppose this individual does not value leisure at all. Without any computation, explain what his optimal leisure choices are each period. (10 points)

Question 2 (40 points)

Time goes on forever. An economy consists of an infinitely-lived individual who values consumption and leisure, and has h units of time each period; an infinitely lived representative firm which is owned by the consumer; and a government. Let us assume that the government plans to spend G every period, where $G > 0$ (i.e., $G_t = G > 0$ for all t), and its tax revenues of T_t levied on the representative consumer is such that its lifetime budget constraint holds. Production at the representative firm is $Y_t = z_t F(K_t, N_t)$.

Suppose also that the economy is initially in an equilibrium. The labour and goods market equilibria in time period 1 are illustrated in figure 1.

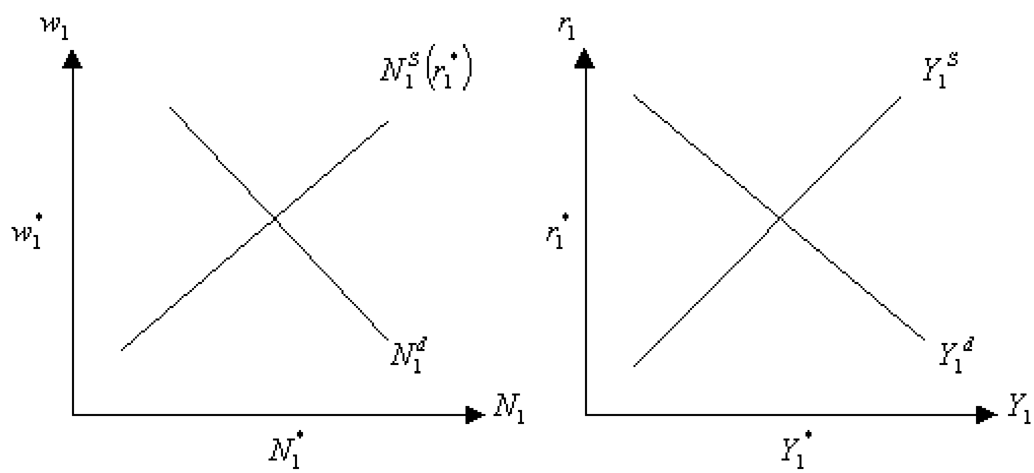

Figure 1

At the beginning of time period 1 there is a decrease in z_1 from z_1 to $\widehat{z}_1$. Explain clearly the impact of this on equilibria in the labour and goods markets in time period 1 as illustrated in figure 1, taking care to explain clearly the impact on the decisions made in time period 1 by the representative consumer and the representative firm.

__________________ End of Midterm __________________

Suggested Solutions to

EC2102 **Macroeconomic Analysis I**

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Question 1 (60 points)

PART A

(i) Letting

$$y_t^d \equiv w_t (h - l_t) - \tau_t,$$

$$\begin{aligned} c_1 + s_1 &= y_1^d && (1) \\ c_2 + s_2 &= y_2^d + (1 + r_1) s_1. && (2) \end{aligned}$$

But observe that the individual will choose $s_2 = 0$ since he dies at the end of the second period. (You can write $s_2 = 0$ either in part (i) or (ii)) (4 points)

(ii) From (2) we can express s_1 as:

$$\frac{c_2}{1 + r_1} - \frac{y_2^d}{1 + r_1} = s_1. \qquad (3)$$

Substituting (3) into (1), we get

$$c_1 + \frac{c_2}{1 + r_1} = \frac{y_2^d}{1 + r_1} + y_1^d \equiv \varpi^d. \qquad (4)$$

(3 points)

(iii)

$$\max_{c_1, c_2, l_1, l_2} u(c_1) + v(l_1) + \beta [u(c_2) + v(l_2)]$$
$$s.t. \ (4).$$

(5 points)

(iv) Since

$$c_1 + \frac{c_2}{1 + r_1} = w_1 (h - l_1) - \tau_1 + \frac{w_2 (h - l_2) - \tau_2}{1 + r_1},$$

we can express

$$l_1 = l_1(c_1, c_2, l_2) = h - \frac{\tau_1}{w_1} + \frac{w_2(h - l_2) - \tau_2}{w_1(1+r_1)} - \frac{c_1}{w_1} - \frac{c_2}{w_1(1+r_1)} \tag{5}$$

The individual's optimization problem is:

$$\begin{aligned} &\max_{c_1,c_2,l_2} u(c_1) + v(l_1(c_1,c_2,l_2)) + \beta[u(c_2) + v(l_2)] \\ &\textit{where } l_1(c_1,c_2,l_2) \text{ is given by } (5). \end{aligned}$$

(8 points)

(v)

$$F.O.C.(c_1) : u'(c_1^*) + v'(l_1^*)\left(\frac{-1}{w_1}\right) = 0 \tag{6}$$

$$F.O.C.(c_2) : v'(l_1^*)\left(\frac{-1}{w_1(1+r_1)}\right) + \beta u'(c_2^*) = 0 \tag{7}$$

$$F.O.C.(l_2) : v'(l_1^*)\left(\frac{-w_2}{w_1(1+r_1)}\right) + \beta v'(l_2^*) = 0 \tag{8}$$

(12 points)

(vi) From (6),

$$\frac{v'(l_1^*)}{u'(c_1^*)} = w_1, \quad -\frac{v'(l_1^*)}{u'(c_1^*)} = -w_1, \tag{9}$$

which is just equation (II).

From (7),

$$\frac{v'(l_1^*)}{\beta u'(c_2^*)} = w_1(1+r_1), \tag{10}$$

so substututing (9) into the above, we get

$$\begin{aligned} \frac{v'(l_1^*)}{\beta u'(c_2^*)} &= \frac{v'(l_1^*)}{u'(c_1^*)}(1+r_1), \text{ or} \\ \frac{u'(c_1^*)}{\beta u'(c_2^*)} &= (1+r_1), \text{ or } -\frac{u'(c_1^*)}{\beta u'(c_2^*)} = -(1+r_1) \end{aligned}$$

which is just equation (I).

From (8),

$$\frac{v'(l_1^*)}{\beta v'(l_2^*)} = \frac{w_1(1+r_1)}{w_2},$$

so substituting (10) into the above, we get

$$\begin{aligned} \frac{v'(l_1^*)}{\beta v'(l_2^*)} &= \frac{v'(l_1^*)}{\beta u'(c_2^*)}\frac{1}{w_2}, \\ \frac{v'(l_2^*)}{u'(c_2^*)} &= w_2, \text{ or } -\frac{v'(l_2^*)}{u'(c_2^*)} = -w_2 \end{aligned}$$

which is just equation (III) . (12 points)

(vii) $RHS\,(I)$ is the price of consumption in time period 1 relative to time period 2 (with a minus sign). $LHS\,(I)$ is the marginal rate of substitution between consumption in time period 1 relative to time period 2 evaluated at the optimum. Equation (I) states that in equilibrium, the agent's rate of tradeoff between consumption in time period 1 relative to time period 2 has to be equal to the relative price of consumption in time period 1 relative to time period 2.

$RHS\,(II)$ is the price of leisure in time period 1 relative to consumption in time period 1 (with a minus sign). $LHS\,(II)$ is the marginal rate of substitution between leisure in time period 1 relative to consumption in time period 1 evaluated at the optimum. Equation (II) states that in equilibrium, the agent's rate of tradeoff between leisure and consumption in time period 1 has to equal the relative price of leisure to consumption in time period 1.

$RHS\,(III)$ is the price of leisure in time period 2 relative to consumption in time period 2 (with a minus sign). $LHS\,(III)$ is the marginal rate of substitution between leisure in time period 2 relative to consumption in time period 2 evaluated at the optimum. Equation (III) states that in equilibrium, the agent's rate of tradeoff between leisure and consumption in time period 2 has to equal the relative price of leisure to consumption in time period 2.

(6 points)

PART B

Since this individual does not value leisure at all, he must be choosing to work all the time in the day, that is, $l_t^* = 0$ for all t. (10 points)