Solutions to EC2102 Macroeconomic Analysis I
Tutorial 10, Week 13, April 12-16, 2010

Question 1:

(i)

$$\begin{aligned}
& \max_{C_1,l_1,S_1} u(C_1,l_1)+\beta u(C_2,l_2)+... \\
s.t.\ C_1+S_1^P &= w_1(h-l_1)+\pi_1+(1+r_0)S_0^P-T_1 \quad (1)\\
C_2+S_2^P &= w_2(h-l_2)+\pi_2+(1+r_1)S_1^P-T_2 \quad (2)\\
& ...
\end{aligned}$$

Assume $S_0^P=0$. From

From (1) and (2),

$$C_2=w_2(h-l_2)+\pi_2-T_2+[w_1(h-l_1)+\pi_1-T_1-C_1](1+r_1)-S_2^P$$

Hence, the representative consumer's problem in period 1 is:

$$\begin{aligned}
& \max_{C_1,l_1} u(C_1,l_1)+\beta u(C_2(C_1,l_1),l_2)+... \\
where\ C_2 &= w_2(h-l_2)+\pi_2-T_2 \\
&\quad +[w_1(h-l_1)+\pi_1-T_1-C_1](1+r_1)-S_2^P
\end{aligned}$$

The F.O.C.s are:

$$\begin{aligned}
F.O.C.(C_1):&\quad u_1(C_1^*,l_1^*)-\beta(1+r_1)u_1(C_2^*,l_2^*)=0,\\
F.O.C.(l_1):&\quad u_2(C_1^*,l_1^*)-w_1\beta(1+r_1)u_1(C_2^*,l_2^*)=0,
\end{aligned}$$

(ii) In time period 1 the representative firm wants to pick (N_1,K_2) (or (N_1,I_1)) to maximize its stream of profits:

$$\begin{aligned}
& \max_{N_1,K_2} \pi_1+\frac{\pi_2}{1+r_1}+...,\ \text{where } \pi_t=z_tF(K_t,N_t)-w_tN_t-I_t \\
& \max_{N_1,K_2} z_1F(K_1,N_1)-w_1N_1-K_2+(1-\delta)K_1+\frac{z_2F(K_2,N_2)}{1+r_1} \\
&\qquad +\frac{(1-\delta)K_2}{1+r_1}-\frac{w_2N_2}{1+r_1}-\frac{K_3}{1+r_1}+...
\end{aligned}$$

$$F.O.C.(N_1) \ : \ z_1F_2(K_1,N_1^*)-w_1=0$$

$$F.O.C.(K_2) \ : \ -1+\frac{z_2F_1(K_2^*,N_2)+1-\delta}{1+r_1}=0,$$

(iii) Given that the government is spending and taxing (G_t, T_t) such that its LBC holds, a competitive equilibrium here is an allocation $\left(C_t^*, S_t^*, N_t^{S*}, N_t^{D*}, I_t^*\right)$, and prices (r_t^*, w_t^*), such that:

- the representative consumer chooses $\left(C_t^*, S_t^*, N_t^{S*}\right)$ in order to maximize lifetime utility, taking (r_t^*, w_t^*) as given;
- the representative firm maximizes its present value of profits by choosing $\left(N_t^{D*}, I_t^*\right)$, taking (r_t^*, w_t^*) as given; and
- the labour market and the goods market clear:
  - the labour market clears when labour supply equals labour demand, i.e., $N_t^* = N_t^{S*} = N_t^{D*}$, and
  - the goods market clears when goods supplied equals goods demanded, $Y_t^* = Y_t^s = Y_t^d = C_t^* + I_t^* + G_t$.

(NOTE: Recall that there are 3 markets here: Labour Market, Goods Market and Credit Market, so by Walras's Law, if goods and labour markets clear each period, then credit market also clears in every period.)

Question 2

(i) Assume there is no long-run inflation, the nominal and real interest rate are equal, or $R_1 = r_1$.The demand for real money balances today is given by $L(Y_1, r_1)$. The real demand for money is increasing in aggregate real output Y_1 ($\partial L(Y_1, r_1)/\partial Y_1 > 0$) and decreasing in the real interest rate r_1 ($\partial L(Y_1, r_1)/\partial r_1 < 0$). So given today's price level $\overline{P}_1$, and income level $\hat{Y}_1$, money demand is downward sloping as a function of real interest rate.

Once the nominal rate of interest has reached zero, money and government bonds are essentially equivalent assets since zero nominal interest rate means the opportunity cost of holding money is zero, so the consumer is indifferent between holding bonds and money. The MD curve therefore becomes horizontal when interest rates are zero. See Figure 2.1.

Given that the nominal money supply is determined exogenously by the government, equilibrium in the money market today is when nominal money supply equals nominal money demand, or $\hat{M}_1 = M_1^s = M_1^d = \bar{P}\,L(\hat{Y}_1, r_1)$, given the price level $\hat{P}_1$ and output level $\hat{Y}_1$, we have an output-interest rate pair $(\hat{Y}_1, \hat{r}_1)$, for which the money market is in equilibrium.

To derive LM_1 curve, suppose that the money market was originally at equilibrium as describe above. Every point on LM_1 denotes an equilibrium in money market. Hence, we already have one point $(\hat{Y}_1, \hat{r}_1)$.

Now let the level of real income rise to $\tilde{Y}_1$. Hence, nominal money demand must increase for any given real interest rate, so $\overline{P}L(\hat{Y}_1, r_1)$ shifts to the right to $\tilde{M}_1^d = \overline{P}_1 L(\tilde{Y}_1, r_1)$, real interest rate thus rises from $\hat{r}_1$ to $\tilde{r}_1$. We have another output interest rate pair $(\tilde{Y}_1, \tilde{r}_1)$, for which the money market is in equilibrium. So we have another LM_1 curve. This part of LM_1 is upward sloping.

We continue decreasing real income, and money demand curve still shifts to the left, and just intersects money supply curve at zero interest rate. So we have a point of $(Y_1^{'}, 0)$ on LM_1 curve.

We continue decreasing real income, and money demand curve still shifts to the left, but the interest rate remains at zero since money supply intersects money demand function when the latter is horizontal at $r_1 = 0$. Any further reduction in income can not restore equilibrium in the money market. The LM_1

curve therefore becomes horizontal at a zero rate of interest.

Continually changing real income levels, we can derive the entire LM_1 curve, which include a flat part when nominal interest rate is zero and an upward-sloping part when the interest is above zero.

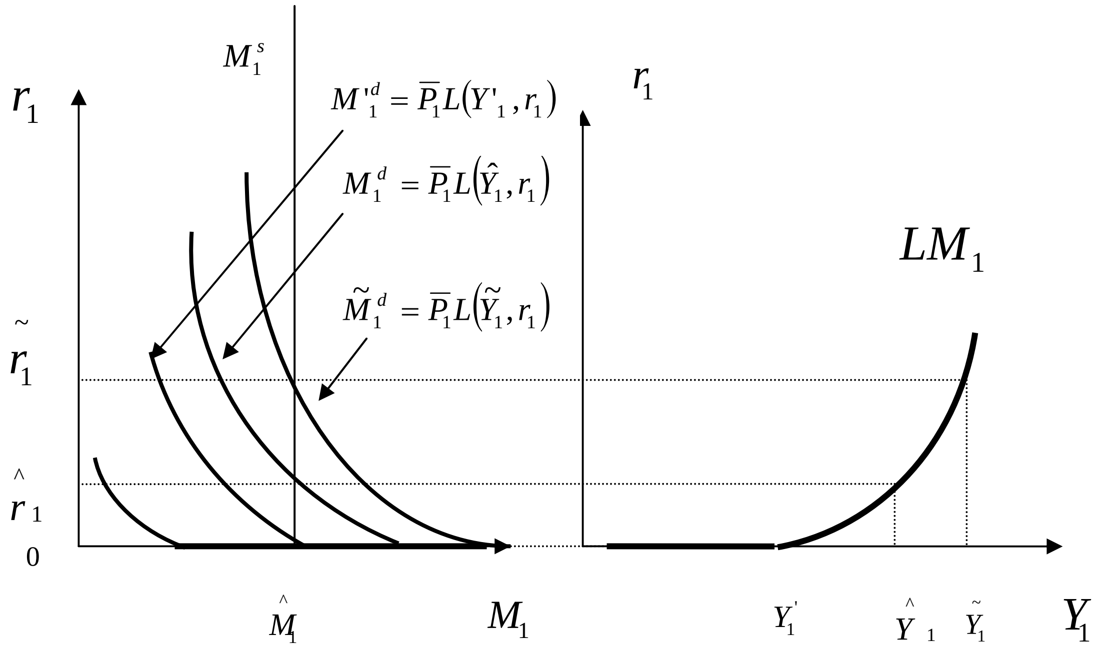

Figure 2.1

(ii) The increase of the money supply shifts LM curve down. However, the intersection of IS and the new LM curve remains at the same position, it has no effect on the interest rate and output level, so the expanding monetary policy becomes powerless in this case. See Figure 2.2.

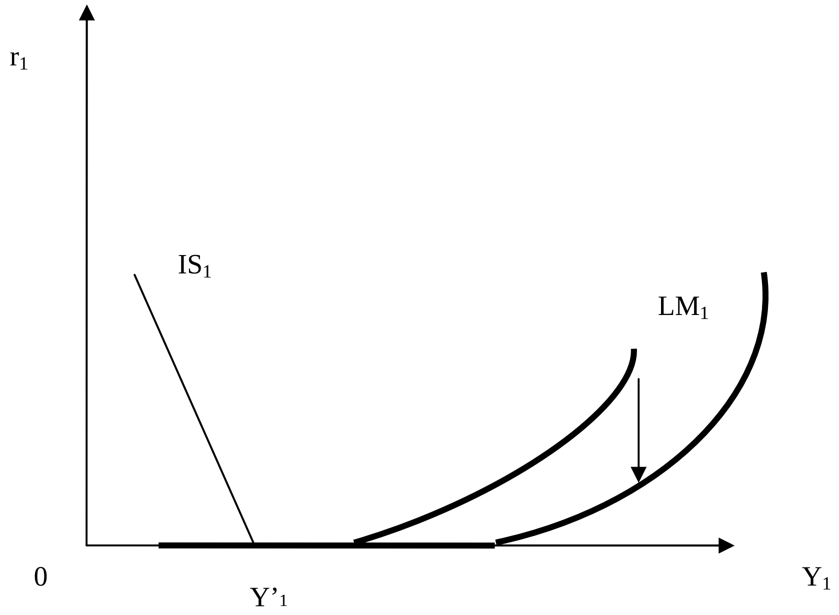

Figure 2.2

(iii) The Keynesian sticky wage model predicts that the monetary policy becomes powerless when the nominal interest rate is zero. It has no effect on the output level or employment rate. The government should adopt other policy (e.g. fiscal policy) to expand the economy.

Question 3:

The central bank learning story is from your lecture notes, so please refer to your notes.