Introducing Money into our Model (1/4)
Reading: Chapter 10, Williamson

- What is money?

Money is anything that has the following properties simultaneously:
- a medium of exchange (accepted in exchange for goods because it can be used in trade for other goods)
- a store of value (can use it to trade current for future goods, like other assets)
- a unit of account (e.g., contracts can be denominated in terms of money),

Examples include:
- fiat money
- commodity money, like cigarettes used by POWs during world wars
- legal tender

Introducing Money into our Model (2/4)

- Why should we study money?

1. changes in price level can have real effects (i.e., effects on real wages, real output, etc)
2. economy seems to function better with money than without
   - imagine a barter economy; require double coincidence of wants
   - Understanding why money exists and how its properties influence people's trading decisions can help us in designing good monetary policy

- How do you introduce money into our model?

Introducing Money into our Model (3/4)

1. Model why people hold money, or
2. Just assume money exists and is used without asking why it exists

Model we study here is like in 2., where money is assumed to exist

We're going to look at a <u>cash-in-advance</u> (CIA) model:

- model transactions as requiring money
- convenient to think of money as currency.

New market: <u>money market</u>

- money demanded by representative consumer
- money supplied by central bank

Introducing Money into our Model (4/4)

We still have an infinitely-lived representative consumer, an infinitely-lived representative firm, just like before. But now we have introduced money, so variables now denoted in nominal terms as opposed to real terms

The Fisher Equation

P_t : price level
R_t : nominal interest rate
r_t : real interest rate
i_t : inflation rate
Fisher relation/equation:

$$1 + r_t = (1 + R_t)\frac{P_t}{P_{t+1}} = \frac{1 + R_t}{1 + i_t}$$

rearranging,

$$\begin{aligned} r_t &= R_t - i_t - r_t i_t \\ R_t &\approx r_t + i_t \text{ if } (r_t i_t) \text{ small} \end{aligned}$$

For instance, if $r_t = 3\%$ and $i_t = 2\%$, the approximated Fisher equation gives $R_t = 5\%$, whereas the true value is $1.02 \times 1.03 - 1 = 5.06\%$. Hence, the difference between the two values are small enough.

Cash-in-advance Model: Timeline (1/3)

Sequence of events (all transactions carried out using money):

1. day begins in period t; representative consumer starts day with two assets carried over from yesterday, M_{t-1} and B_{t-1}, each measured in units of money
- Note that B_t are nominal bonds; previously bonds denoted in real terms

2. consumer pays lump sum tax to government: T_t (real), or $P_t T_t$ (nominal), where P_t: current price of consumption good in terms of money, or current price level.

3. consumer goes to credit market,
- gets $B_{t-1}(1 + R_{t-1})$ in money; R_{t-1} : yesterday's nominal interest rate;

Cash-in-advance Model: Timeline (2/3)

- chooses how much bonds to buy today B_t^d, using money

    $B_t^d > 0$ means consumer is saving/lending; $B_t^d < 0$ means consumer is borrowing. who is consumer buying from? Government.

4. consumer goes to work, again choosing how much labour to supply $(h - l_t)$. real wage is w_t, so nominal wage he gets is $P_t w_t$

    - wages are not received until after firm sells goods

5. consumer gets off work and goes to goods market to purchase consumption goods from representative firm, using money. Can only use what money he has on hand to pay for goods, thus <u>cash-in-advance constraint</u>:

$$P_t C_t \leq M_{t-1} + B_{t-1}(1 + R_{t-1}) - P_t T_t - B_t^d$$

Cash-in-advance Model: Timeline (3/3)

At the same time, government also enters goods market and purchases consumption goods from firm, using money.

6. day comes to an end; firm has sold its goods, now pays consumer its wages $P_t w_t (h - l_t)$ and dividend income $P_t \pi_t$, both using money. this then determines how much money balances he carries over tomorrow, M_t^d

Implicit assumption: consumer cannot be in more than one market at the same time

Budget constraint for period t:

$$\begin{aligned} & P_t C_t + B_t^d + M_t^d \\ = \; & M_{t-1} + B_{t-1} (1 + R_{t-1}) + P_t w_t (h - l_t) + P_t \pi_t - P_t T_t \end{aligned}$$

CIA Model: Consumer's Max. Problem (1/2)

Consumer's maximization problem today (period 1) is:

$$\max_{C_1, l_1, B_1^d, M_1^d} u(C_1, l_1) + \beta u(C_2, l_2) + \ldots$$

$s.t.$ for all $t \geq 1$ the CIA constraints and BCs are satisfied, i.e.,

$$\begin{aligned} P_t C_t &\leq M_{t-1} + B_{t-1}(1 + R_{t-1}) - P_t T_t - B_t^d \\ P_t C_t + B_t^d + M_t^d &= M_{t-1} + B_{t-1}(1 + R_{t-1}) + P_t w_t (h - l_t) \\ &\quad + P_t \pi_t - P_t T_t \end{aligned}$$

Note: wage $P_t w_t$ and dividend income $P_t \pi_t$ cannot be spent on period t consumption goods but must be held as money until period $(t+1)$

CIA Model: Consumer's Max. Problem (2/2)

Assume $R_t > 0$. Therefore no reason to hold money unless it's absolutely necessary: Money is return-dominated by bonds. Hold just enough to purchase consumption goods in goods market, so cash-in-advance constraint holds with equality.

The Demand for Money(1/2)

But how much money is held? Demand for money in real terms today:

$$\frac{M_1^d}{P_1} = L\left(Y_1, R_1\right)$$

$\frac{\partial L(Y_1, R_1)}{\partial Y_1} > 0$ ($\nearrow$ in Y_1, taking it as a proxy for lifetime wealth, implies consume$\nearrow$, need $\nearrow$ money

$\frac{\partial L(Y_1, R_1)}{\partial R_1} < 0$ (R_1 is opportunity cost of holding money)

Can write nominal demand for money:

$$M_1^d = P_1 L\left(Y_1, R_1\right)$$

The Demand for Money(2/2)

Using Fisher's equation, $R_1 \approx r_1 + i_1$. All experiments we consider do not affect i_1, so i_1 is a constant, set it to zero for convenience. Hence

$$M_1^d = P_1 L \left(Y_1, r_1\right)$$

If Y_1 increases or r_1 decreases, then M_1^d curve shifts to the right (pivoting at origin).

So we have our M_1^d curve, see "figures 10.5" and 10.6"

What about M_1^S? From Central Bank.

Money Supply: Central Banker

Your textbook assumes for simplicity that government is also the central banker. Not an innocuous assumption, so we will drop it.

We will consider an independent central bank which is in charge of monetary policy; let our government be in charge of fiscal policy.

In the US, the Fed (Federal Reserve System) is the central bank; and the Treasury implements fiscal policy.

In Singapore, MAS is the central bank; and the government decides fiscal policy.

Central bank here has to decide what M_t is every period.

Government

In nominal terms, period t's BC is:

$$P_t G_t + (1 + R_{t-1})\, B_{t-1} = P_t T_t + B_t$$

We can write government's lifetime BC:

$$P_1 G_1 + \frac{P_2 G_2}{1 + R_1} + \ldots = P_1 T_1 + \frac{P_2 T_2}{1 + R_1} + \ldots \tag{27}$$

Competitive Equilibrium (1/3)

- Four markets: Labour Market, Goods Market, Credit Market, and Money Market
  - Remember that we only need to care about 3 markets (fourth market clears by Walras' Law): drop the credit market

Competitive Equilibrium (2/3)

A competitive equilibrium for the cash-in-advance economy, given $\{G_t, T_t, M_t\}$, is an allocation $\left\{C_t^*, M_t^*, B_t^*, N_t^{S*} I_t^*, N_t^{D*}\right\}$ and prices $\{P_t^*, r_t^*, w_t^*\}$ such that:

- the representative consumer chooses $\left\{C_t^*, M_t^*, B_t^*, N_t^{S*}\right\}$ each period in order to maximize lifetime utility, subject to cash-in-advance constraint and per period budget constraint, taking as given (P_t^*, r_t^*, w_t^*),

- the representative firm maximizes its present value of profits by choosing each period $\left\{I_t^*, N_t^{D*}\right\}$, taking as given (P_t^*, r_t^*, w_t^*)

Competitive Equilibrium (3/3)

- the labour market, goods market, and money market clear:
  - * labour market equilibrium, where labour supply equals labour demand, pins down the equilibrium wage rate and equilibrium quantity of employment
    * goods market equilibrium, where goods supplied equals goods demanded, pins down the equilibrium interest rate and equilibrium output
    * money market equilibrium, where money demanded equals money supplied, pins down the equilibrium price level ("figure 10.7")

"Figure 10.8"

Increase in Money Supply

Suppose money supply has always been M every period, and now, it has been decided that money supply today increases to $\widehat{M}$ in period 1 and stays constant at $\widehat{M}$ (level increase in money supply)

How can we obtain a level increase in money supply?

1. Print money. When given to consumers as transfers, Milton Friedman calls it "helicopter drop"

2. Open Market Operation (OMO): reduce the quantity of bonds, and therefore debt, circulated. OMOs in practice are carried out this way: Fiscal authority issues government bonds (debt), and the central bank buys this debt by issuing money, this is called open market purchase.

Money Neutrality (1/3)

A model displays **money neutrality** when a change in the quantity of money leaves the real variables unchanged, and therefore only the nominal variables are affected. **The CIA model developed displays money neutrality**. Money neutrality is also called the **Classical dichotomy**.

Classical dichotomy in our model because real variables are solved for in the labor and goods markets, and the price level is then determined, given real output, in the money market. So **real** variables are determined separately from **nominal** variables.

Suppose that when money supply rises from M to $\widehat{M}$, given as helicopter drops, and suppose we were in an equilibrium before the rise in money supply as in "figure 10.8" where in time period 1 goods market cleared at $\left(Y_1^*, r_1^*\right)$, labour market cleared at $\left(N_1^*, w_1^*\right)$, and money market cleared at $\left(M_1^*, P_1^*\right)$.

Money Neutrality (2/3)

It can be shown that with a rise in money supply

- the only equilibrium when the model is solved is that an increase in money supply changes prices proportionately (for example, doubling money supply doubles price level)

All an increase in money supply does is to increase the equilibrium price level. Goods and labor market unaffected by increase in money supply.

Since Y_1^* and r_1^* are unaffected this means that $L\left(Y_1^*, r_1^*\right)$ is unchanged. But $\frac{M_1^d}{P_1} = L\left(Y_1^*, r_1^*\right)$, so if money supply increases price will increase.

Money Neutrality (3/3)

More precisely the price must increase by exactly the same proportion as the increase in money supply. In fact, money neutrality implies that

$$\frac{M}{P_1} = L\left(Y_1^*, r_1^*\right) = \frac{\widehat{M}}{\widehat{P}_1} = \frac{M + \Delta M}{P_1 + \Delta P_1},$$

so $\frac{P_1+\Delta P_1}{P_1} = \frac{M+\Delta M}{M}$, which simplifies to $1 + \frac{\Delta P_1}{P_1} = 1 + \frac{\Delta M}{M}$, that is

$$\underbrace{\frac{\Delta P_1}{P_1}}_{\text{\% change in prices}} = \underbrace{\frac{\Delta M}{M}}_{\text{\% change in money supply}}.$$

This result makes sense: if there is twice as much money in the economy, prices double to “absorb” the extra liquidity.

See "figure 10.10"

Money Neutrality: A Check (1/3)

To get some intuition for this result, let us look at consumer's CIA_1 constraint, and BC_1 before money supply rises. To simplify, let us assume that there are no bond holdings.

First observe that CIA_1 has to hold with equality. Thus we have:

$$\begin{aligned} CIA_1 &: P_1 C_1 = M - P_1 T_1 \\ BC_1 &: P_1 C_1 + M_1^d = M + P_1 w_1 (h - l_1) + P_1 \pi_1 - P_1 T_1 \end{aligned} \tag{28}$$

Combining the two equations above, we can write

$$\begin{aligned} M_1^d &= P_1 w_1 (h - l_1) + P_1 \pi_1, \text{ or} \\ \frac{M_1^d}{P_1} &= w_1 (h - l_1) + \pi_1, \end{aligned}$$

Money Neutrality: A Check (2/3)

In equilibrium in the money market, $M_1^d = M_1^S = M$, so

$$\frac{M}{P_1} = w_1 (h - l_1) + \pi_1 \tag{29}$$

If money supply now doubles, and price doubles as a consequence, LHS (29) is unchanged, and hence, RHS (29) has to be unchanged.

From equation (28),

$$C_1 = \frac{M}{P_1} - T_1 \tag{28$'$}$$

Money Neutrality: A Check (3/3)

so if money supply doubles from M to $\widehat{M}$, for instance, and consumer gets a transfer of the entire amount of money printed, so he is able to bring into goods market $2(M) = \widehat{M}$ to purchase goods, and price doubles, $LHS\,(28')$ is unchanged, and $RHS\,(28')$ is also unchanged.

All that has happened is that the price level has doubled, but real variables are unchanged.

Comments

1. Money may be neutral, but money matters, because all transactions are carried out using money.

2. Money neutrality is subject to a lot of debate. Economists generally agree that money is neutral in the long run. The question is whether money is neutral in the short run. In this model, money is neutral in the short and long run.

3. The equilibrium in goods and labour market are not the same equilibrium as in the model without CIA constraint that we studied in previous lectures!

Monetary Policy: Targets and Policy Rules

If money is neutral, then why should we care about monetary policy? Well, referring to point 2 above, money may be neutral in the long run, but it may not be neutral in the short run? Besides, inflation does matter for consumers' welfare. Furthermore, central banks, who have control of money supply, often have mandates to achieve certain "targets."

In the US, Fed has a dual mandate: price stability (inflation) and full-employment For the ECB (European Central Bank), their mandate is to target inflation ($\leq 2\%$)

Jean-Claude Trichet (President of ECB) and his team perceived as "hawkish" when not keen on lowering interest rates during the financial crisis as inflation is above target in many EU countries; compare it to Ben Bernanke.

Suppose central bank cares about **inflation targeting**. In particular, let us assume they want inflation rate to be zero. See "figure 10.17", "figure 10.18", and "figure 10.19"

If the central bank can do precisely the "right" thing, then they can indeed target inflation.

What about **money supply targeting** as proposed by Milton Friedman? Say, increase M1 by x% a year, every year?

In reality, money supply targeting was very popular in 1970s and 1980s, and then abandoned, because if central bank cares about prices stability, you don't get price stability with money supply targeting, and monetary policy cannot respond to shocks. In fact, if money supply target is high, what you get at the end of the day is a high rate of inflation

The Fed seems to try for **nominal interest rate targeting**. Every six weeks, the FOMC (Federal Open Market Committee) meets to decide on a new nominal fed funds target rate (federal funds rate), which is the interest rate on interbank overnight lending (effectively close to 0% now: [0, 0.25%]). By the Fisher equation, if inflation rate is zero, then this is like targeting the real rate. Question is when they will raise this. They have already raised the discount window from 0.5% to 0.75%.

What about the **Taylor Rule**? Requires that the central bank set nominal interest rate target based on observed inflation rate and aggregate output relative to an inflation target and "potental" output respectively. That is, there should be a higher nominal interest rate target the larger the "output gap", i.e., difference between actual and potential output, and the larger the difference between and actual inflation rate and target inflation rate.

What's going on now in UK and US?

In UK and US, target interest rates of central banks are close to zero; and in the US at least, in practice it is effectively zero. So does it mean they is no more room for expansionary monetary policy?
- new expansionary policy the US tried last year and two years ago: **quantitative easing** as attempted in Japan in the 1990s.