Suggested Solutions to EC2102 Macroeconomic Analysis I
Tutorial 2, Week 4 (February 1-5, 2010)

Question 1 (This is a continuation of Question 1 of Tutorial 1)

(v) We know that at the equilibrium real interest rate of r^*,

$$s_1^{A*}(r^*) + s_1^{B*}(r^*) = 0.$$

Substituting Mr A and Mr B's savings functions from part (i) of this question into the above, we obtain:

$$10 - \frac{10(1+r^*)+50}{\left[\beta^2(1+r^*)+1\right](1+r^*)} + 50 - \frac{50(1+r^*)+10}{\left[\beta^2(1+r^*)+1\right](1+r^*)} = 0,$$

which you can show, after algebraic manipulation, that

$$r^* = \frac{1}{\beta} - 1.$$

Differentiating r^* with respect to β

$$\frac{dr^*}{d\beta} = -\frac{1}{\beta^2} < 0.$$

As β increases (both individuals are more patient), a lower interest rate is needed, in equilibrium, to compensate individuals' for their savings, and vice versa. This is because as β increases given some initial real interest rate r^* that cleared the credit market, both Mr. A and B are more patient. At this original interest rate of r^*, the saver would like to save more and the borrower would like to borrow less. If r^* remained at its original level, their new desired savings in total would exceed zero, and hence, the credit market cannot be in equilibrium. There are now too much savings in the economy. For the credit market to be in equilibrium, the equilibrium interest rate needs to be lower, so that the saver will choose to save less, and the borrower will choose to borrow more. The credit market will be in equilibrium when the interest rate has decreased sufficiently so that Mr. A and Mr. B's desired savings choice at this new value of β summed up to zero.

(vi) Given $\beta = 0.8$,

$$r^* = 0.25.$$

It is easy to check that

$$
\begin{aligned}
c_1^{A*} &= 27\frac{7}{9};\ s_1^{A*} = -17\frac{7}{9};\ c_2^{A*} = 27\frac{7}{9}; \\
c_1^{B*} &= 32\frac{2}{9};\ s_1^{B*} = 17\frac{7}{9};\ c_2^{B*} = 32\frac{2}{9}.
\end{aligned}
$$

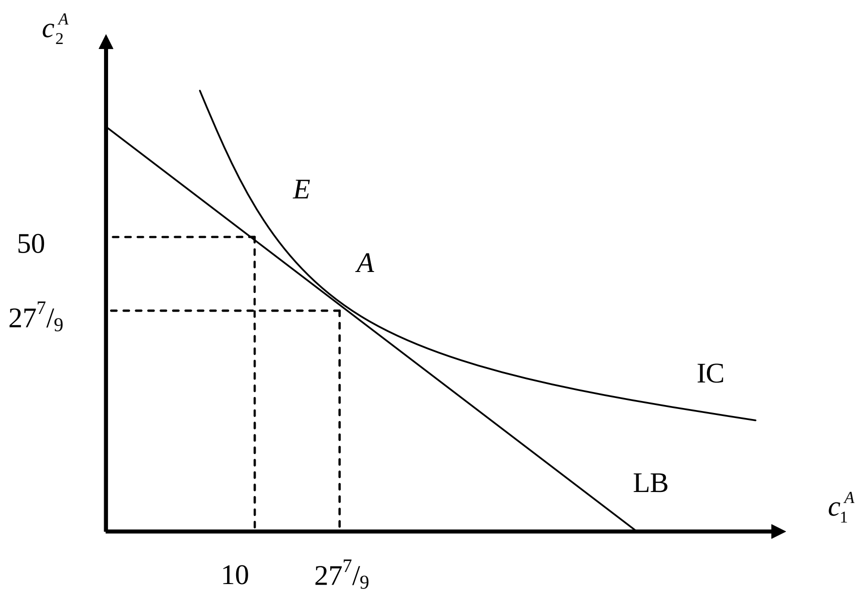
Hence, Mr A is a borrower while Mr B is a lender.

Their consumption profiles are flat because

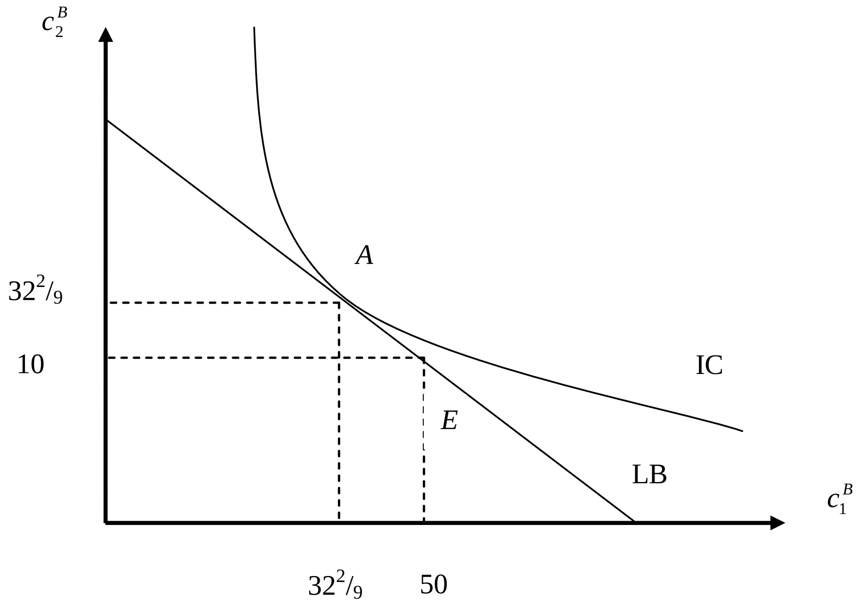
$$\rho = \frac{1}{\beta} - 1 = 0.2 = r^*$$

(vii) Let E denote the endowment point and let A denote the optimal consumption bundle.

For Mr A, since he is a borrower:



For Mr B, since he is a lender:



$(viii)$ A competitive equilibrium for the economy considered here is an allocation $\left(c_1^{j*}, c_2^{j*}, s_1^{j*}\right)$, $j = A, B$, and an interest rate r^*, such that:

- for each individual j, taking the interest rate r^* as given, $\left(c_1^{j*}, c_2^{j*}, s_1^{j*}\right)$ is the solution to his lifetime utility maximization problem;

- the credit market clears at the interest rate r^*, i.e.,

$$s_1^{A*}\left(r^*\right) + s_1^{B*}\left(r^*\right) = 0.$$

(ix) The equilibrium is said to be "competitive" as each individual takes the interest rate r^* as given when optimizing.

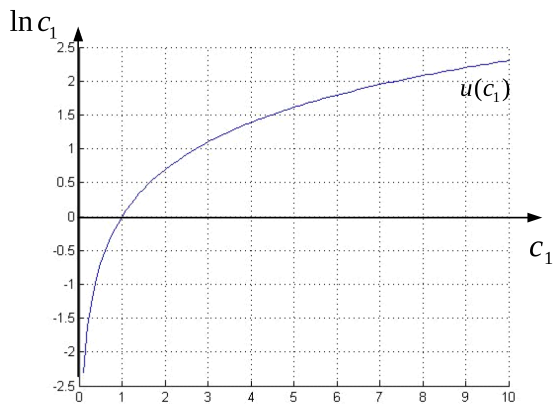
We only need to check that either the good or credit market clears because of Walras' law: if the credit market clears, it implies the good market clears, and vice versa.

Question 2

(i)

$$\begin{aligned} u(c_t) &= \ln(c_t); \\ u'(c_t) &= \frac{1}{c_t} > 0 \text{ for } c_t > 0; \\ u''(c_t) &= -\frac{1}{c_t^2} < 0 \text{ for } c_t > 0. \end{aligned}$$

(ii) Since utility is ordinal, only the relative ranking matters, it does not matter when the level of utility is negative.



(iii) It must be that

$$\begin{aligned} c_1^{A*} &= c_2^{A*} = c_1^{B*} = c_2^{B*} = 50, \text{ and} \\ s_1^{A*} &= s_1^{B*} = 0. \end{aligned}$$

(iv) Since the two agents are identical, it must be that $s_1^{A*} = s_1^{B*}$. For the credit market to clear, $s_1^{A*} + s_1^{B*} = 0$, so it must be that $s_1^{A*} = s_1^{B*} = 0$.