

Multiple Choice:

1. Which of the following statements is true?
 - a. $P=120-3Q$ is an inverse demand function
 - b. $P=120-3Q$ is a demand function
 - c. For every demand function, there is an equivalent inverse demand function.
 - d. Both a and c.

2. Authors typically prefer lower book prices than publishers because:
 - a. authors are more sympathetic to the budget issues faced by students.
 - b. authors maximize revenue, while publishers maximize profit.
 - c. authors do not bear the cost of producing the extra books that would be sold at the lower price.
 - d. Both b and c (many authors write books that are not purchased by students)

3. The demand curve faced by a firm that is a “price taker” is
 - a. $P = A$; where A is the market-determined price
 - b. $Q = A$; where A is the market-determined quantity
 - c. $P = Q$
 - d. $P = 0$

4. Worked-out problem 9.3 states that you can “find the new efficient scale of production by determining the output level at which:
 - a. marginal revenue = average cost
 - b. marginal cost = marginal revenue
 - c. marginal cost = average cost
 - d. none of the above

5. The import fee for softwood lumber, causes the supply curve in the domestic market to shift
 - a. up and to the left (“in”)
 - b. down and to the right (“out”)

6. The discussion of the behavior of U.S. oil producers (in application 9.4) assumes that these firms are:
 - a. price takers
 - b. perfectly competitive firms
 - c. firms with market power
 - d. Both a and b

7. Profit is equal to:
 - a. revenue – total cost
 - b. producer surplus – sunk cost
 - c. Both a and b
 - d. None of the above

8. True or false? A firm’s producer surplus equals the area between a horizontal line drawn at the level of its price P and its supply curve.
 - a. True
 - b. False

9. True or false? When a firm's profit-maximizing sales level is positive, its marginal revenue equals its marginal cost at that quantity.

- a. True
- b. False

10. True or false? A firm's profit equals its producer surplus less its sunk costs.

- a. True
- b. False

Answers to Multiple Choice Quiz

- 1. d
- 2. d
- 3. a
- 4. c
- 5. a
- 6. d
- 7. c
- 8. a
- 9. a
- 10. a

Answers to In-Text Questions

9.1

It is easier to answer the last question first. The inverse demand function is the demand function rearranged and solved for P :

$$Q^d = 450 - 2P$$

$$2P = 450 - Q^d$$

$$P = 225 - (Q^d/2)$$

If Q^d is 100, P is $225 - (100/2) = \$175$. If Q^d is 150, P is $225 - (150/2) = \$150$.

9.2

Dan's best positive sales quantity satisfies the condition that $P = MC$.

$$15 = 5 + (Q/40)$$

$$10 = Q/40$$

$$Q = 400$$

Students should verify that producing 400 units provides positive profits. There are three possible ways to do this:

- 1. By calculating profit and seeing that it is positive. $\Pi = R(Q) - C(Q) = (15 \times 400) - [(5 \times 400) + (400^2/80)] = \mathbf{\$2,000} > \mathbf{\$0}$.
- 2. By observing that average cost is below the price. From above, $C(Q) = \$4,000$, so $AC(Q) = C(Q)/Q = 4000/400 = \10 . $\mathbf{\$10} < \mathbf{\$15}$.

3. By observing that the price is greater than the minimum of $AC(Q)$. The average cost function is $C(Q)/Q = 5 + (Q/80)$. This hits its minimum when Q is very small (almost zero), so the minimum of $AC(Q)$ is just about \$5. **\$5 < \$15**.

If Dan has an avoidable fixed cost of \$750, this is not enough to discourage him from producing. It would reduce his profits to \$1,250, but these profits are still positive, so producing is still the best option (and 200 is still the best quantity). We could also say that since $C(Q) = \$4,000 + \$750 = \$4,750$, $AC(Q)$ is now $\$4,750/400 = \11.88 , which is still lower than the price of \$15.

9.3

Dan maximizes profit by producing where $P = MC$, which yields: $P = Q$. This formula tells us how Dan will produce in response to different prices. This is his supply curve (when he produces). We need to know his shut down price, however. Dan's cost function is the sum of his fixed and variable costs:

$$C(Q) = VC(Q) + FC$$

$$C(Q) = Q^2/2 + 50$$

And his average cost is $C(Q)/Q$, or: $AC(Q) = Q/2 + 50/Q$. We need to know what the minimum of $AC(Q)$ is so that we know Dan's shutdown price. This can be done with calculus by advanced students, or we can remember that when $AC(Q)$ is at its minimum, it must be equal to $MC(Q)$. Solving the equation $MC(Q) = AC(Q)$ for Q will give us the quantity at which $AC(Q)$ is minimized, which we can then plug into $AC(Q)$ to find the minimum $AC(Q)$.

$$MC(Q) = AC(Q)$$

$$Q = Q/2 + 50/Q$$

$$(\frac{1}{2})Q = 50/Q$$

$$(\frac{1}{2})Q^2 = 50$$

$$Q^2 = 100$$

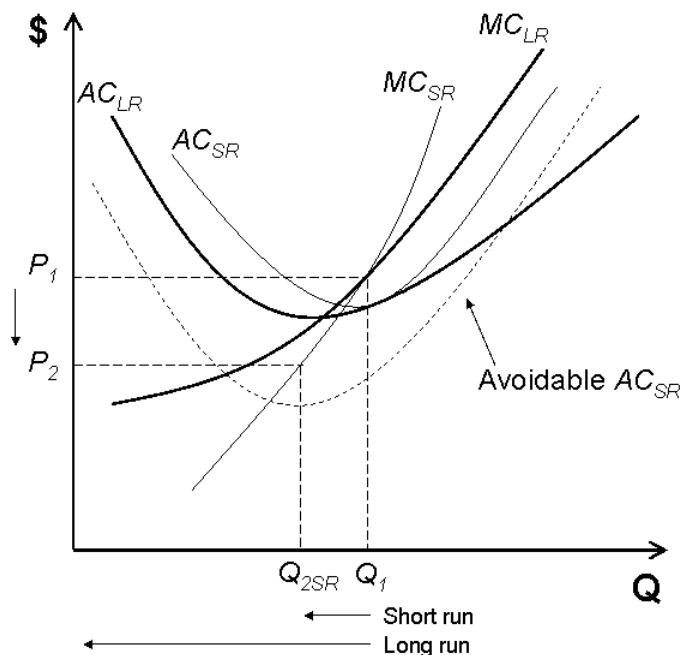
$$Q = 10$$

Plugging this into $AC(Q)$ gives $(10/2) + (50/10) = \$10.00$. This is the minimum of $AC(Q)$, and therefore the shutdown price. So Dan's supply function is:

$$S(P) = \begin{cases} P & \text{if } P \geq \$10.00 \\ 0 & \text{if } P < \$10.00 \end{cases}$$

9.4

In the drawing to the right, the original price is P_1 . The firm is producing quantity Q_1 at this price and is earning a profit ($P_1 > AC$). When the price falls to P_2 , the firm is no longer earning a profit. However, it continues to produce because $P_2 > \text{avoidable } AC$. In other words, it is more profitable to produce than not to. In the short run, the firm reduces output to quantity Q_{2SR} , where $P_2 = MC_{SR}$. In the long run, however, all costs are avoidable and this firm would



not continue to produce at a loss. The firm will produce zero units in the long run. A price decrease is more likely to cause a long-run decision to produce zero since some firms may still find it in their best interest to produce at a loss in the short run, while *no* firm will find it so in the long run. If the avoidable AC curve had been drawn higher, it could have been that this firm would shutdown in the short run as well.

9.5

First we consider the long-run supply function. If the $MC_{LR} = 200$, then for prices above \$200, the condition that $P = MC_{LR}$ can never be satisfied. In this situation, P will always be greater than MC_{LR} , so the firm would want to sell an infinite amount of the good. Conversely, if P is less than \$200, then P will always be less than MC_{LR} and the firm would not want to produce anything. For prices exactly equal to \$200, the firm will supply any amount that consumers are willing to buy.

In the short run, the quantity rule can apply. This requires that $P = MC$. Since the MC_{SR} is $2Q$, this means that $P = 2Q$ or that $Q = P/2$. The avoidable part of their $C_{SR}(Q)$ is Q^2 . As long as revenue from producing is higher than the avoidable costs, they will produce. Revenue from producing is $P \times Q$, or $P^2/2$. The avoidable costs of production are Q^2 or $P^2/4$. Since $P^2/2 > P^2/4$, revenue is always higher than avoidable costs. The short-run supply function is $S_{SR}(P) = P/2$.

9.6

We must apply the quantity rule to both goods, so that for each good $P = MC$. We have to find Q_B and Q_C such that the following two equations hold:

$$\begin{aligned} 110 &= 50 + 2Q_B - Q_C \\ 55 &= 25 + 2Q_C - Q_B \end{aligned}$$

This system is easily solved by substitution. Solving the first equation for Q_C gives the condition that $Q_C = 2Q_B - 60$, which can be substituted into the second equation:

$$\begin{aligned} 55 &= 25 + 2(2Q_B - 60) - Q_B \\ 55 &= 25 + 4Q_B - 120 - Q_B \\ 55 &= -95 + 3Q_B \\ 3Q_B &= 150 \\ Q_B &= 50 \end{aligned}$$

Then, plugging this into the condition above shows that Q_C is 40. Producing this combination of output yields profit of:

$$[110(50) + 55(40)] - [50(50) + (50)^2 - (50)(40)] - [25(40) + (40)^2 - (50)(40)] = \$4,100$$

Applying the shutdown rule, we must consider whether this level of production is better than shutting down production of one or both goods. If they stop producing garden chairs, so that $Q_C = 0$, their cost function for benches becomes $C_B(Q_B) = 50Q_B + Q_B^2$ and the marginal cost is $MC_B = 50 + 2Q_B$. To find profit-maximizing quantity of Q_B , we set $P_B = MC_B$:

$$\begin{aligned} 110 &= 50 + 2Q_B \\ 2Q_B &= 60 \\ Q_B &= 30 \end{aligned}$$

In this case, profit would be $110(30) - [50(30) + (30)^2] = \900 .

Similarly, if they stop producing garden benches, so that $Q_B = 0$, their cost function for benches becomes $C_C(Q_C) = 25Q_C + Q_C^2$ and the marginal cost is $MC_C = 25 + 2Q_C$. To find profit-maximizing quantity of Q_C , we set $P_C = MC_C$:

$$55 = 25 + 2Q_C$$

$$30 = 2Q_C$$

$$Q_C = 15$$

In this case, profit would be $55(15) - [25(15) + (15)^2] = \225 .

Both of these cases (and also the case of producing none of both kinds of goods) give Noah and Naomi less profit than the first solution where $Q_B = 50$ and $Q_C = 40$. This must be the profit-maximizing set of sales quantities.

Answers to End-of-Chapter Questions

9.1

$$Q^d = \frac{1,000}{\sqrt{P}}$$

$$\sqrt{P} = \frac{1,000}{Q^d}$$

$$P = \frac{1,000,000}{(Q^d)^2}$$

9.2

In many cases, an author gets paid a percentage of total revenue. If this is so, the author would prefer a price that maximizes total revenue. This price will be the price where $MR = 0$. The publisher would prefer a price that maximizes profit. This will be the price where $MR = MC$. In the case of an electronic book, where the marginal cost of production is almost zero, the publisher would be satisfying the same condition that the author would want satisfied, $MR = 0$. Therefore, in this case, the author and the publisher are likely to agree on the right price and sales quantity. The lower the marginal cost of production, the more alike their preferences.

9.3

All profit-maximizers will choose a situation where $MR = MC$. If the author were setting the price, he would choose a price where $MR_A^* = MC_A$. However, the costs associated with writing a book are all fixed costs, and do not vary with output. Therefore, the author would choose a price such that the sales quantity would cause MR_A^* to be 0. The publisher would choose a price where $MR_P^* = MC_P$. Since, for the publisher, there are actual marginal costs of production, $MC_P > 0$. In this case, the desired level of marginal revenue from the author's point of view, MR_A^* , is less than the desired level of marginal revenue from the publisher's point of view, MR_P^* . Since MR (for a price taking firm) is always decreasing as quantity increases, the author's desired sales quantity, Q_A^* , will be higher (so that MR_A^* is lower) than the publisher's desired sales quantity, Q_P^* . Because of the law of demand, a higher sales quantity requires a lower price, so that P_A^* is lower than P_P^* .

9.4

So long as the author's profit is based on revenue, the author will want revenue to be maximized. Revenue maximization is the same in both cases, so these two authors would agree on price.

9.5

The price reduction would raise profit if:

$$\begin{aligned}(2.00)Q_A - C(Q_A) &> (2.20)Q_B - C(Q_B) \\ (2.00)Q_A - C(Q_A) - (2.20)Q_B + C(Q_B) &> 0 \\ (2.00)(Q_A - Q_B) - [C(Q_A) + C(Q_B)] - (0.20)Q_B &> 0 \\ (2.00)(Q_A - Q_B) - [C(Q_A) + C(Q_B)] &> (0.20)Q_B\end{aligned}$$

Using the assumptions from Application 9.2 about the average variable cost in the industry, this can be rewritten as:

$$\begin{aligned}(2.00)(Q_A - Q_B) - [(1)Q_A + (1)Q_B] &> (0.20)Q_B \\ Q_A - Q_B &> (0.20)Q_B \\ \frac{Q_A - Q_B}{Q_B} &> 0.20\end{aligned}$$

In other words, sales would only have to increase by 20% to make this price reduction profitable.

9.6

The solution must satisfy the quantity rule:

$$\begin{aligned}P &= MC \\ 243 &= 3Q^2 \\ 81 &= Q^2 \\ Q &= 9\end{aligned}$$

If the fixed cost is unavoidable, we have to compare revenue from production to the variable cost of production and make sure that revenue is greater. The revenue from producing and selling nine units is $\$243 \times 9 = \$2,187$. The variable cost from producing nine units is Q^3 , or $(9)^3$, which is \$729. The revenue is greater than the variable cost and producing leaves the firm \$1,458 better off than if it did not produce, so producing nine units is the best choice.

If the fixed cost is avoidable, then we compare revenue (\$2,187) to the sum of variable costs (\$729) and fixed cost (\$3,000). Total cost is \$3,729. This is greater than revenue; if the firm produces, its best possible profit is -\$1,542. The firm is better off producing nothing in this case, earning a profit of zero.

Another way to look at it is to realize that, whether the fixed cost is avoidable or not, the actual profit from producing nine units is -\$1,542. If the fixed cost is unavoidable, the profit from not producing is -\$3,000, which makes producing a better choice. If the fixed cost is avoidable, the profit from not producing is \$0, making not producing a better choice.

9.7

If the firm produces, it produces where $P = MC$, regardless of whether the fixed costs are sunk or not. Only the shutdown price changes when the fixed costs go from being sunk to being nonsunk. $P = MC$ means that $P = 3Q^2$, so that $Q = (P/3)^{1/2}$.

With sunk fixed costs, it is only necessary that P be greater than the minimum of AVC . The VC function is Q^3 , so the AVC function is Q^2 . Its minimum is obviously zero (when $Q = 0$). Therefore, in the face of sunk fixed cost, the firm will produce at any positive price, and its supply function is $S(P) = (P/3)^{1/2}$.

In the face of avoidable fixed costs, the price must be higher than the minimum of average cost. The total cost function is the sum of variable and fixed cost, and the average cost function is the total cost function divided by Q :

$$\begin{aligned} C(Q) &= VC(Q) + FC \\ C(Q) &= Q^3 + 3,000 \\ AC(Q) &= C(Q)/Q \\ AC(Q) &= Q^2 + (3,000/Q) \end{aligned}$$

To find the minimum of $AC(Q)$, we need to find where it crosses the $MC(Q)$ curve:

$$\begin{aligned} AC(Q) &= MC(Q) \\ Q^2 + (3,000/Q) &= 3Q^2 \\ (3,000/Q) &= 2Q^2 \\ (1,500/Q) &= Q^2 \\ 1,500 &= Q^3 \\ Q &= 11.447 \end{aligned}$$

We can find that $AC(11.447) = MC(11.447) = \393.11 . This means that if the price is less than \$393.11, the firm is better off producing nothing. So, in the face of the avoidable fixed cost, its supply function is:

$$S(P) = \begin{cases} 0 & \text{if } P < \$393.11 \\ \sqrt{\frac{P}{3}} & \text{if } P \geq \$393.11 \end{cases}$$

9.8

If Dan produces, he will choose a quantity that satisfies the quantity rule:

$$P = MC$$

$$P = 4 + (Q/20)$$

$$P - 4 = Q/20$$

$$20P - 80 = Q$$

Dan will only produce, however, if the price is higher than the minimum of average cost. Since cost is $C(Q) = 4Q + (Q^2/40)$, then $AC(Q)$, which is just $C(Q)/Q$ must be $4 + (Q/40)$. Just from looking at the function, we can tell that the minimum of $AC(Q)$ occurs at a quantity of zero (or just about zero) and that the minimum $AC(Q)$ is \$4. Therefore, Dan will produce at any price greater than or equal to \$4, and his supply function is:

$$S(P) = \begin{cases} 0 & \text{if } P < \$4 \\ 20P - 80 & \text{if } P \geq \$4 \end{cases}$$

If Dan has an avoidable fixed cost of \$10, then his cost function for producing becomes $C(Q) = 4Q + (Q^2/40) + 10$, and his average cost becomes $4 + (Q/40) + (10/Q)$. To find the minimum of this $AC(Q)$ function, we have to set it equal to $MC(Q)$.

$$AC(Q) = MC(Q)$$

$$4 + (Q/40) + (10/Q) = 4 + (Q/20)$$

$$(Q/40) + (10/Q) = (Q/20)$$

$$(10/Q) = (Q/40)$$

$$Q^2 = 400$$

$$Q = 20$$

We can then see that $AC(20) = MC(20) = \$5$. This means that Dan will only produce in this case if the price is greater than or equal to \$5. If he does produce, he still uses the same condition as above, so his supply function would be:

$$S(P) = \begin{cases} 0 & \text{if } P < \$5 \\ 20P - 80 & \text{if } P \geq \$5 \end{cases}$$

9.9

If Dan produces, he will still choose a quantity that satisfies the quantity rule:

$$P = MC$$

$$P = 6 + (Q/20)$$

$$P - 6 = Q/20$$

$$20P - 120 = Q$$

Dan will only produce, however, if the price is higher than the minimum of average cost. Since cost is $C(Q) = 6Q + (Q^2/40)$, then $AC(Q)$, which is just $C(Q)/Q$ must be $6 + (Q/40)$. Just from looking at the function, we can tell that the minimum of $AC(Q)$ occurs at a quantity of zero (or just about zero) and that the minimum $AC(Q)$ is \$6. Therefore, Dan will produce at any price greater than or equal to \$6, and his supply function is:

$$S(P) = \begin{cases} 0 & \text{if } P < \$6 \\ 20P - 120 & \text{if } P \geq \$6 \end{cases}$$

The supply curve has experienced a parallel shift upwards.

In the case where Dan has an avoidable fixed cost of \$10, his cost function for producing becomes $C(Q) = 6Q + (Q^2/40) + 10$, and his average cost becomes $6 + (Q/40) + (10/Q)$. To find the minimum of this $AC(Q)$ function, we have to set it equal to $MC(Q)$.

$$\begin{aligned} AC(Q) &= MC(Q) \\ 6 + (Q/40) + (10/Q) &= 6 + (Q/20) \\ (Q/40) + (10/Q) &= (Q/20) \\ (10/Q) &= (Q/40) \\ Q^2 &= 400 \\ Q &= 20 \end{aligned}$$

We can then see that $AC(20) = MC(20) = \$7$. This means that Dan will only produce in this case if the price is greater than or equal to \$7. If he does produce, he still uses the same condition as above, so his supply function would be:

$$S(P) = \begin{cases} 0 & \text{if } P < \$7 \\ 20P - 120 & \text{if } P \geq \$7 \end{cases}$$

Again, the supply curve has just experienced a parallel shift upward.

9.10

His supply curve from End-of-Chapter exercise 9.8 with an avoidable fixed cost of \$10 was the following:

$$S(P) = \begin{cases} 0 & \text{if } P < \$5 \\ 20P - 80 & \text{if } P \geq \$5 \end{cases}$$

If his avoidable fixed cost increases to \$22.50, then his cost function for producing becomes $C(Q) = 4Q + (Q^2/40) + 22.5$, and his average cost becomes $4 + (Q/40) + (22.5/Q)$. To find the minimum of this $AC(Q)$ function, we have to set it equal to $MC(Q)$.

$$\begin{aligned} AC(Q) &= MC(Q) \\ 4 + (Q/40) + (22.5/Q) &= 4 + (Q/20) \\ (Q/40) + (22.5/Q) &= (Q/20) \\ (22.5/Q) &= (Q/40) \\ Q^2 &= 900 \\ Q &= 30 \end{aligned}$$

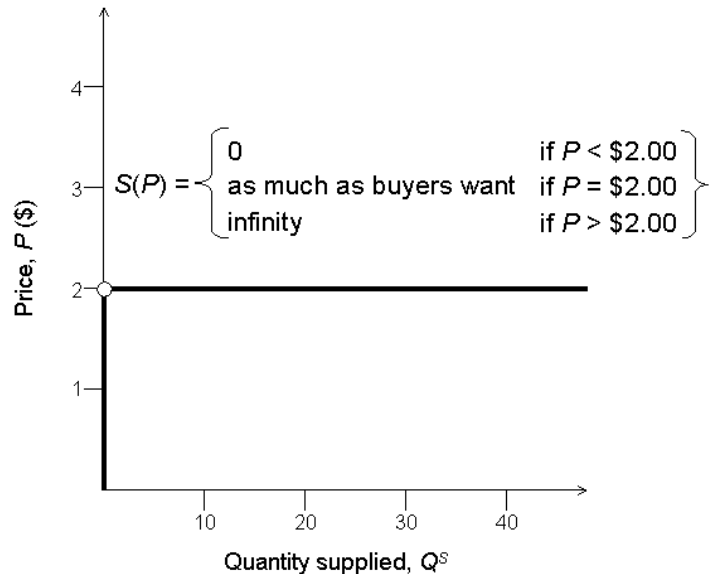
We can then see that $AC(30) = MC(30) = \$5.50$. This means that Dan will only produce in this case if the price is greater than or equal to \$5.50. If he does produce, he still uses the same condition as above, so his supply function would be:

$$S(P) = \begin{cases} 0 & \text{if } P < \$5.50 \\ 20P - 80 & \text{if } P \geq \$5.50 \end{cases}$$

Nothing has changed about his supply function, other than his shutdown price.

9.11

If it costs a firm \$2 to produce each unit of its output, then $MC = 2$. If price is greater than \$2.00, the quantity rule that $P = MC$ will never be satisfied because price will always be greater than marginal cost. In this situation, the firm would want to produce an infinite number of units in order to maximize profit. If the price were lower than \$2, then the quantity rule could never be satisfied because marginal cost would always be greater than the price. This firm would not be willing to produce anything at price lower than \$2. If the price were exactly equal to \$2, the firm would be willing to produce any amount that buyers were willing to buy. The firm would be indifferent between producing and not producing.



The supply curve would be a horizontal line at the price of \$2. At prices below \$2, quantity supplied would be equal to zero.

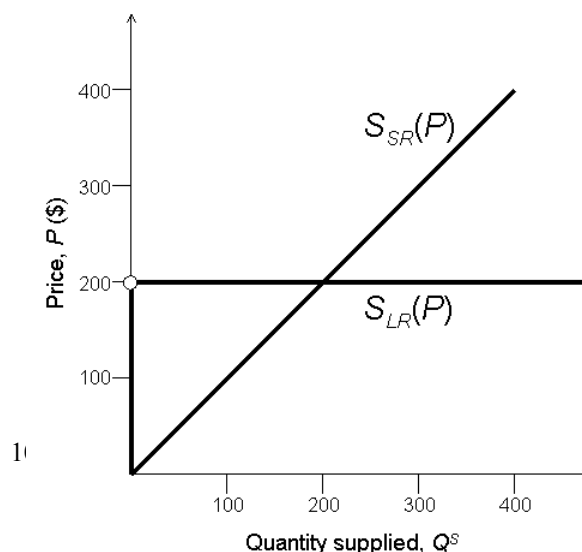
9.12

From End-of-Chapter exercise 8.14 on page 292, the short-run cost function was:

$$C_{SR}^{200}(Q) = 20,000 + \frac{Q^2}{2}$$

Students will have to know, either by calculus or by recognizing it from previous problems, that the MC_{SR} associated with the above short-run cost function is $MC = Q$. Given this, we can satisfy the quantity rule: $P = MC$ means that $P = Q$.

The short run supply function will be $S(P) = P$ when Hannah and Sam produce. We must figure out their shutdown price, which is the minimum of their average variable cost. The variable part of their short-run cost is the $Q^2/2$ part. The average variable cost is just this expression divided by Q , which is $Q/2$. It is clear that AC_{min} occurs when $Q = 0$ and that $AC_{min} = 0$. Therefore, Hannah and Sam will produce in the short run at any positive price.



Again, from exercise 8.14, the long run cost function was $C_{LR}(Q) = 200Q$. The MC_{LR} is easy to spot: $MC_{LR} = 200$. We have run into the situation of a constant marginal cost before. If price is higher than \$200, the firm will want to produce and sell an infinite amount of the good. If the price is lower than \$200, the firm will want to produce and sell nothing. If the price is exactly \$200, the firm will produce and sell any amount—whatever buyers ask for.

The two supply curves are graphed to the right.

9.13

From End-of-Chapter exercise 8.15 on page 292, the short-run cost function was:

$$C_{SR}^{100}(Q) = 100,000 + \frac{Q^4}{1,000}$$

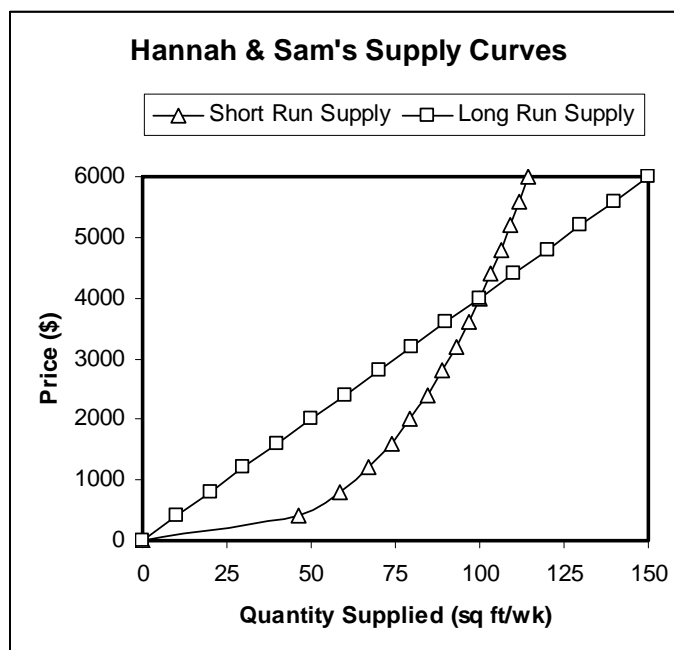
Students will have to know, either by calculus or by some other way, that the MC_{SR} associated with the above short-run cost function is $MC = Q^3/250$. Given this, we can satisfy the quantity rule: $P = MC$ means that $P = Q^3/250$. When we solve this for Q we find that $Q = (250P)^{1/3}$.

The short run supply function will be $S(P) = (250P)^{1/3}$ when Hannah and Sam produce. We must figure out their shutdown price, which is the minimum of their average variable cost. The variable part of their short-run cost is the $Q^4/1000$ part. The average variable cost is just this expression divided by Q , which is $Q^3/1000$. It is clear that AVC_{min} occurs when $Q = 0$ and that $AVC_{min} = 0$. Therefore, Hannah and Sam will produce in the short run at any positive price.

Again, from exercise 8.15, the long run cost function was $C_{LR}(Q) = 20Q^2$. Students will have to know by calculus or some other way that the $MC_{LR} = 40Q$. Given this, we can satisfy the quantity rule: $P = MC$ means that $P = 40Q$. When we solve this for Q we find that $Q = P/40$.

The long run supply function will be $S(P) = P/40$ when Hannah and Sam produce. We must figure out their shutdown price, which is the minimum of their average cost. The average cost is just the cost function above divided by Q , which gives $20Q$. It is clear that AC_{min} occurs when $Q = 0$ and that $AC_{min} = 0$. Therefore, Hannah and Sam will produce in the long run at any positive price.

The more complicated short run supply curve calls for using something like Excel to make the graphs. Both curves are graphed to the right.



9.14

Producer surplus, as defined in the text, is revenue less avoidable costs. Since marginal costs only show costs that change with the level of output, the MC curve could be thought of as a Marginal Avoidable Cost curve. Therefore, since MR equals the change in revenue from an increase in output, and MC equals the change in avoidable cost from an increase in output, it must be that $MR - MC$ equals the change in producer surplus from an increase in output. The area in between the MR and MC curves will be the sum of all of the incremental changes in producer surplus from increases in output. The sum of all of the increases in producer surplus would be equal to total producer surplus, so the area between the MR and MC curves is a valid measure of producer surplus. As long as $MR > MC$, then increasing output will increase producer surplus. When they are equal, producer surplus has stopped growing.

9.15

We must apply the quantity rule to both goods, so that for each good $P = MC$. We have to find Q_B and Q_C such that the following two equations hold:

$$\begin{aligned} 120 &= 50 + 2Q_B + Q_C \\ 75 &= 25 + 2Q_C + Q_B \end{aligned}$$

This system is easily solved by substitution. Solving the first equation for Q_C gives the condition that $Q_C = 70 - 2Q_B$, which can be substituted into the second equation:

$$\begin{aligned} 75 &= 25 + 2(70 - 2Q_B) + Q_B \\ 75 &= 25 + 140 - 4Q_B + Q_B \\ 75 &= 165 - 3Q_B \\ 3Q_B &= 90 \\ Q_B &= 30 \end{aligned}$$

Then, plugging this into the condition above shows that Q_C is 10. Producing this combination of output yields profit of:

$$[120(30) + 75(10)] - [50(30) + (30)^2 + (30)(10)] - [25(10) + (10)^2 + (30)(10)] = \$1,000$$

Applying the shutdown rule, we must consider whether this level of production is better than shutting down production of one or both goods. If they stop producing garden chairs, so that $Q_C = 0$, their cost function for benches becomes $C_B(Q_B) = 50Q_B + Q_B^2$ and the marginal cost is $MC_B = 50 + 2Q_B$. To find profit-maximizing quantity of Q_B , we set $P_B = MC_B$:

$$\begin{aligned} 120 &= 50 + 2Q_B \\ 2Q_B &= 70 \\ Q_B &= 35 \end{aligned}$$

In this case, profit would be $120(35) - [50(35) + (35)^2] = \$1,225$.

Similarly, if they stop producing garden benches, so that $Q_B = 0$, their cost function for benches becomes $C_C(Q_C) = 25Q_C + Q_C^2$ and the marginal cost is $MC_C = 25 + 2Q_C$. To find profit-maximizing quantity of Q_C , we set $P_C = MC_C$:

$$\begin{aligned} 75 &= 25 + 2Q_C \\ 50 &= 2Q_C \end{aligned}$$

$$Q_C = 25$$

In this case, profit would be $75(25) - [25(25) + (25)^2] = \625 .

The highest profit (\$1,225) comes when Noah and Naomi produce 35 garden benches and no garden chairs.

To analyze the case where $P_B = \$135$, we repeat all of the above steps.

$$135 = 50 + 2Q_B + Q_C$$

$$75 = 25 + 2Q_C + Q_B$$

This system is easily solved by substitution. Solving the first equation for Q_C gives the condition that $Q_C = 85 - 2Q_B$, which can be substituted into the second equation:

$$75 = 25 + 2(85 - 2Q_B) + Q_B$$

$$75 = 25 + 170 - 4Q_B + Q_B$$

$$75 = 195 - 3Q_B$$

$$3Q_B = 120$$

$$Q_B = 40$$

From above, we see that in this case $Q_C = 5$. As above we compare the profit from producing 40 benches and five chairs (which is \$1,625) to the profit from producing only garden benches or only garden chairs. The profit from producing only garden chairs will not have changed from before, and since it is only \$625, it is already ruled out. The only other case that must be considered is the case where $Q_C = 0$. As before, we set $P_B = MC_B$:

$$135 = 50 + 2Q_B$$

$$2Q_B = 85$$

$$Q_B = 42.5$$

If they produced 42.5 garden benches and no garden chairs, profit would be \$1,806.25; once again the best choice is to produce only garden benches, but no garden chairs.