Multiple Choice:

1. A strategy for deploying inputs to produce output is inefficient if
a. it would be possible to replace some of the workers with machinery or electronic equipment.
b. it is possible to produce more output with the same inputs by organizing or deploying the same inputs in a different manner.

2. A firm's production function is $Q = 2KL$, where K is the number of square feet of workspace and L is the number of man hours available to produce output. Which of the following statements is correct?
a. With 1000 square feet, 3 workers can produce 6,000 items.
b. With 2000 square feet, 5 workers can produce 6,000 items
c. With 1000 square feet, 5 workers can produce 10,000 items
d. Both a and c.

3. The Law of Diminishing Marginal Returns can be described as the Law of Eventually Diminishing Returns because it states that
a. marginal product of the variable input decreases at every possible value of the variable input..
b. marginal product of the variable input eventually begins to decrease as the quantity of the variable input is increased.
c. marginal product of the fixed input eventually begins to decrease as the quantity of the variable input is increased.
d. none of the above

4. Sally has two art projects due tomorrow. She has 5 hours to complete both projects. She plans to spend 3 hours working on the first project and 2 hours working on the second project. She believes that the last minute spent working on the first project will add 3 points to Project # 1 score, and the last minute spent working on the second project will add 20 points to the Project # 2 score. Which of the following statements is accurate?
a. Assuming that Sally's goal is to maximize the total number of points, Sally's plan will allocate her time efficiently.
b. Sally would earn a higher point total if she increased the time allocated to Project # 1.
c. Sally would earn a higher point total if she decreased the time allocated to Project # 1.
d. None of the above

5. If $MP_L = 7$ and $MP_K = 4$, then $MRTS_{LK}$ is equal to
a. 7/4
b. 4/7
c. 4*7
d. none of the above

6. For any Cobb Douglas production function, MRTS =
a. $\dfrac{\alpha AL^{(\alpha-1)} K^{\beta}}{\beta AL^{\alpha} K^{(\beta-1)}}$
b. $\dfrac{AL^{\alpha} K^{\beta}}{AL^{\alpha} K^{\beta}}$
c. Neither of the above

7. Suppose an industry is dominated by a large monopolist. If this industry exhibits increasing returns to scale,
a. splitting the large monopolist into several small competitive firms would increase efficiency.
b. splitting the large monopolist into several small competitive firms would decrease efficiency
c. splitting the large monopolist into several small competitive firms would not affect efficiency
d. None of the above.

8. Syverson analyzed concrete plant efficiency, and found large efficiency differences across plants. Which of the following provide potential explanations for the fact that all plants are not equally efficient?
a. The data might fail to capture differences in the quality of inputs used in the plants.
b. Workers at different plants might have different levels of experience.
c. Owners and managers might have different levels of ability for organizing production.
d. All of the above.

9. Benkard analyzed production data for the L-1011 airplane. He found that
a. worker marginal product increased as the workers gained experience by building more airplanes.
b. Some of the productivity gains were lost when the airplane design was modified.
c. Both of the above
d. None of the above.

10. Technological change
a. can make either labor or capital more productive
b. always increases the MP of capital
c. does not impact MRTS
d. none of the above

Answers to Multiple Choice Quiz

1. b
2. d
3. b
4. c
5. a
6. a
7. b
8. d
9. c
10. a

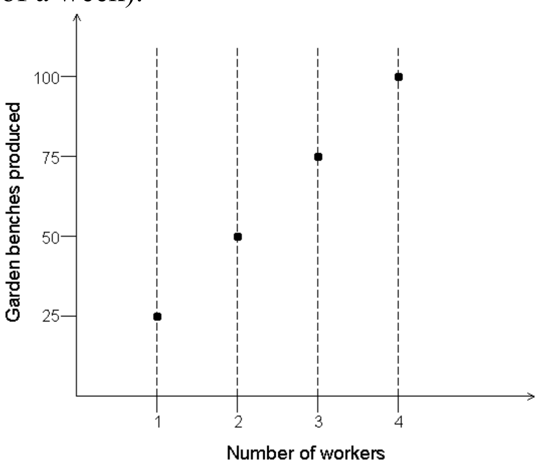
Answers to In-Text Questions

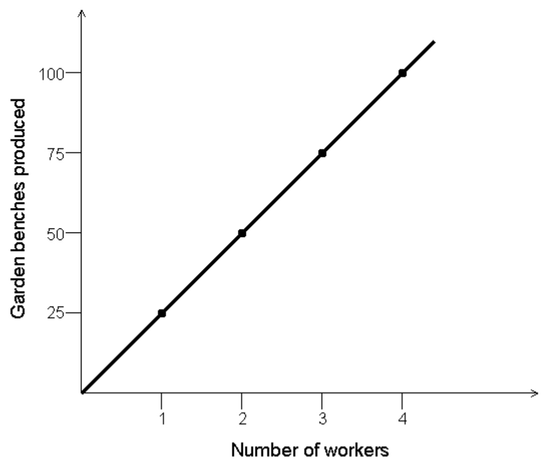
7.1 (BW p.212)

Production Method	Number of Assembly Workers	Garden Chairs Produced per Week	Efficient?
A	2	24	No
B	2	36	Yes
C	3	30	No
D	3	44	Yes
E	3	24	No

Method *B* is efficient because it produces the most garden chairs with 2 workers. Method *D* is also efficient because it creates the greatest number of chairs with 3 workers.

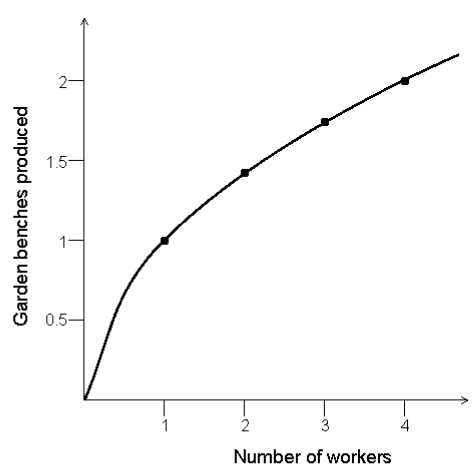
7.2 (BW p.215)
If $Q = F(L) = 25L$, then $F(1) = 25(1) = 25$, $F(2) = 25(2) = 50$, $F(3) = 25(3) = 75$, and finally $F(4) = 25(4) = 100$. Two drawings of this production function are below. On the left, the number of workers is not divisible (workers must be hired for the entire week), on the right, the number of workers is finely divisible (workers may be hired for fractions of a week).





If the production function was $Q = F(L) = \sqrt{L}$, then $F(1) = \sqrt{1} = 1$, $F(2) = \sqrt{2} = 1.414$, $F(3) = \sqrt{3} = 1.732$, and $F(4) = \sqrt{4} = 2$.

The production function for the case where workers are finely divisible is shown to the right.

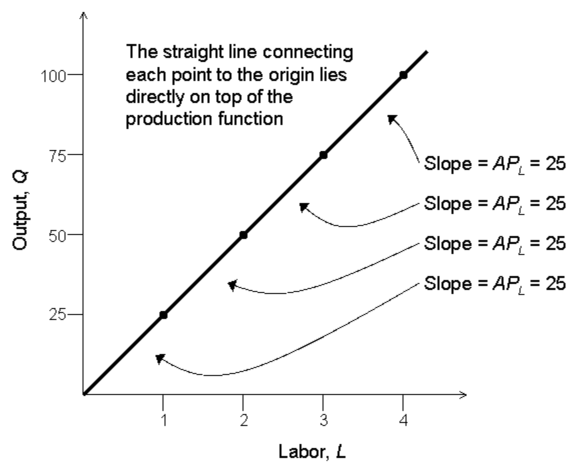


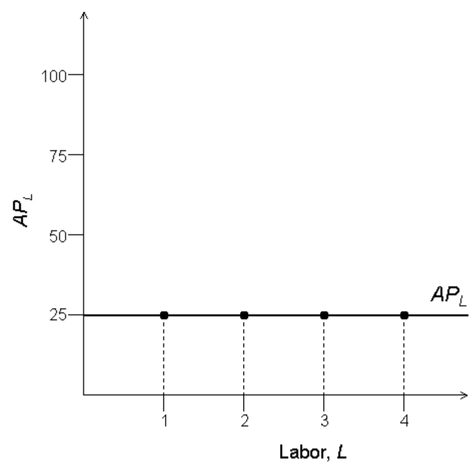
7.3 (BW p.217)
When K is fixed at 10 units, the short run production function is
$F(L,\overline{K}) = Q = 10\sqrt{L}\sqrt{10}$, which students may write as $10\sqrt{10L}$ or $31.623\sqrt{L}$.

Simply plugging 1, 2 and 3 into the short run production function yields the following:

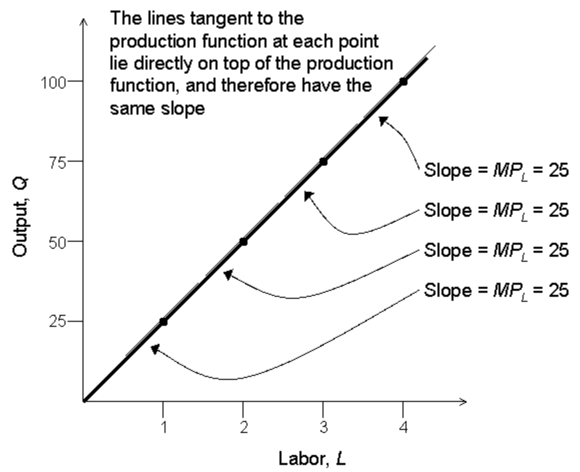
$$F(1,10) = 10\sqrt{1}\sqrt{10} = 32.623$$
$$F(2,10) = 10\sqrt{2}\sqrt{10} = 44.721$$
$$F(3,10) = 10\sqrt{3}\sqrt{10} = 54.772$$

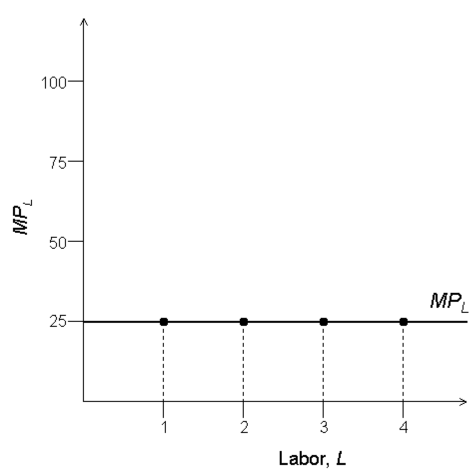
7.4 (BW p.223)
This is the first pair of graphs, used to derive AP_L.



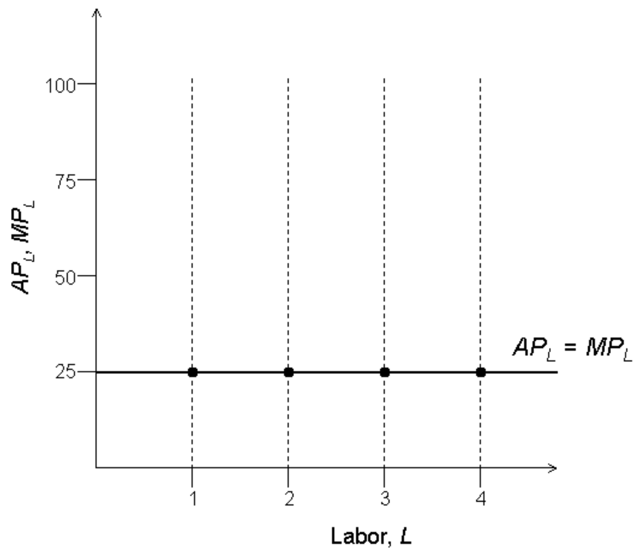


This is the second pair of graphs, used to derive MP_L.





The last graph, below, shows the relationship between the AP_L and MP_L for this production function. They are equal. This makes sense, because the production function suggests that for every 1 unit increase in L, there is a 25 unit increase in Q. This means that every worker is exactly the same—they are all *average* workers. And since they are all the same, every next worker is also average.



7.5 (BW p.226)
The first step is to write the marginal product in plant 2 as a function of the number of crates of oranges assigned to plant 1, O_1:

$MP^2_O = 1200 – 2O_2$
$MP^2_O = 1200 – 2(700 – O_1)$
$MP^2_O = 1200 – 1400 + 2O_1$
$MP^2_O = 2O_1 – 200$

Then find the level of O_1 that equates the marginal product of oranges in each plant. Set $MP^1_O = MP^2_O$ and solve for O_1.

$MP^1_O = MP^2_O$
$1{,}000 – O_1 = 2O_1 – 200$
$1{,}200 = 3O_1$
$\boldsymbol{O^*_1 = 400}$

Therefore, they should allocate 400 crates of oranges to plant 1 and the rest (300 crates of oranges) to plant 2.

7.6 (BW p.232)
The formula for marginal product given on page 218 will be most helpful here. When calculating the marginal product of labor in the case of more than one input, all other inputs need to be held constant. We must rewrite this formula to have two inputs:

$$MP_L = \frac{\Delta Q}{\Delta L} = \frac{F(L,K) - F(L - \Delta L, K)}{\Delta L}$$

Since we want to know about the marginal product of the third worker (L = 3) and ΔL is given as 1, we can re-write the formula for MP_L as:

MP_L of the third worker = $\dfrac{F(3,K) - F(3-1,K)}{1} = F(3,K) - F(2,K)$

This makes things very simple. Now, we just calculate MP_L of the third worker for the three different garage sizes (the three different values of K):

Garage size, *K*	***F*(3,*K*)**	***F*(2,*K*)**	**MP_L = *F*(3,*K*) – *F*(2,*K*)**
1,000 sq. ft.	111	74	37
1,500 sq. ft.	140	105	35
2,000 sq. ft.	167	130	37

Calculating MP_K is very similar:

$$MP_K = \frac{\Delta Q}{\Delta K} = \frac{F(L,K) - F(L,K-\Delta K)}{\Delta K}$$

Using 1,500 square feet of garage space, ΔK = 500 and L = 4 workers:

$$MP_K = \frac{F(4,1500) - F(4,1500-500)}{500} = \frac{F(4,1500) - F(4,1000)}{500}$$

$$MP_K = \frac{170-132}{500} = \frac{38}{500}$$

MP_K = 0.076

7.7 (BW p.240)

The production function $Q = F(L,K) = (L + K)^2$ is not Cobb-Douglas, so we cannot simply look at exponents to determine returns to scale. Be careful that students correctly FOIL this function out if they are looking to simplify it. The simplest way to do this problem is as in Worked-Out Problem 7.2, by considering what would happen if the firm were to double its inputs of labor and capital from L and K to $2L$ and $2K$.

$F(L, K) = (L + K)^2$
$F(L, K) = L^2 + 2KL + K^2$

$F(2L, 2K) = (2L + 2K)^2$
$F(2L, 2K) = 4L^2 + 8KL + 4K^2$
$F(2L, 2K) = 4 \times [L^2 + 2KL + K^2]$
$F(2L, 2K) = 4 \times F(L, K)$

Since doubling both inputs more than doubles output (in fact, here it quadruples it), this firm experiences **increasing returns to scale**.

Answers to End-of-Chapter Questions (BW p.246-248)

7.1

In order to graph a production possibilities set, we need to have a standardized measure of output, like claims per hour, claims per day, or claims per week. In the solution that follows, I will calculate claims per eight hour work day. The different methods of production available to this firm involve combinations of lawyers using the three different break schedules. First we need to calculate how many claims a lawyer could process under each break schedule in an eight hour work day. We start by calculating how long it takes to process one claim:

Break schedule (1): 6 hours, plus 3 breaks = 6 + 3(20)/60 = 7 hours/claim
Break schedule (2): 8 hours, plus 2 breaks = 8 + 2(20)/60 = 8.667 hours/claim
Break schedule (3): 5 hours, plus5 breaks = 5 + 5(20)/60 = 6.667 hours/claim

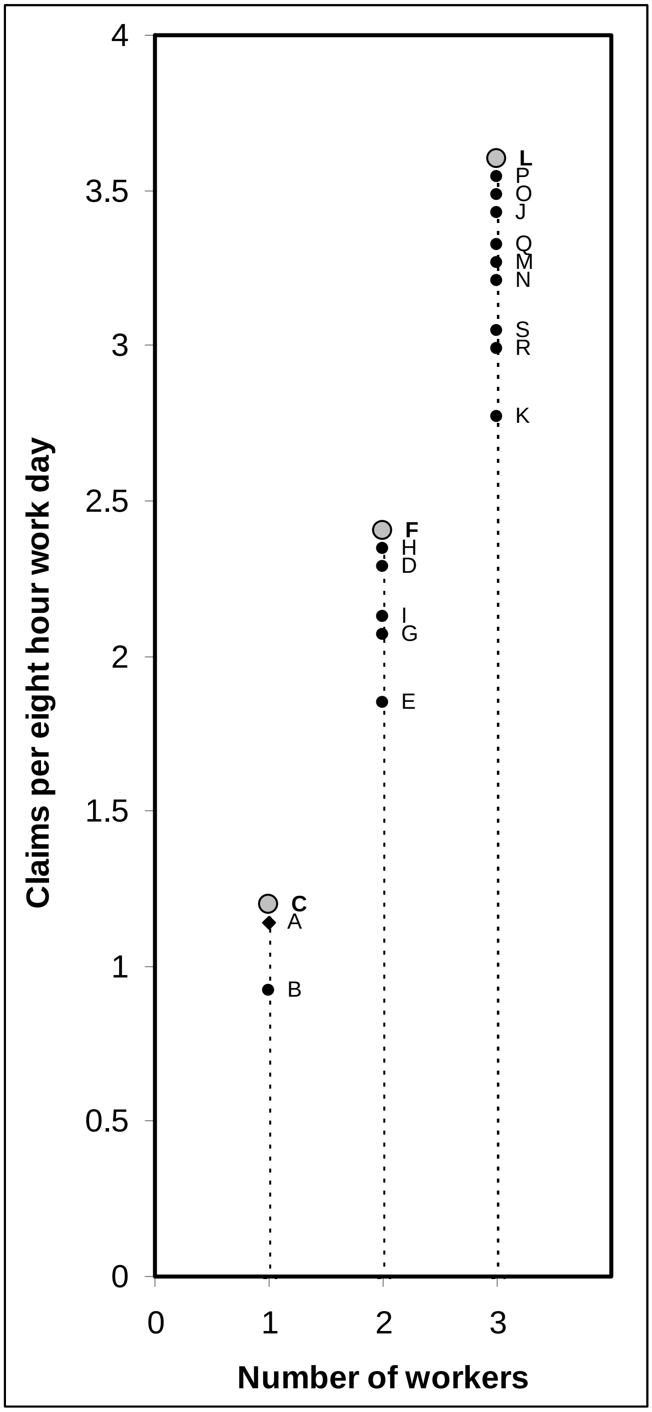
To figure out the number of claims processed in an eight hour work day with each break schedule, we divide each time above into eight hours:

Break schedule (1): 8 / 7 = 1.143 claims per eight hour work day
Break schedule (2): 8 / 8.667 = 0.923 claims per eight hour work day
Break schedule (3): 8 / 6.667 = 1.200 claims per eight hour work day

Then, we define some different production methods (to follow the chapter's methodology). Student methods may vary, but here's a complete set of methods with one, two and three workers on varying schedules (efficient methods in bold):

Method	Total lawyers	Lawyers on schedule (1)	Lawyers on schedule (2)	Lawyers on schedule (3)	Claims processed per work day
A	1	1	0	0	1.143
B	1	0	1	0	0.923
C	**1**	**0**	**0**	**1**	**1.2**
D	2	2	0	0	2.286
E	2	0	2	0	1.846
F	**2**	**0**	**0**	**2**	**2.4**
G	2	1	1	0	2.066
H	2	1	0	1	2.343
I	2	0	1	1	2.123
J	3	3	0	0	3.429
K	3	0	3	0	2.769
L	**3**	**0**	**0**	**3**	**3.6**
M	3	1	1	1	3.266
N	3	2	1	0	3.209
O	3	2	0	1	3.486
P	3	1	0	2	3.543
Q	3	0	1	2	3.323
R	3	1	2	0	2.989
S	3	0	2	1	3.046

The graph of this production possibilities set would look like the one below. This one contains all of the possible methods for one, two or three workers, but your students don't



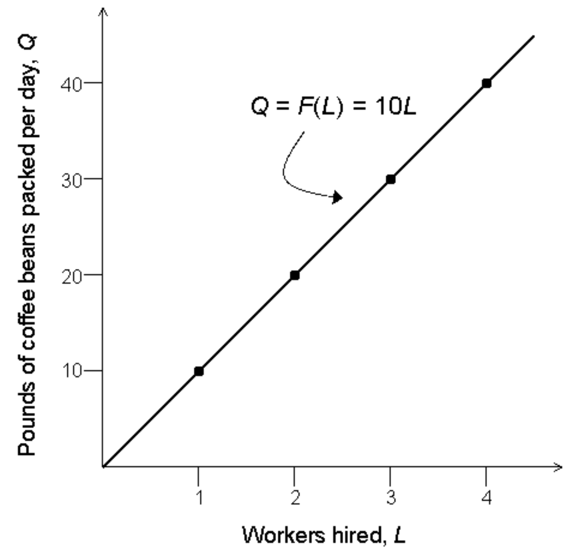
necessarily have to include every method. Also, this one is stretched out so the instructor can see each method. The efficient method for each number of workers is at the top and is indicated by a larger grey circle with a black border.

The production function is an equation that gives output as a function of inputs using only efficient methods of production. Therefore, the relevant production function here would only include methods *C*, *F* and *L* for one, two and three workers respectively. Looking at the table we created, we notice that methods *C*, *F* and *L* all involve only using break schedule (3). This makes sense because workers on break schedule (3) process more claims in a day (or, similarly, they use less time in the processing of one claim).

So the only break schedule that should be used is (3). This means that every worker will be able to process 1.2 claims per eight hour work day, and that the production function would look like: $\boldsymbol{Q = F(L) = 1.2L}$, where Q is number of claims per eight hour work day and L is the number of lawyers hired to work eight hour days.

Remember that students may use a different definition of Q, so answers may vary. Some possibilities include $\boldsymbol{Q = .15L}$, where Q is the number of claims per hour and L is number of lawyer hours, or $\boldsymbol{Q = 6L}$, where Q is the number of claims per week and L is the number of lawyers hired to work for a week.

7.2
Their production function is a straight line with a slope of 10 like the one drawn below.



7.3
Students should recall that MP_L is equal to the change in output divided by the change in labor. Since we are increasing labor by one unit here, it is simply the change in output caused by the unit of labor in question. The AP_L is output divided by labor.

Number of Assembly Workers	Garden Benches Produced per Week	MP_L	AP_L
0	0	—	—
1	35	35	35
2	70	35	35
3	99	29	33
4	112	13	28

The relationship between MP_L and AP_L satisfies the properties discussed in section 7.2. When MP_L is constant, APL is constant and equal to MP_L (see the rows for one and two workers above), but when MP_L falls below AP_L, the AP_L begins to fall (see the rows for three and four workers above).

7.4
To use information about AP_L to find out about MP_L, we must first use the AP_L information to find out about total output. Since $AP_L = Q/L$, then we also know that $Q = AP_L \times L$. If we use this formula to find Q, we can then find MP_L in the usual way.

Hours worked, L	AP_L	$Q = AP_L \times L$	MP_L
0	—	0	—
1	5	5	5
2	4	8	3
3	3	9	1
4	2.5	10	1

7.5
Theoretically, the marginal product of labor *can* be negative. A negative MP_L implies that total output decreases with an additional unit of labor. If workers are so numerous that they begin to get in one another's way or they become too hard to manage, this could be true. In practice, however, we never have to be concerned with this situation because a firm would not hire a worker if the MP_L was negative. Why would the firm pay a worker to reduce production? As long as firms are profit-maximizers and workers have a non-negative wage, this situation will not arise. (If firms are forced to hire workers with a negative MP_L, in the case of a contract for example, a negative MP_L would still never happen in practice because the counter-productive worker could be told to sit in the public library, where he or she would not exert a negative impact on production.)

7.6
If the production function is $Q = F(L,K) = L \times \sqrt{L+K}$ and K is fixed at 10 units in the short run, then the short run production function is $Q = F(L,\overline{K}) = L \times \sqrt{L+10}$. To find the output at one, two and three workers, we just plug these numbers into the production function.

When $L = 1$, short run production equals $1 \times \sqrt{1+10} = \sqrt{11} = 3.317$.
When $L = 2$, short run production equals $2 \times \sqrt{2+10} = 2\sqrt{12} = 6.928$.
When $L = 3$, short run production equals $3 \times \sqrt{3+10} = 3\sqrt{13} = 10.817$.

7.7
First we need to construct an expression for AP_L, which we do by dividing Q by L:
$$AP_L = \frac{Q}{L} = \frac{L^3 - 200L^2 + 10{,}000L}{L} = L^2 - 200L + 10{,}000$$

Then, to find where AP_L and MP_L are equal, we set them equal to each other and solve algebraically for L.

$AP_L = MP_L$
$L^2 - 200L + 10{,}000 = 3L^2 - 400L + 10{,}000$
$0 = 2L^2 - 200L$
$0 = L^2 - 100L$

In most applications, dividing by L in the next step causes us to "lose a root," namely 0. However, since $L = 0$ cannot be our solution (there is no AP_L or MP_L at $L = 0$), we don't mind losing this root, so dividing by L is acceptable.

$0 = L - 100$
$\boldsymbol{L = 100}$

Therefore, AP_L and MP_L are equal when L is 100. To verify that AP_L and MP_L satisfy the properties discussed in the text, we should evaluate them both at a level of L less than 100 and a level of L greater than 100. Using the formulas above, we can get results like the ones that follow. (Students' methods may vary greatly here, but they should be trying to investigate how AP_L and MP_L behave when not equal.)

L	AP_L	MP_L
90	100	-1,700
95	25	-925
100	0	0
105	25	1,075
110	100	2,300

Although the MP_L oddly starts negative, then becomes positive, we notice that the relationship between the AP_L and MP_L makes sense here. At $L = 90$ and $L = 95$, the MP_L is less than the AP_L and the AP_L is falling (compare it to when $L = 95$ and $L = 100$). After $L = 100$, the MP_L is greater than the AP_L, and the AP_L is rising.

7.8
The first step is to write the marginal product in plant 2 as a function of the number of crates of oranges assigned to plant 1, O_1:

$MP^2_O = 1200 - 2O_2$
$MP^2_O = 1200 - 2(8500 - O_1)$
$MP^2_O = 1200 - 1700 + 2O_1$
$MP^2_O = 2O_1 - 500$

Then find the level of O_1 that equates the marginal product of crates of oranges in each plant. Set $MP^1_O = MP^2_O$ and solve for O_1.

$MP^1_O = MP^2_O$
$1{,}000 - O_1 = 2O_1 - 500$
$1{,}500 = 3O_1$
$\boldsymbol{O^*_1 = 500}$

Therefore, they should allocate 500 crates of oranges to plant 1 and the rest (350 crates of oranges) to plant 2.

7.9
Starting with the first worker, each worker should be assigned to the plant where his or her marginal product is highest. In practice, this will usually involve assigning some workers to both plants. Specifically, the manager will distribute the fixed number of workers between both plants in a way that makes the marginal product of labor equal at both plants. If by saying "plant B is more productive than plant A" means that, *at every given level of labor*, plant B is more productive than plant A, then this will definitely involve assigning more workers to plant B. Diminishing marginal product will require that plant B, having higher marginal product at each level of L, have more workers in order to lower its marginal product of labor to be equal to that of plant A. It could be, given the amount of available L and the different marginal product functions, that all of

the labor should be assigned to plant B. Resources will always go where they are most productive, so there is no case, interpreting the conditions given in this question correctly, in which plant A should receive more workers.

7.10
You should allocate each minute where it does the most good. Let's define marginal productivity of study time as additional points earned per hour. We can arrange the information from this problem as follows:

Hours spent studying micro	**Marginal Productivity of studying micro**	**Hours spent studying macro**	**Marginal Productivity of studying macro**
0 – 25	1	0 – 15	1.333
26 – 100	.333	16 – 60	.667

(Notice that the most time you can spend studying for macro is 60 hours, this is because there are only 50 points on each part of the test, so studying more than 60 hours for macro does not raise your score.)

The highest marginal productivity is for the first 15 hours of macro study time, so your first 15 hours of study time go to that. This earns you 20 extra points on the macro section of your test. After that, the next highest marginal productivity is for the first 25 hours of studying for micro, so your next 25 hours go to studying micro. This earns you 25 points on the micro part of your test. Next, the third highest marginal productivity is for the next 45 hours of studying for macro, which earn you 30 more points on the macro section (for 50 points total on macro). Since there's no more macro to study, you spend your last 15 hours studying for micro. These 15 hours earn you an additional 5 points on that section of the test (for 30 total points on micro).

In summary: study macro for 60 hours and micro for 40 hours. Total score: 80 points.

7.11
The first step is to write the marginal product in plant 2 as a function of the number of workers assigned to plant 1, L_1:

$MP^2_L = 50/L_2$
$MP^2_L = 50/(90 - L_1)$

Then find the level of L_1 that equates the marginal product of labor in each plant. Set $MP^1_L = MP^2_L$ and solve for L_1.

$MP^1_L = MP^2_L$
$100/L_1 = 50/(90 - L_1)$

Cross multiply to get:

$9000 - 100L_1 = 50L_1$

$9000 = 150L_1$
$\boldsymbol{L_1 = 60}$

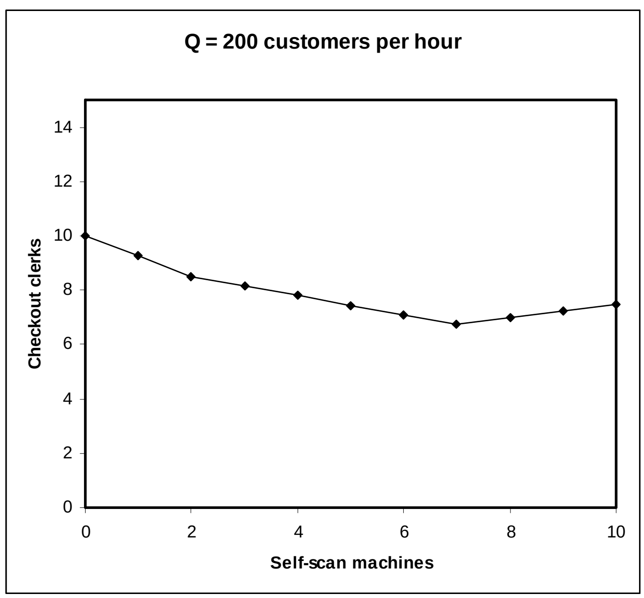
Therefore, Beta Inc. should allocate 60 workers to plant 1 and the rest (30 workers) to plant 2.

7.12
Since half of the store's customers refuse to use the new machines, then 100 customers per hour (when output is fixed at 200 customers per hour) will require a checkout clerk. Since checkout clerks can each checkout 20 customers per hour, the minimum number of checkout clerks (not counting those needed to monitor the self-scan machines) is five. The maximum number of customers that will use the self-scan machines is 100. Forty of them take 3 minutes each, requiring 120 self-scan machine minutes. Since we're interested in servicing these 40 customers in one hour, we would need two machines just for them. Sixty of the customers would take five minutes apiece, requiring 300 self-scan machine minutes, or 5 machines. The maximum number of machines, then, is 7. These seven machines require two clerks to monitor them.

All of the possible ways of checking out 200 customers an hour are summarized below (assuming that the customers who are best at using the self-scan machines are the first to use them and that, since clerks can do other productive things if not checking out customers, we can treat clerk hours as divisible):

(1) Self-scan machines	**Clerk hours spent monitoring**	**Customers using self-scan machines**	**Customers being checked out by clerks**	**Clerk hours spent checking customers out**	**(2) Total clerks**
0	0	0	200	10	10
1	0.25	20	180	9	9.25
2	0.5	40	160	8	8.5
3	0.75	52	148	7.4	8.15
4	1	64	136	6.8	7.8
5	1.25	76	124	6.2	7.45
6	1.5	88	112	5.6	7.1
7	1.75	100	100	5	6.75

Columns (1) and (2) above are used to construct the isoquant shown to the right. It slopes upwards after seven self-scan machines (data not shown on table above) because no more customers can be helped by more than seven machines, but each additional machine requires another quarter hour of checkout clerk time.



7.13
No. If the marginal rates of technical substitution are not equal, this means that one input is more productive at one plant and the other input is more productive at the other plant. I am missing out on opportunities to produce more with the same amount of inputs. In this case, I should reallocate the resources so that more of each goes where it is more productive. By doing this, because of the principle of diminishing marginal productivity, the productivity of these resources will fall at their respective plants and the marginal rates of technical substitution will get closer together. I will continue this reallocation until the marginal rates of technical substitution are equal. (This assumes that resource prices are the same, which is reasonable given the problem's mention of the two plants being in the same city.)

7.14
The relationship between these exponents and returns to scale is the same as it is for the two-input case discussed in the text. An example like the one given on page 239 makes this clear. If we double each of the inputs we get:

$$F(2L, 2K, 2M) = A(2L)^{\alpha}(2K)^{\beta}(2M)^{\gamma} = 2^{\alpha+\beta+\gamma}(AL^{\alpha}K^{\beta}M^{\gamma}) = 2^{\alpha+\beta+\gamma}F(L,K,M)$$

If $\alpha+\beta+\gamma = 1$, then output exactly doubles, exhibiting *constant* returns to scale.
If $\alpha+\beta+\gamma > 1$, then output more than doubles, exhibiting *increasing* returns to scale.
If $\alpha+\beta+\gamma < 1$, then output less than doubles, exhibiting *decreasing* returns to scale.

7.15
This technological advance doubles the firms output but does not change the relative productivities of its inputs. Because output has doubled at *every* combination of its two inputs, the marginal products of those inputs has also doubled everywhere. *MRTS* is a ratio of marginal products, and the doubling of the top and bottom of a fraction leave its value unchanged. This is a factor-neutral technological change and therefore the *MRTS* (the rate at which labor can be substituted with capital) does not change.

7.16
The formula for AP_L is Q/L. From the story told in the problem, Q doubles at every level of L, but L does not change. Doubling the numerator of a fraction and leaving the denominator unchanged doubles the value of the fraction. Therefore, AP_L would double for every level of L.

The formula for MP_L is change in Q divided by change in L. For every one unit change in L, the level of output has doubled, and so has the previous level of output. This means that the change between them has doubled as well. Once again, this change doubles the numerator but not the denominator, so the value of the fraction doubles. MP_L doubles for every level of L.

7.17
The condition under which Technology B is more productive than Technology A depends on how much is going to be produced in total. Suppose that I want to produce Q^* units of

output. The technology that can produce my desired output using the fewest units of labor is the one that is most productive. (I assume that Q^* is greater than 300 because if Q^* is equal to or less than 300, then it is obvious that Technology B is the most productive technology.) To proceed, I must derive an equation for the amount of labor used under each technology as a function of output.

Technology A	Technology B
$Q_A = 2L_A$	$Q_B = 3(100) + \beta(L_B - 100)$
$L_A = Q_A/2$	$Q_B = 300 + \beta L_B - 100\beta$
	$Q_B - 300 + 100\beta = \beta L_B$
	$L_B = (Q_B - 300)/\beta + 100$

In order for Technology B to be more efficient, L_B must be less than L_A for $Q_A = Q_B = Q^*$:

$$L_B < L_A$$

$$\frac{Q^* - 300}{\beta} + 100 < \frac{Q^*}{2}$$

$$\frac{Q^* - 300}{\beta} < \frac{Q^*}{2} - 100$$

$$\frac{Q^* - 300}{Q^*/2 - 100} < \beta$$

Or, written differently: $\beta > 2 \times \left(\frac{Q^* - 300}{Q^* - 200} \right)$.

As long as this condition is satisfied, then Technology B is more efficient than Technology A. For example, if I desire to produce 400 units, β must be greater than 1 for Technology B to be the better choice. Advanced students may notice that the limit of this critical value of β is 2 as Q^* approaches infinity.

Note: students could also solve this problem holding the amount of labor constant at L^* and then solving for β that makes Q_B greater than Q_A. Doing so gives a critical value of β as a function of L^* that is similar in form. Again, the limit is 2 as L^* approaches infinity.

The solution in this case is: $\beta > 2 \times \left(\frac{L^* - 150}{L^* - 100} \right)$.