

Multiple Choice:

1. Section 8.1 presents a discussion of fixed and variable costs, sunk costs and avoidable costs. The take-home message of this section is:
 - a. It is not possible to classify any cost (such as the purchase price of a license to operate a taxi cab) unless you know the context of the decision and the terms of the sale. If you buy a one-year license, the purchase price is fixed for one year after you buy the license. It is only a sunk cost if you cannot legally sell the license to another party (to recover your money). It was avoidable prior to moment at which you actually committed to purchase it.
 - b. All fixed costs are sunk.
 - c. Fixed costs cannot be avoidable.
 - d. None of the above.
2. JetBlue has several different types of contracts for leasing gates at airports. At some airports, JetBlue must rent each gate for an entire day, regardless of the number of flights per day. At these airports, the gate rental expense is a:
 - a. variable cost.
 - b. fixed cost.
 - c. avoidable cost.
 - d. sunk cost.
3. Coors uses rice to brew beer, and it grows its own rice. Coors could have chosen to purchase this rice in the rice market at market prices, but – instead – Coors prefers to grow it's own rice. Does this strategy reduce the cost of obtaining this input?
 - a. yes, if the cost of growing the rice is lower than the cost to purchase the rice in the market
 - b. no, because the opportunity cost of using the rice it grows is equal to the market price
 - c. none of the above
4. If a long-lived asset is “expensed”,
 - a. the asset is very costly.
 - b. the firm records the full cost of the asset in the year the expenditure occurs.
 - c. the expense records the true cost of the asset.
 - d. all of the above
5. Calpine’s average cost function slopes up because:
 - a. Producing more electricity requires more natural gas. As Calpine increases output, it must use more natural gas, so the cost increases.
 - b. Calpine has diseconomies of scale.
 - c. Calpine uses its most efficient generators first. If that does not provide sufficient capacity, it adds generators that are less efficient. (Less efficient generators produce electricity at higher cost).
 - d. None of the above

6. Budget lines are one specific type of isocost line. The difference between the indifference curve/ budget line graphs used to analyze consumer decisions and the isoquant/isocost graphs used to analyze firm input purchase decisions is:
- The household is more careful about how it spends its money.
 - A household has one budget line because the household has a specific income at any point in time. In contrast, a firm looks at the entire family of isocost lines because the firm chooses the production quantity, buys inputs, and then sells the output to generate revenue. The firm is not restricted to one specific level of expenditure.
 - None of the above
7. When the firm is using the least-cost combination of inputs,
- the ratio of the marginal products of the two inputs is equal to the ratio of the prices of the two inputs.
 - the marginal product of the last dollar spent on input A is equal to the marginal product of the last dollar spent on input B.
 - the ratio of the prices of the two inputs is equal to MRTS.
 - all of the above
8. Dell noticed that computer manufacturers produced their products long before consumers bought them. This delay between producing the product and selling the product generates two types of costs in the computer industry. Identify the two that are noted in Application 8.4.
- The money spent to purchase the inputs and produce the product is not sitting in a bank. The opportunity cost of this capital is one of these costs.
 - The cost of computer parts was falling rapidly at that time. If input purchases could be delayed, the unit price would be lower.
 - Warehouses must be rented or purchased to store the assembled computers prior to sale.
9. Economies of scale occur when
- total cost falls as output increases.
 - average cost falls as output increases.
 - the cost curve shifts as output increases
 - all of the above.
10. Frederick Smith founded Federal Express in 1973. This company is successful because Frederick Smith identified a segment of consumers that were not well-served by existing firms. He developed this insight when he was:
- a high school student
 - an undergraduate student
 - a graduate student
 - an employee of one of the existing firms

Answers to Multiple Choice Quiz

1. a
2. b
3. b
4. b
5. c
6. b
7. d
8. a, b
9. b
10. b

Answers to In-Text Questions

8.1

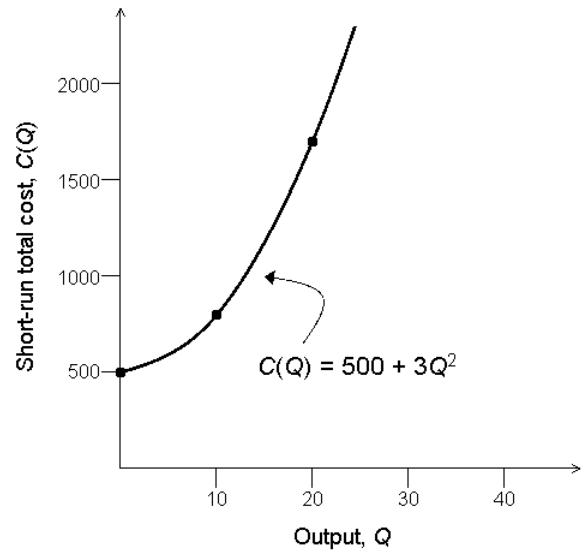
The relationship between the labor input and output is given by $Q = 2\sqrt{L}$. Solving for L gives a function for the amount of labor needed to produce Q units of output.

$$Q = 2\sqrt{L}$$

$$\frac{Q}{2} = \sqrt{L}$$

$$L = \frac{Q^2}{4}$$

If we multiply this by the wage rate, it becomes the variable cost function: $VC = 3Q^2$. The short run cost function is therefore $C(Q) = 500 + 3Q^2$. The graph of this function is given to the right.



8.2

Recall that the $MRTS_{LK}$ for a Cobb-Douglas production function is:

$$MRTS_{LK} = \left(\frac{\alpha}{\beta} \right) \left(\frac{K}{L} \right)$$

The first condition that must hold for an interior solution is that the $MRTS_{LK}$ equal the price ratio W/R . We can substitute in α , β , W and R , all of which we know from the problem.

$$\left(\frac{\alpha}{\beta} \right) \left(\frac{K}{L} \right) = \frac{W}{R}$$

$$\left(\frac{0.5}{0.5} \right) \left(\frac{K}{L} \right) = \frac{\$1,000}{\$250}$$

$$\left(\frac{K}{L} \right) = 4$$

$$\mathbf{K = 4L}$$

The second condition that must hold is that this point be on the isoquant for $Q = 200$. So we plug $K = 4L$ into the 200-unit isoquant:

$$Q = 10L^{0.5}K^{0.5}$$

$$Q = 10L^{0.5}(4L)^{0.5}$$

$$Q = 20L$$

$$(200) = 20L$$

$$\mathbf{L = 10}$$

The least-cost input combination therefore uses 10 units of labor and 40 units of capital. Total cost is \$20,000 ($10 \times \$1,000 + 40 \times \250) per week.

8.3

To begin, we satisfy the first condition that $MRTS_{LK}$ equal the price ratio:

$$\left(\frac{\alpha}{\beta}\right)\left(\frac{K}{L}\right) = \frac{W}{R}$$

$$\left(\frac{0.5}{0.5}\right)\left(\frac{K}{L}\right) = \frac{\$1,000}{\$1,000}$$

$$\left(\frac{K}{L}\right) = 1$$

$$K = L$$

Then we plug this into the production function:

$$Q = 10L^{0.5}K^{0.5}$$

$$Q = 10L^{0.5}(L)^{0.5}$$

$$Q = 10L$$

The two conditions are $Q = 10L$ and $K = L$. Solving the first for L gives $L = Q/10$.

Plugging that into the second gives $K = Q/10$. To construct a cost function, we multiply these input requirement functions by the appropriate prices. The wage rate is \$1,000 and the cost of capital is, in this problem, also \$1,000. So the cost of producing Q units is:

$$C(Q) = 1,000(Q/10) + 1,000(Q/10)$$

$$C(Q) = 100Q + 100Q$$

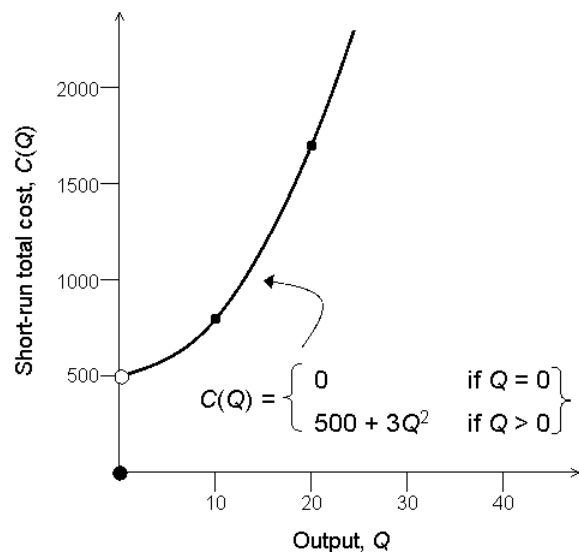
$$C(Q) = 200Q$$

8.4

The relationship between Noah and Naomi's labor input and their output is the same as in In-Text Exercise 8.1. The only difference is that in the long run, they can avoid the fixed cost of the garage space, so their long-run weekly cost function is

$$C(Q) = \begin{cases} 0 & \text{if } Q = 0 \\ 500 + 3Q^2 & \text{if } Q > 0 \end{cases}$$

The graph is given to the right.



8.5

Noah and Naomi need to divide production between the two plants so that their marginal costs are equal. To do this, they must choose Q_1 and Q_2 so that $MC_1 = MC_2$. Their total output is 100, we can substitute $100 - Q_1$ for Q_2 in the equation for MC_2 .

$$MC_2 = 4Q_2$$

$$MC_2 = 4(100 - Q_1)$$

$$MC_2 = 400 - 4Q_1$$

Now, we just find the Q_1 that equates MC_1 and MC_2 :

$$MC_1 = MC_2$$

$$12Q_1 = 400 - 4Q_1$$

$$16Q_1 = 400$$

$$Q_1 = 25$$

Since 25 units ought to be produced at plant 1, this means that the other 75 ought to be produced at plant 2. The total cost is the sum of the cost of production at each plant:

$$C = C_1 + C_2$$

$$C = 6Q_1^2 + 2Q_2^2$$

$$C = 6(25)^2 + 2(75)^2$$

$$C = 6(525) + 2(5,625)$$

$$C = \$15,000$$

8.6

The solution to In-Text Exercise 8.3 dealt with the situation where $W = R = \$1,000$. In this case, the solution was to use $Q/10$ units of capital and $Q/10$ units of labor. Since they are remodeling 100 square feet per week, this means they choose 10 units of capital and 10 units of labor.

In the short run, their capital is fixed at this quantity of 10, so the production function becomes: $Q = 10L^{0.5}(10)^{0.5}$, which can be solved for L :

$$Q = 10\sqrt{L}\sqrt{10}$$

$$\frac{Q}{10\sqrt{10}} = \sqrt{L}$$

$$L = \frac{Q^2}{1000}$$

The fixed cost is $K = 10$ multiplied by the price of capital, \$1,000. The variable cost is the function for L above multiplied by the price of labor, \$1,000. So the short run cost function is:

$$C_{SR}^{100}(Q) = (1,000)(10) + (1,000)\left(\frac{Q^2}{1,000}\right)$$

$$C_{SR}^{100}(Q) = (10,000) + Q^2$$

The long-run cost function was given in the solution to In-Text Exercise 8.3, and is:

$$C_{LR} = 200Q$$

Answers to End-of-Chapter Questions

8.1

Friedman's simple contention is that there are costs involved in every decision—including the decision to have a free lunch. Instead of being treated to a free lunch, you could have gone to your favorite restaurant to reconnect with a friend or have ordered (and paid for) a better lunch. Perhaps you could have skipped lunch altogether and studied for an exam. There are opportunity costs associated with every action.

8.2

By staying open hotels do not have to incur the costs of closing down and starting up every year. So, they avoid costs involved in hiring and training as well as maintenance and fees. Further, some costs must still be incurred even if the hotel closes; properties must be maintained, secured and managed. The resorts consider these costs to be sunk and so do not consider them when making a decision to stay open in the off season. It must be that the non-sunk costs of remaining open are smaller than the revenues earned in the off-season.

8.3

First, we turn the production function into a labor requirement function by solving for L , which gives $L = (2/3)Q$. Then, we multiply this by the wage rate to get the variable cost function.

$$VC = (15)(2/3)Q = 10Q$$

The short run cost function is the sum of this variable cost function and the fixed costs, if there are any.

$$C_{SR}(Q) = 10Q + FC$$

8.4

The implications of this confusing production function are that if $L \geq 2$, output equals zero. If $L < 2$, then output is negative. (Output can never be positive.) If output equals zero, then a typical cost function cannot be derived because any amount of labor greater than two produces the same amount of output (zero). The cost does not depend on output here, but on the number of units of labor hired to produce this no output. If $L < 2$, then $Q = L - 2$, so that $L = Q + 2$. This can be multiplied by the wage rate to give the variable cost function $VC = 15Q + 30$. The short run cost function implied by this bizarre production function (with any fixed cost denoted as FC) is:

$$C_{SR} = \begin{cases} 15L + FC & \text{if } Q = 0 \\ 15Q + 30 + FC & \text{if } Q < 0 \end{cases}$$

8.5

True. There are many combinations of inputs that are efficient methods of production. All of the combinations of inputs on an isoquant are efficient. It is the input price ratio that determines which one is least-cost. If this input price ratio were to change, the least-cost combination would become one of the other efficient combinations on the same isoquant.

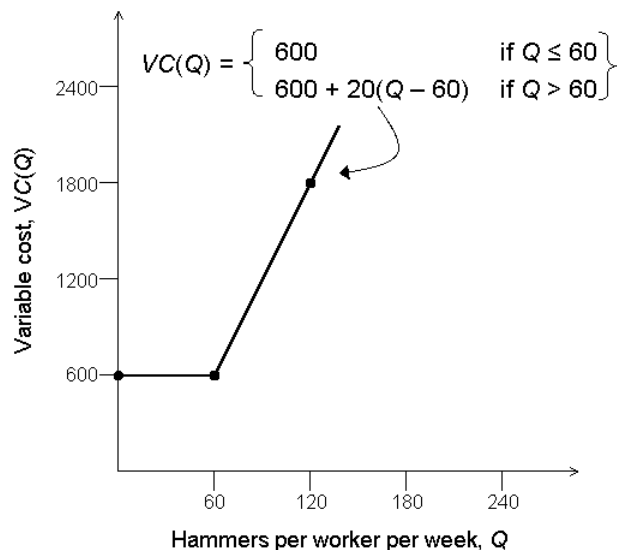
8.6

Johnson Tools must pay its employees for at least 30 hours of work each week, regardless of what is produced. So these first 30 hours may be thought of as a fixed cost, equal to $\$20 \times 30$, or \$600 per worker. Since the company can produce two hammers per hour, each worker could produce up to 60 hammers in the first 30 hours of work each week.

If Johnson Tools wants to produce more than 60 hammers per worker per week, it must pay the overtime rate of \$40 per hour, meaning that hammers over 60 (per worker) cost \$20 each, so the variable cost of these hammers is $20Q$.

Therefore, the variable cost function where Q is hammers *per worker* per week is:

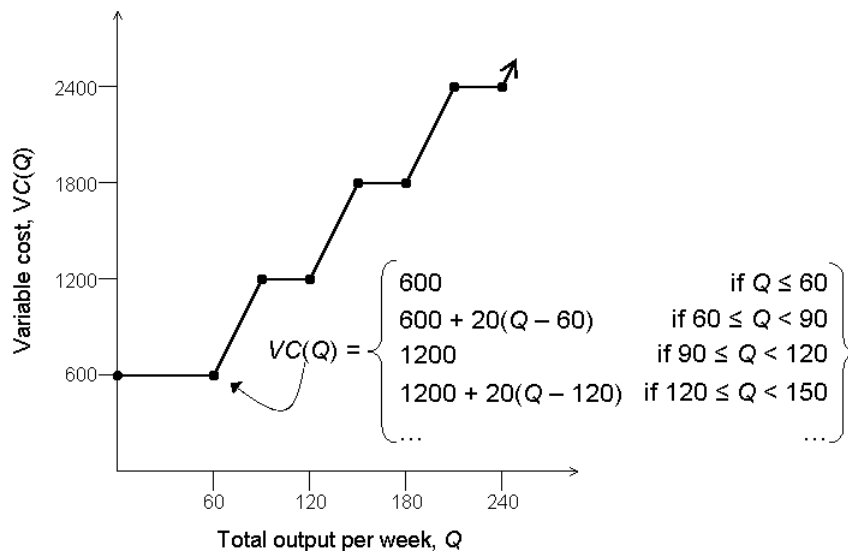
$$VC(Q) = \begin{cases} \$600 & \text{if } Q \leq 60 \\ \$600 + \$20(Q - 60) & \text{if } Q > 60 \end{cases}$$



However, if we think of Q as being total output (in the normal way), then we realize that we don't know how many employees Johnson Tools has. That is okay since we then realize that Johnson Tools will hire enough workers to avoid paying overtime to any of them.

Thinking through this case, we see that Johnson Tools pays \$600 for the first 60 hammers it wants to produce (by signing a contract with one worker). For hammers over 60, Johnson Tools pays this first worker over time to produce additional hammers until it would simply be cheaper to hire another worker and give neither of the two overtime. Since the next worker costs \$600, the maximum number of overtime hours Johnson will give the first worker is $\$600/\$40 = 15$. In these 15 hours, this worker will have produced 30 more hammers. This continues like this, so that the variable cost function looks like this:

$$VC(Q) = \left\{ \begin{array}{ll} \$600 & \text{if } Q \leq 60 \\ \$600 + \$20(Q - 60) & \text{if } 60 < Q \leq 90 \\ \$1200 & \text{if } 90 < Q \leq 120 \\ \$1200 + \$20(Q - 120) & \text{if } 120 < Q \leq 150 \\ \dots & \dots \end{array} \right\}$$



8.7

High school graduates have a marginal productivity of four hammers per hour and they cost \$15 per hour to employ. If we divide W_{HS} by MP_{HS} , we will have a new number that tells us the marginal cost of a hammer if it is produced by a high school graduate. $W_{HS}/MP_{HS} = \$15/4 = \3.75 per hammer produced by a high school graduate. College graduates produce five hammers per hour but cost \$25 per hour to employ. We can construct the same figure as above. The marginal cost of a hammer produced by a college graduate is $\$25/5 = \5.00 per hammer. It is easy to see that a hammer produced by a high school graduate is cheaper, so the least-cost method of producing involves hiring only high school graduates.

If the productivity of a high school graduate were only two hammers per hour, then the marginal cost of a hammer produced by a high school graduate would increase to $\$15/2 = \7.50 per hammer. In this case, college graduates would produce the least-cost hammers.

The critical difference in productivity is the one that would make the firm indifferent to hiring college graduates or high school graduates; in this case, the marginal cost of hammers produced by both types of workers would be equal.

$$\begin{aligned}\frac{W_{HS}}{MP_{HS}} &= \frac{W_C}{MP_C} \\ \frac{\$15}{MP_{HS}} &= \frac{\$25}{MP_C} \\ \frac{\$15}{\$25} &= \frac{MP_{HS}}{MP_C} \\ 0.60 &= \frac{MP_{HS}}{MP_C}\end{aligned}$$

If high school graduates are 60% as productive as college graduates, then the firm is indifferent between the two worker types. If MP_{HS}/MP_C is greater than 60%, then high school graduates are hired. If MP_{HS}/MP_C is less than 60%, then only college graduates are hired.

8.8

As usual, the first condition requires that the $MRTS_{LK}$ equal the input price ratio.

$$\left(\frac{\alpha}{\beta}\right)\left(\frac{K}{L}\right) = \frac{W}{R}$$

$$\left(\frac{0.2}{0.3}\right)\left(\frac{K}{L}\right) = \frac{\$1,500}{\$1,000}$$

$$\left(\frac{2}{3}\right)\left(\frac{K}{L}\right) = \frac{3}{2}$$

$$\left(\frac{K}{L}\right) = \frac{9}{4}$$

$$K = 2.25L$$

The second condition is the production function.

$$Q = F(L, K) = 10L^{0.2}K^{0.3}$$

$$Q = 10L^{0.2}(2.25L)^{0.3}$$

$$Q = 10(2.25)^{0.3}(L)^{0.5}$$

$$Q = 12.754(L)^{0.5}$$

$$(L)^{0.5} = Q/12.754$$

$$L = Q^2/162.671$$

Since Q is 100, L equals $10,000/162.671$, or **61.474**. Therefore, K equals **138.316**.

$$TC = \$1,500(61.474) + \$1,000(138.316) = \$92,211 + \$138,316 = \mathbf{\$230,527}$$

8.9

Since we know that the production function exhibits constant returns to scale, $\alpha + \beta$ must equal 1. Furthermore, since we know XYZ spends twice as much on labor as it does on capital, α (the exponent on L) must be twice as large as β (the exponent on K). This means that α is $\frac{2}{3}$ and β is $\frac{1}{3}$.

8.10

Noah and Naomi need to divide production between the two plants so that their marginal costs are equal. To do this, they must choose Q_1 and Q_2 so that $MC_1 = MC_2$. Their total output is 200, we can substitute $200 - Q_1$ for Q_2 in the equation for MC_2 .

$$MC_2 = 4Q_2$$

$$MC_2 = 4(200 - Q_1)$$

$$MC_2 = 800 - 4Q_1$$

Now, we just find the Q_1 that equates MC_1 and MC_2 :

$$MC_1 = MC_2$$

$$6Q_1 = 800 - 4Q_1$$

$$10Q_1 = 800$$

$$Q_1 = 80$$

Since 80 units ought to be produced at plant 1, this means that the other 120 ought to be produced at plant 2. The total cost is the sum of the cost of production at each plant:

$$C = C_1 + C_2$$

$$C = 3Q_1^2 + 2Q_2^2$$

$$C = 3(80)^2 + 2(120)^2$$

$$C = 3(6,400) + 2(14,400)$$

$$C = \$48,000$$

8.11

Noah and Naomi need to divide production between the two plants so that their marginal costs are equal. To do this, they must choose Q_1 and Q_2 so that $MC_1 = MC_2$. Their total output is 100, we can substitute $100 - Q_1$ for Q_2 in the equation for MC_2 .

$$MC_2 = 650 + 4Q_2$$

$$MC_2 = 650 + 4(100 - Q_1)$$

$$MC_2 = 1,050 - 4Q_1$$

Now, we just find the Q_1 that equates MC_1 and MC_2 :

$$MC_1 = MC_2$$

$$6Q_1 = 1,050 - 4Q_1$$

$$10Q_1 = 1,050$$

$$Q_1 = 105$$

The solution we get suggests that 105 of the 100 units be produced at plant 1. This would mean that -5 units should be produced at plant 2. This is a corner point solution. The high fixed cost at plant 2 makes it so that, given $Q = 100$, marginal cost at each plant cannot be made equal. MC_1 will always be lower. Therefore, all 100 units should be produced at plant 1.

The total cost is the sum of the cost of production at each plant:

$$\begin{aligned}C &= C_1 + C_2 \\C &= 6Q_1^2 + 2Q_2^2 \\C &= 6(100)^2 + 2(0)^2 \\C &= 6(10,000) \\C &= \mathbf{\$60,000}\end{aligned}$$

8.12

Noah and Naomi need to divide production between the two plants so that their marginal costs are equal. To do this, they must choose Q_1 and Q_2 so that $MC_1 = MC_2$. Their total output is 100, we can substitute $100 - Q_1$ for Q_2 in the equation for MC_2 .

$$\begin{aligned}MC_2 &= 650 - 4Q_2 \\MC_2 &= 650 - 4(100 - Q_1) \\MC_2 &= 250 + 4Q_1\end{aligned}$$

Now, we just find the Q_1 that equates MC_1 and MC_2 :

$$\begin{aligned}MC_1 &= MC_2 \\600 - 6Q_1 &= 250 + 4Q_1 \\350 &= 10Q_1 \\Q_1 &= \mathbf{35}\end{aligned}$$

Be careful. This solution suggests that 35 units be produced at plant 1 and the other 65 be produced at plant 2. But a closer look at the marginal cost functions reveals something interesting: at both plants, marginal cost falls as Q increases. This means that the best option will be to produce all of the output at one of the two plants, to take full advantage of this falling marginal cost. To figure out which, we can just calculate the total cost of producing all 100 units at each plant (and the total cost of the above solution, just for comparison.)

$$\begin{aligned}C &= C_1 + C_2 \\C(Q_1, Q_2) &= 600Q_1 - 3Q_1^2 + 650Q_2 - 2Q_2^2 \\C(35, 65) &= 600(35) - 3(35)^2 + 650(65) - 2(65)^2 = \$51,125 \\C(100, 0) &= 600(100) - 3(100)^2 + 650(0) - 2(0)^2 = \$30,000 \\C(0, 100) &= 600(0) - 3(0)^2 + 650(100) - 2(100)^2 = \$45,000\end{aligned}$$

The least-cost solution is to produce all 100 units at plant 1. Total cost is \$30,000.

8.13

To find the short run cost function when Hannah and Sam are initially remodeling 200 square feet per week, we first need to figure out how much capital they are using when they do this. This is the amount of capital that they will be stuck with in the short run. The problem starts as usual:

$$\begin{aligned}\left(\frac{\alpha}{\beta}\right)\left(\frac{K}{L}\right) &= \frac{W}{R} \\ \left(\frac{0.5}{0.5}\right)\left(\frac{K}{L}\right) &= \frac{\$1,000}{\$250} \\ \left(\frac{K}{L}\right) &= 4 \\ \mathbf{K} &= \mathbf{4L}\end{aligned}$$

The second condition to be satisfied is that they produce 200 units:

$$\begin{aligned}200 &= 10L^{0.5}K^{0.5} \\ 200 &= 10L^{0.5}(4L)^{0.5} \\ 200 &= 20L \\ \mathbf{L} &= \mathbf{10}\end{aligned}$$

Therefore, K is 40. This is their fixed amount of capital in the short run, which is a fixed cost of $\$250(40)$, or $\$10,000$. When their capital is fixed at 40 units, the production function becomes $Q = 10\sqrt{L}\sqrt{40}$, which, when solved for L yields: $L = Q^2/4000$. Multiplying this equation for L by the wage rate of $\$1,000$ gives the variable cost function $VC(Q) = Q^2/4$. The short run total cost function is the sum of the fixed and variable cost:

$$C_{SR}^{200}(Q) = 10,000 + \frac{Q^2}{4}$$

The long-run cost has not changed from the answer given in Worked-Out problem 8.3 on page 267 (since the production function and input prices have not changed).

$$C_{LR}(Q) = 100Q$$

8.14

To find the short run cost function when Hannah and Sam are initially remodeling 200 square feet per week, we first need to figure out how much capital they are using when they do this. This is the amount of capital that they will be stuck with in the short run. The problem starts as usual:

$$\begin{aligned}\left(\frac{\alpha}{\beta}\right)\left(\frac{K}{L}\right) &= \frac{W}{R} \\ \left(\frac{0.5}{0.5}\right)\left(\frac{K}{L}\right) &= \frac{\$1,000}{\$1,000} \\ \left(\frac{K}{L}\right) &= 1 \\ \mathbf{K} &= \mathbf{L}\end{aligned}$$

The second condition to be satisfied is the production function:

$$\begin{aligned}Q &= 10L^{0.5}K^{0.5} \\ Q &= 10L^{0.5}(L)^{0.5} \\ Q &= 10L \\ \mathbf{L} &= \mathbf{Q/10}\end{aligned}$$

With $Q = 200$, $L = 20$.

Therefore, K is also 20. This is their fixed amount of capital in the short run, which is a fixed cost of $\$1,000(20)$, or $\$20,000$. When their capital is fixed at 20 units, the production function becomes $Q = 10\sqrt{L}\sqrt{20}$, which, when solved for L yields: $L = Q^2/2000$. Multiplying this equation for L by the wage rate of $\$1,000$ gives the variable cost function $VC(Q) = Q^2/2$. The short run total cost function is the sum of the fixed and variable cost:

$$C_{SR}^{200}(Q) = 20,000 + \frac{Q^2}{2}$$

To figure out the long-run cost function, we return to the result for L above, where Q was not fixed, $L = Q/10$. Since $K = L$, this means that K is also equal to $Q/10$. Therefore, the long-run cost function is:

$$\begin{aligned}C_{LR}(Q) &= RK + WL \\ C_{LR}(Q) &= 1,000(Q/10) + 1,000(Q/10) \\ C_{LR}(Q) &= 200Q\end{aligned}$$

8.15

To find the short run cost function when Hannah and Sam are initially remodeling 200 square feet per week, we first need to figure out how much capital they are using when they do this. This is the amount of capital that they will be stuck with in the short run. The problem starts as usual:

$$\begin{aligned}\left(\frac{\alpha}{\beta}\right)\left(\frac{K}{L}\right) &= \frac{W}{R} \\ \left(\frac{0.25}{0.25}\right)\left(\frac{K}{L}\right) &= \frac{\$1,000}{\$1,000} \\ \left(\frac{K}{L}\right) &= 1 \\ \mathbf{K} &= \mathbf{L}\end{aligned}$$

The second condition to be satisfied is the production function:

$$\begin{aligned}Q &= 10L^{0.25}K^{0.25} \\ Q &= 10L^{0.25}(L)^{0.25} \\ Q &= 10L^{0.5} \\ L^{0.5} &= Q/10 \\ \mathbf{L} &= \mathbf{Q^2/100}\end{aligned}$$

With $Q = 100$, $L = 100$.

Therefore, K is also 100. This is their fixed amount of capital in the short run, which is a fixed cost of $\$1,000(100)$, or $\$100,000$. When their capital is fixed at 100 units, the production function becomes:

$$\begin{aligned}Q &= 10L^{0.25}100^{0.25} \\ Q &= 31.623L^{0.25} \\ L^{0.25} &= Q/31.623 \\ \mathbf{L} &= \mathbf{Q^4/1,000,000}\end{aligned}$$

Multiplying this equation for L by the wage rate of $\$1,000$ gives the variable cost function $VC(Q) = Q^4/1,000$. The short run total cost function is the sum of the fixed and variable cost:

$$C_{SR}^{100}(Q) = 100,000 + \frac{Q^4}{1,000}$$

To figure out the long-run cost function, we return to the result for L above, where Q was not fixed, $L = Q^2/100$. Since $K = L$, this means that K is also equal to $Q^2/100$. Therefore, the long-run cost function is:

$$\begin{aligned}C_{LR}(Q) &= RK + WL \\C_{LR}(Q) &= 1,000(Q^2/100) + 1,000(Q^2/100) \\C_{LR}(Q) &= 20Q^2\end{aligned}$$

To derive average cost functions, we simply divide by Q . So they are:

$$\begin{aligned}AC_{SR}^{100}(Q) &= \frac{100,000}{Q} + \frac{Q^3}{1,000} \\AC_{LR}(Q) &= 20Q\end{aligned}$$

8.16

This firm will exhibit diseconomies of scale because the change in cost increases faster than a change in quantity. For example, doubling quantity more than doubles cost (in fact, it quadruples it). Since cost functions show the cost of the most efficient means of producing any given quantity, we can be certain that doubling output means increasing labor and capital by more than two times (assuming that W and R are constant).

8.17

If we think of service to different airports as different products, then this phenomenon is quite easily explained in terms of economies of scope. It is cheaper for the airline to produce a variety of products at one more efficient plant (the hub) than it is to divide products up to be produced at various plants. The hubs produce a great variety of products (service to many other airports), while the non-hubs produce only one product: service to the hub.