Competitive Equilibrium: An example (1/2)

Assume all individuals are identical (with the same preference and same income process).

Then all agents will choose to save the same amount in each period.

It must be that $S_1^P(r^*) = 0$ in equilibrium because no one saves and borrows and just consumes whatever income they have in each period, and in the aggregate, $S_1^P(r^*) = 0$.

In other words, each individual i will choose to save $s_1^{i*} = 0$, and $c_1^{i*} = y_1^{i*}$ and $c_2^{i*} = y_2^{i*}$

Since everyone is identical, we can just talk about a **representative consumer**

Competitive Equilibrium: An example (2/2)

A competitive equilibrium here is an allocation $\left\{c_1^{i*}, c_2^{i*}, s_1^{i*}\right\}_{i=1}^{N}$ and price, r^*, such that

- the representative consumer chooses $\left(c_1^* = y_1, s_1^* = 0, c_2^* = y_2\right)$ to maximize lifetime utility, taking as given real interest rate r^*
- the credit market clears at r^*. That is, $S_1^P(r^*) = 0$, i.e., no aggregate savings or dissavings.

WARNING: definition of competitive equilibrium differs according to the economy being described. Do NOT memorize; UNDERSTAND what exactly it is that makes the economy be in equilibrium!

Introducing Govt. into our Two-Period Model (1/5)

Now we introduce a Government (notice the variables are in caps)
- Expenditures: G_1 and G_2
- Levy **lump-sum** taxes: T_1 and T_2

Can issue bonds at the end of first period: B_1, so first period budget constraint is

$$G_1 = T_1 + B_1.$$

- If $G_1 > T_1$, then the government is running a deficit, and it must issue bonds to finance it, and hence, $B_1 > 0$. That is, the government is in debt.
- but if $G_1 < T_1$, then the government is running a surplus, and the government can lend out this surplus by buying bonds issued by individuals, so $B_1 < 0$.

Introducing Govt. into our Two-Period Model (2/5)

In the second period, government's budget constraint is

$$G_2 = T_2 - (1 + r)B_1$$

What is assumed here is that govt does not default.

Like an individual, we can write the government's lifetime budget constraint:

$$G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r}, \tag{14}$$

and like an individual, the government also has to finance its expenditures based on how much lifetime "earnings" it has.

Homework: Verify that the government's budget constraint holds by manipulating its first and second period budget constraints.

Introducing Govt. into our Two-Period Model (3/5)

If we think of this economy as having N people, with each person paying the same amount of taxes t_1 in period 1 and t_2 in period 2, then $T_1 = Nt_1$; $T_2 = Nt_2$;

$$G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r} = Nt_1 + \frac{Nt_2}{1+r}.$$

For consumers, since each of them pays taxes of t_1 and t_2 in periods 1 and 2 respectively, there is now a distinction between income y and disposable income $y_t^d = y_t - t_t$

An individual's budget constraints:
- first period: $c_1 + s_1 = y_1 - t_1 = y_1^d$
- second period: $c_2 - (1+r)\, s_1 = y_2 - t_2 = y_2^d$,

Introducing Govt. into our Two-Period Model (4/5)

where disposable income in each period t as income net of taxes:

$$y_t^d = y_t - t_t$$

The new LBC for the consumers is

$$c_1 + \frac{c_2}{1+r} = (y_1 - t_1) + \frac{(y_2 - t_2)}{1+r} = y_1^d + \frac{y_2^d}{1+r},$$

which can be rewritten as

$$c_1 + \frac{c_2}{1+r} = \underbrace{y_1 + \frac{y_2}{1+r}}_{PV\text{ of lifetime income}} - \underbrace{\left(t_1 + \frac{t_2}{1+r}\right)}_{PV\text{ of taxes}} = \omega - \tau,$$

where $\tau = t_1 + \frac{t_2}{1+r}$ is the present value of taxes consumer pays.

Introducing Govt. into our Two-Period Model (5/5)

So if we define disposable wealth as

$$\omega^d = \omega - \tau,$$

then the individual's LBC is

$$c_1 + \frac{c_2}{1+r} = \omega^d \tag{15}$$

From (15), it is clear that an individual's consumption decisions are based on wealth, but now we have to be more careful: it is based on <u>disposable wealth</u> since the individual could be paying taxes

Note that PV of taxes affect consumer's decision.

Competitive Equilibrium (1/2)

Given that the government in this two-period economy with N individuals is spending and taxing $\left(G_1, G_2, \left\{t_1^i, t_2^i\right\}_{i=1}^N\right)$ such that the government's LBC holds, a competitive equilibrium is an allocation $\left(\left\{c_1^{i*}, c_2^{i*}, s_1^{i*}\right\}_{i=1}^N\right)$, and an interest rate r^*, such that

- each individual i chooses $\left(c_1^{i*}, c_2^{i*}, s_1^{i*}\right)$ optimally, given the real interest rate r^*; and

- the credit market clears at r^*, that is,

$$S_1^P\left(r^*\right) = B_1. \tag{16}$$

Competitive Equilibrium (2/2)

Why equation (16)? Because $S_1^g = -B_1$, and $S_1 = S_1^P + S_1^g$,
where S_1 : aggregate savings

So if $S_1(r^*) = S_1^P(r^*) + S_1^g(r^*) = 0 \Rightarrow S_1^P(r^*) = B_1$

But this also implies that goods market clears. The goods market clears if

$$Y_t = C_t + G_t,\ t \in \{1, 2\}$$

Note: To see this for $t = 1$, since $S_1^P = Y_1 - C_1 - T_1$, and $B_1 = G_1 - T_1$, so saying that $S_1^P = B_1$ is equivalent to saying that $Y_1 = C_1 + G_1$.

Question:
Can we apply what we have learnt so far to the real world? Is it actually useful?

What we have learnt so far sounds very theoretical.
Some of you may be thinking that we have a lot of "symbols" all over the slides, so you may be thinking: "So what? I'm never going to see all this stuff, much less use it, in the real world."

But all this theory is very useful. What is the message you have learnt so far?

Current issues: "global recession"

Another: tax cuts, especially before elections

The Ricardian Equivalence Theorem (1/5)

Theorem 1 *For a given level of government expenditures for the first and second periods, the exact timing of taxes has no impact on the real economy, i.e., the consumption choices of the individuals and the real interest rate do not change when the timing of taxes changes.*

Intuition: if govt is going to spend G_1 and G_2, so long the govt's own LBC, (14) , holds, it doesn't matter when the govt levies taxes; the individual's lifetime disposable wealth is unaffected, and since each individual consumes and saves out of lifetime disposable wealth, his consumption choices are unchanged. Also, real interest rate is unaffected. That is, all **real** variables are unchanged

Message: A tax cut is not a free lunch!

The Ricardian Equivalence Theorem (2/5)

Suppose initially the government **balances** its budget in each period: $G_t = T_t$, so that $t_t = T_t/N = G_t/N$

Suppose now the govt still wants to spend (G_1, G_2), but decides to implement a tax cut in the first period so that $\widehat{T}_1 < G_1$:
- hence, $\widehat{t}_1 < t_1$,
- and govt therefore runs a deficit in the first period since $(G_1 - \widehat{T}_1) > 0$
- hence, govt must issue bonds $B_1 > 0$.

Then second period taxes must increase in order to pay back debt issued in first period, that is, $\widehat{T}_2 = G_2 + B_1 (1 + r)$, so each individual now pays

The Ricardian Equivalence Theorem (3/5)

$$
\begin{aligned}
\widehat{t}_2 &= \frac{\widehat{T}_2}{N} = \frac{G_2 + B_1(1+r)}{N} = \frac{G_2}{N} + \frac{B_1(1+r)}{N} \\
&= t_2 + (1+r)\frac{B_1}{N} = t_2 + (1+r)\frac{\left(G_1 - \widehat{T}_1\right)}{N} \\
&= t_2 + (1+r)\left(\frac{G_1}{N} - \frac{\widehat{T}_1}{N}\right) = t_2 + (1+r)\left(t_1 - \widehat{t}_1\right),
\end{aligned}
$$

and the PV of taxes paid by each individual is:

$$
\begin{aligned}
\widehat{\tau} &= \widehat{t}_1 + \frac{\widehat{t}_2}{1+r} \\
&= \widehat{t}_1 + \frac{t_2 + (1+r)\left(t_1 - \widehat{t}_1\right)}{1+r} = \widehat{t}_1 + \frac{t_2}{1+r} + \left(t_1 - \widehat{t}_1\right) \\
&= t_1 + \frac{t_2}{1+r} = \tau!
\end{aligned}
$$

The Ricardian Equivalence Theorem (4/5)

So PV of taxes unchanged! But this means that disposable wealth unchanged!
So each individual's maximization problem unchanged!
So consumption choices unchanged!

What is going on here? Govt cuts taxes in first period, but individuals know they have to pay for this shortfall in taxes in the second period, so they save the entire amount of the tax cut $\left(t_1 - \widehat{t}_1\right)$ by buying bonds from govt. (If they were dissaving, they will dissave less.) In the second period, the govt will increase taxes to the tune of $\left(t_1 - \widehat{t}_1\right)(1+r)$, so individuals will use what they have saved, plus interest, to pay off this increase in taxes.

Since PV of taxes unchanged, LBC unchanged, so consumers do not modify their optimal consumption choices, and hence c_1^{i*}, and c_2^{i*} not affected by the change of timing of taxes for every individual

The Ricardian Equivalence Theorem (5/5)

But s_1^{i*} changes because each consumer saves more now to pay more taxes later.
- (see figure 10)

Intuition as to why r^* is unchanged:
- private savings increase today since each individual saves more,
- but govt is borrowing more; in fact, the exact increase in govt dissavings equals increase in private savings,
- hence, credit market is still clearing at original interest rate of r^*

(See Figure 8.16, but in our example, $B_1 = 0$)

Note: What is above is has very different predictions from keynesian consumption function: $c_t = a + by_t^d$; a, b exogenous.

When does the Ricardian Equivalence fail?

Ricardian Equivalence result relies on several assumptions. And when some of these assumptions are relaxed, it can fail; however, it can also be robust to modifications of the assumptions.

Four instances under which the Ricardian Equivalence fails

1. Intragenerational Transfers

2. Intergenerational Transfers

3. Distortionary Taxes

4. Credit Market Imperfections
- credit constraints

Ricardian Equivalence and Intragenerational Transfers

Within each generation, tax cuts unevenly distributed, and subsequent tax increase unevenly distributed. In other words, it could be that some people get a tax cut, but others pay for it in second period. In this case, an individual's LBC changes, because his wealth is different, so optimal consumption choices are different.

Ricardian Equivalence and Intergenerational Transfers (1/2)

4 periods

Two generations who each live two periods
- first generation born in period one
- second generation born in period three

If $\widehat{T}_1 < T_1$, $\widehat{T}_2 = T_2$, $\widehat{T}_3 > T_3$and $\widehat{T}_4 = T_4$ - redistribution from gen 1 to gen 2
- first generation: increase in its disposable lifetime wealth so c_1^* and c_2^* increase
- second generation: decrease in disposable lifetime wealth so c_1^* and c_2^* decrease, and r^* will change

Ricardian Equivalence and Intergenerational Transfers (2/2)

Restore Ricardian Equivalence by **altruism**

- if you benefit from a tax cut and your children will see an increase in tax because of that, you save more to give them a higher bequest. Hence, no change in your consumption and no change in your children's consumption following the change in the timing of taxes
- string of generations behave as if it were one person living forever

Ricardian Equivalence and Distortionary Taxes

- If proportional taxes, i.e., $t = \theta y$
  - distort labour supply choice

- Changes in timing of taxes affect labour supply choices: no longer Ricardian equivalence

Ricardian Equivalence and Credit Constraints

- If some individuals are credit constrained: would like to consume more today but cannot because they cannot borrow more

- If tax cut today, can consume more: get closer to their desired consumption path
  - consumption in the first period increases and decreases in the second period
  - if increases the welfare of constrained agents without decreasing the welfare of the non-constrained ones, there is a **Pareto improvement**

Social Security

Two systems (can have combination of these two)

- Pay-as-you-go: working people pay for retirees (a lot of European countries)
- Fully funded: individual savings account like CPF

PayGo System and Overlapping Generations (1/3)

- time horizon is infinite

- each generation lives for two periods
  - t-th generation born in period t
  - population grows at rate n: $N_t = (1 + n)N_{t-1}$

- At each date t:
  - N_t young who are born at date t
  - N_{t-1} old who were born at date $t - 1$

PayGo System and Overlapping Generations (2/3)

- Assume initially no retirement system and then switched to a pay-as-you-go system at date T
  - old receive subsidy b
  - young must pay taxes to balance the SS budget:

$$\begin{aligned} N_t \times t &= N_{t-1} \times b \\ \text{so that } t &= \frac{N_{t-1} b}{N_t} = \frac{b}{1+n} \end{aligned}$$

  - Assume that SS has no impact on real interest rate

PayGo System and Overlapping Generations (3/3)

- Initial old are better off: no tax when they are young and get subsidy when they are old (fig. 11)

- Other generations:

  - better off if $n > r$: Pareto impovement (fig. 12)

Fully Funded Systems

- either SS does not bind and does not make any difference or it binds and people are worse off (fig 8.19 in textbook)

- talk about transition from pay-as-you-go to fully funded system. Will current old lose? See "Macroeconomics in Action", pp. 294-295, textbook