Add-On 5A

MAXIMIZING A UTILITY FUNCTION

Ellen's income is \$M per month. She spends it all on soup and bread. As in Section 5.1, we can write her budget constraint mathematically:

$$P_S S + P_B B \leq M \qquad \textbf{(1)}$$

Here, S stands for pints of soup, P_S is the price of soup per pint, B stands for ounces of bread, and P_B is the price of bread per ounce.

A utility function assigns a utility index to each consumption bundle. Let's assume that the utility function $U(S, B)$ summarizes Ellen's preferences. Making the best choice is equivalent to finding, among the affordable bundles, the one with the highest utility index. In other words, we can represent Ellen's problem mathematically as follows:

$$\text{Choose } S \text{ and } B \text{ to maximize } U(S,B) \qquad \textbf{(2)}$$

$$\text{subject to the constraint (1)}$$

This problem involves *constrained maximization*—that is, the maximization of a function (known as the *objective function*) subject to a constraint. This Add-On describes two methods that are commonly used to solve these types of problems. To keep the explanation simple, we'll focus throughout on interior solutions, using the same utility function to illustrate each method:

$$U(S, B) = 2\log(S) + 3\log(B) \qquad \textbf{(3)}$$

Computational Methods

Even if you do not know calculus, you can solve Ellen's problem using spreadsheet software, such as Excel. To illustrate, imagine that $M = 100$, $P_S = 3$, and $P_B = 2$. We'll also assume that Ellen's preferences correspond to the utility function in (3).

The spreadsheet on the next page compares some of Ellen's possible choices. Column A tells us how much she spends on soup. The lowest soup expenditure listed is \$10; each successive choice adds another \$10 up to \$90. Column B tells us how much she spends on bread. This amount is the difference between \$100 and the number in column A. Column C indicates the amount of soup purchased. It equals the number in column A divided by the price of soup (\$3 per pint). Column D indicates the amount of bread purchased. It equals the number in column B divided by the price of bread (\$2 per ounce). Columns E and F compute the logarithms of the numbers in columns C and D, respectively. Column G computes utility using the expression in (3) (that is, twice the number in column E plus three times the number in column F).

	A	B	C	D	E	F	G
1	\$ on *S*	\$ on *B*	*S*	*B*	log(*S*)	log(*B*)	Utility
2	10	90	3.33	45	0.52	1.65	6.01
3	20	80	6.67	40	0.82	1.60	6.45
4	30	70	10.00	35	1.00	1.54	6.63
5	40	60	13.33	30	1.12	1.48	6.68
6	50	50	16.67	25	1.22	1.40	6.64
7	60	40	20.00	20	1.30	1.30	6.51
8	70	30	23.33	15	1.37	1.18	6.26
9	80	20	26.67	10	1.43	1.00	5.85
10	90	10	30.00	5	1.48	0.70	5.05

By inspecting this spreadsheet, we can see that the best choice among those listed is to spend \$40 on soup (for 13.33 pints) and \$60 on bread (for 30 loaves). This bundle is associated with a utility index of 6.68, the highest number in column G.

Is bundle 5 the best affordable bundle? Let's see whether Ellen can do better by shifting one or two pennies from soup to bread, or vice versa. To do so, we'll add some alternatives to the spreadsheet. We'll also increase the precision of the utility index computed in column G, so we can distinguish among fine gradations of well-being.

	A	B	C	D	E	F	G
1	\$ on *S*	\$ on *B*	*S*	*B*	log(*S*)	log(*B*)	Utility
11	39.98	60.02	13.33	30.01	1.12	1.48	6.68124106
12	39.99	60.01	13.33	30.01	1.12	1.48	6.68124119
13	40	60	13.33	30.00	1.12	1.48	6.68124124
14	40.01	59.99	13.34	30.00	1.13	1.48	6.68124119
15	40.02	59.98	13.34	29.99	1.13	1.48	6.68124106

Once again, the best choice among those listed is to spend \$40 on soup and \$60 on bread. If Ellen shifts even one or two pennies from soup to bread or vice versa, her utility index declines slightly.

Real-world problems of this nature are usually quite complex and are sometimes difficult or impossible to solve with simple spreadsheets (or even with calculus). Fortunately, sophisticated computational tools are available. Harnessing the power of computers, decision makers (and economists) can solve an extremely wide range of practical problems.

Solving the Problem with Calculus

Basic calculus tells us we can maximize the types of objective functions commonly encountered in economics by taking derivatives and setting them equal to zero (the *first-order conditions* for maximization).[1] The problem described in (2) is a bit more complicated, in that we are trying to maximize a function while respecting a constraint. How is this done? In this section, we describe two alternative approaches. The first is simpler, but the second is more powerful.

[1]This works as long as the function is concave. A function is concave if its second derivative is negative.

The Substitution Method Sometimes, when we are asked to maximize a function of several variables subject to a constraint, we can solve the constraint for one variable as a function of the others. This allows us to substitute for that variable in the objective function, and then maximize it over the remaining variables. This procedure is known as the *substitution method*.

Let's apply the substitution method to the problem described in expression (2). As long as the More-Is-Better Principle holds, we know that Ellen's best affordable choice must lie on her budget line. This means we can replace the $\leq$ symbol in (1) with the $=$ symbol. We can then use that formula to solve for B, the amount of bread, in terms of S, the amount of soup:

$$B = \frac{M}{P_B} - \frac{P_S}{P_B} S \qquad \textbf{(4)}$$

Formula (4) tells us how much bread Ellen can purchase with her remaining cash once she has bought S pints of soup.

Substituting formula (4) into the utility function gives us the following expression:

$$U\left(S, \frac{M}{P_B} - \frac{P_S}{P_B} S\right) \qquad \textbf{(5)}$$

When Ellen buys S pints of soup and spends the rest of her money on bread, her consumption bundle delivers the utility value described by expression (5).

Now let's think about the following problem:

$$\text{Choose } S \text{ to maximize } U\left(S, \frac{M}{P_B} - \frac{P_S}{P_B} S\right) \qquad \textbf{(6)}$$

This is just another way of writing the problem in (2). However, in (6), there is no constraint. We've eliminated it by substituting it for one of the variables, B. At the same time, we've eliminated the variable B, so that we can maximize utility using only one variable, S, instead of two variables, S and B.

To solve the problem in (6), we take the derivative with respect to the variable S and set the result equal to zero:

$$\frac{\partial U}{\partial S} - \frac{\partial U}{\partial B} \frac{P_S}{P_B} = 0, \qquad \textbf{(7)}$$

where $\frac{\partial U}{\partial S}$ and $\frac{\partial U}{\partial B}$ are the partial derivatives of the function U with respect to S and B, respectively. Since $\frac{\partial U}{\partial S}$ measures the rate at which U changes as S increases, it's equivalent to the marginal utility of soup, MU_S. Similarly, $\frac{\partial U}{\partial B}$ is equivalent to the marginal utility of bread, MU_B. That means we can rewrite formula (7) as follows:

$$\frac{MU_S}{P_S} = \frac{MU_B}{P_B} \qquad \textbf{(8)}$$

This is, of course, the same as formula (6) in Section 5.3.

Example 5A.1

Utility Maximization with the Substitution Method

Let's use the substitution method to maximize the utility function described in (3) while respecting the consumer's budget constraint. Substituting (4) into (3) gives us

$$U(S, B) = 2\log(S) + 3\log\left(\frac{M}{P_B} - \frac{P_S}{P_B}S\right) \quad \textbf{(9)}$$

We differentiate this expression with respect to S and set the result equal to zero:

$$\frac{2}{S} - \frac{3}{\frac{M}{P_B} - \frac{P_S}{P_B}S}\left(\frac{P_S}{P_B}\right) = 0 \quad \textbf{(10)}$$

We can rewrite this as

$$2\left(\frac{M}{P_B} - \frac{P_S}{P_B}S\right) = 3S\frac{P_S}{P_B} \quad \textbf{(11)}$$

After multiplying through by P_B and rearranging, we discover that

$$P_S S = \frac{2}{5}M \quad \textbf{(12)}$$

In other words, Ellen always spends two-fifths of her income on soup. She must therefore spend three-fifths of her income on bread:

$$P_B B = \frac{3}{5}M \quad \textbf{(13)}$$

(Mathematically, we can obtain formula (13) by substituting formula (12) into formula (4)). This is the same answer we obtained computationally for $M = 100$ and $P_B = 2$. To determine the amount of soup purchased, we divide both sides of formula (12) by P_S; likewise, to determine the amount of bread purchased, we divide both sides of formula (13) by P_B.

We've used the substitution method to solve a problem that involves only two goods, but it can also be used to solve problems that involve more than two goods. All we need to do is focus on two goods at a time, holding spending on all other goods fixed.

The Method of Lagrange Multipliers Next we turn to a second and more powerful tool for solving constrained maximization problems: the method of Langrange multipliers. Whenever we are asked to maximize a function of N variables subject to a collection of K binding constraints, this method allows us to convert the problem into one of maximizing a function of $N+K$ variables without constraints. The objective function for this new maximization problem equals the original objective function plus one new term for each constraint. Each new term consists of a new variable, called a *Lagrange multiplier*, times an expression that summarizes the constraint. The method of Lagrange multipliers works even when it's impossible to solve a constraint for one variable as a function of the others, as required for the substitution method.

Assuming that Ellen's preferences satisfy the More-Is-Better Principle, we know that the budget constraint will bind. Therefore, we create a Lagrangian multiplier for the budget constraint, λ (the Greek letter lambda), and consider a new objective function:

$$U(S, B) + \lambda(M - P_S S - P_B B) \tag{14}$$

The first part of the new objective function, $U(S, B)$, is the original objective function. The second part involves the product of the Lagrangian multiplier and the expression $M - P_S S - P_B B$, which always equals zero whenever the budget constraint is satisfied. The method of Lagrange multipliers instructs us to maximize the function in expression (14) over the variables S, B, and λ, without imposing any constraints. A powerful and important mathematical theorem tells us that the solution to this unconstrained maximization problem also solves the constrained maximization problem in (2).

Since the Lagrangian problem involves no constraints, we find the solution by taking derivatives with respect to each of the variables and setting them equal to zero. For S, the first-order condition is

$$\frac{\partial U}{\partial S} - \lambda P_S = 0 \tag{15}$$

Recalling that $\frac{\partial U}{\partial S} = MU_S$, we can rewrite formula (15) as

$$\frac{MU_S}{P_S} = \lambda \tag{16}$$

Similarly, we can write the first-order condition for B as

$$\frac{MU_B}{P_B} = \lambda \tag{17}$$

From formulas (16) and (17), we immediately see that

$$\frac{MU_S}{P_S} = \frac{MU_B}{P_B} \tag{18}$$

which is, of course, the same as formula (8) above, and formula (6) from Section 5.3 (page 141). Finally, the first-order condition for λ is

$$M - P_S S - P_B B = 0 \tag{19}$$

which tells us that the bundle must lie on the budget line. In other words, Ellen's best choice is the bundle on the budget line that satisfies (18).

Example 5A.2

Utility Maximization with the Method of Lagrange Multipliers

Let's use the method of Lagrange multipliers to maximize the utility function described in (3) while respecting the consumer's budget constraint. The Lagrangian problem is:

$$\text{Choose } S, B, \text{ and } \lambda \text{ to maximize} \tag{20}$$

$$2\log(S) + 3\log(B) + \lambda(M - P_S S - P_B B)$$

For S, the first-order condition is

$$\frac{2}{S} - \lambda P_S = 0 \tag{21}$$

which we can rewrite as

$$\frac{P_S S}{2} = \frac{1}{\lambda} \tag{22}$$

Similarly, we can write the first-order condition for B as

$$\frac{P_B B}{3} = \frac{1}{\lambda} \tag{23}$$

Combining (22) and (23), we discover that

$$\frac{2}{3} P_B B = P_S S \tag{24}$$

In other words, Ellen should spend two-thirds as much on soup as on bread. The first-order condition for λ is still (19), the formula for the budget line. So we look for values of S and B that satisfy both (19) and (24). Using (24) to substitute for $P_S S$ in (19) gives us

$$\frac{2}{3} P_B B + P_B B - M = 0 \tag{25}$$

Solving this for $P_B B$ delivers formula (13), as before. Combining (13) with the budget constraint delivers (12) as the solution for $P_S S$.

Add-On 5B

APPLICATION: SUBSTITUTION BETWEEN DOMESTIC AND IMPORTED AUTOMOBILES

If a foreign automobile manufacturer increases the prices of cars sold in the United States, how many fewer cars will it sell? Will sales by U.S. auto makers rise, and if so, by how much? These issues are important to both foreign and domestic auto makers.

To answer these questions, we'll examine the historical relationship between U.S. automobile sales and an indicator of imported auto prices: the dollar-yen exchange rate.[1] An exchange rate is the rate at which people can swap one currency for another. For example, when the dollar-yen exchange rate is 0.01, one Japanese yen buys 0.01 U.S. dollars (1 cent). We chose the Japanese yen rather than some other foreign currency (such as the French franc or the Italian lira) because Japan exports significantly more automobiles to the U.S. than any other country. When the dollar-yen exchange rate is low, the dollar is "strong," and the prices of Japanese goods are low in U.S. dollars. When the dollar-yen exchange rate is high, the dollar is "weak," and the prices of Japanese goods are high in U.S. dollars.

Here's an example. Suppose a Japanese automobile sells for 3 million yen. This converts to $30,000 when the dollar-yen exchange rate is 0.010, and to $36,000 when the exchange rate is 0.012. If the dollar-yen exchange rate were to rise from 0.010 to 0.012, the Japanese auto maker probably wouldn't raise the U.S. price of the car from $30,000 to $36,000—it might, for example, settle for $33,000.[2] Still, a higher dollar-yen exchange rate would be associated with a higher dollar price for the car.

In Figure 5B.1, we've used historical data (from 1990 to 2001) to plot domestic and imported auto sales in the United States against the dollar-yen exchange rate. The blue line shows the average relationship between the exchange rate and U.S. sales of imported autos.[3] Since we can interpret the exchange rate as a measure of import prices, this is essentially a demand curve. Notice that it slopes downward—higher import prices reduce the U.S. demand for imported autos. The red line shows the average relationship between the exchange rate and U.S. sales of domestic autos. Notice that it slopes upward—higher import prices increase the demand for domestic autos—which means that foreign and domestic autos are substitutes.

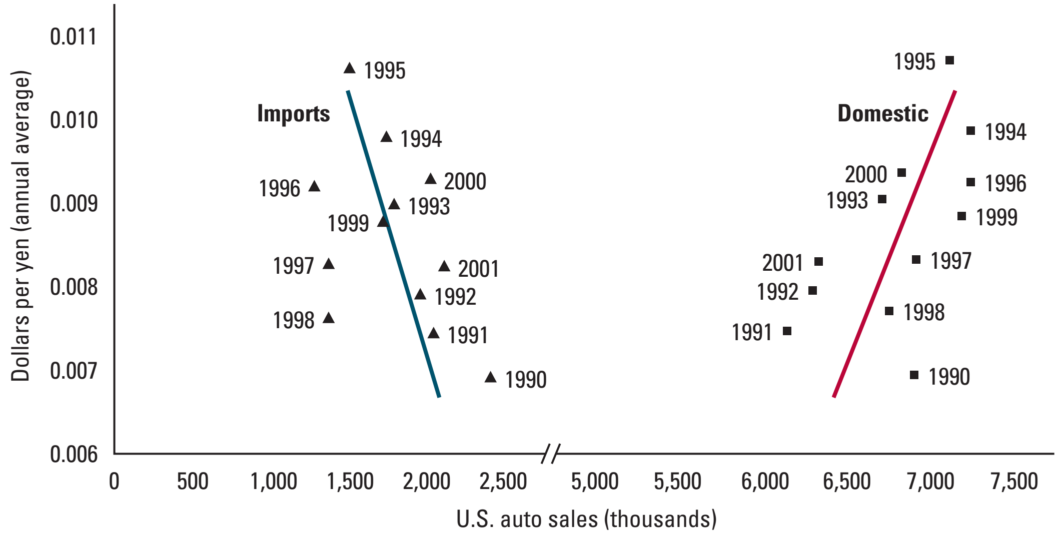
[1]Why not simply examine the relationship between the U.S. prices of foreign autos and the numbers of cars purchased? As discussed in the appendix to Chapter 2, the historical relationship between prices and quantities doesn't reliably reveal the shape of the demand curve. If, for example, an outward shift in the demand for imported cars causes foreign auto makers to increase their prices, we might observe a *positive* relationship between prices and sales, even though demand curves slope downward. The dollar-yen exchange rate qualifies as an "instrumental variable" because it shifts the U.S. supply curve of Japanese auto makers (since it alters the number of yen they receive per car at a fixed U.S. dollar price), but leaves the demand curves of U.S. consumers unchanged. This is why we can use it to measure the effect of imported automobile prices on U.S. demand. See the appendix to Chapter 2 for further discussion.

[2]This reflects a phenomenon known as *exchange rate pass-through*. In this example, the U.S. price rises by 10 percent, while the dollar-yen exchange rate rises by 20 percent. Since 10 percent is half of 20 percent, exchange rate pass-through is 50 percent.

[3]We obtained both the blue line and the red line through linear regression analysis, which we mentioned in the appendix to Chapter 2.

Figure 5B.1

Substitution between Domestic and Imported Autos. When the dollar-yen exchange rate increases, Japanese imports become more expensive in U.S. dollars. U.S. sales of imported cars fall, and U.S. sales of domestic cars rise.



Data sources: *Ward's Automotive Yearbook* (auto sales) and Federal Reserve Board (exchange rates).

According to these data, the elasticity of U.S. automobile demand with respect to the dollar-yen exchange rate is −1.06 for imports, and 0.36 for domestic models. To convert these figures into price elasticities, we would also need to know the relationship between the U.S. prices of imported cars and the exchange rate. If, for example, import prices rise by 0.5 percent for every 1 percent increase in the exchange rate, the price elasticities would be twice as large as the exchange rate elasticities. (Why?)

According to Figure 5B.1, the effect of exchange rates on U.S. automobile sales is quite large. When the dollar-yen exchange rate rises by 0.001 points (say from 0.008 to 0.009), domestic manufacturers sell an additional 195,000 cars per year. This estimate suggests that a 0.004 point rise in the dollar-yen exchange rate—comparable to the change that occurred between 1990 and 1995—can shift roughly 10 percent of U.S. auto sales (roughly 800,000 cars annually) from imported to domestic models.

Add-On 5C

WHAT MAKES A GOOD NORMAL OR INFERIOR?

What makes a good normal or inferior? Let's think again about the consumption of potatoes and beef. We'll assume that each of the consumer's indifference curves has a declining MRS. Suppose we add beef to a consumption bundle, holding the amount of potatoes fixed. If the marginal rate of substitution for potatoes with beef (MRS_{PB}) rises, potatoes are normal. If MRS_{PB} falls, potatoes are inferior. Let's see why.

Like Figures 5.18 and 5.19, Figure 5C.1 shows choices involving the consumption of potatoes and beef at different levels of income. Once again, the consumer's budget allows him to choose any bundle on or below L_1, and he selects A. We'll refer to the indifference curve that runs through bundle A as I_1. With higher income, the consumer's budget line is L_2, which is parallel to L_1. Bundle E lies on the new budget line directly above A. We'll refer to the indifference curve that runs through E as I_2. If the best choice lies to the right of E, potatoes are normal. If it lies to the left, potatoes are inferior.

In moving from bundle A to bundle E, the consumer adds beef, holding the amount of potatoes fixed. Let's suppose that adding beef increases the MRS_{PB}, making the indifference curve steeper. We see this in Figure 5C.1(a). Since I_1 has the same slope as the budget line at A, I_2 must be steeper than the budget line at E. That means the tangency condition is satisfied at some bundle to the right of E, such as F. Potato consumption rises with income, so potatoes are normal.

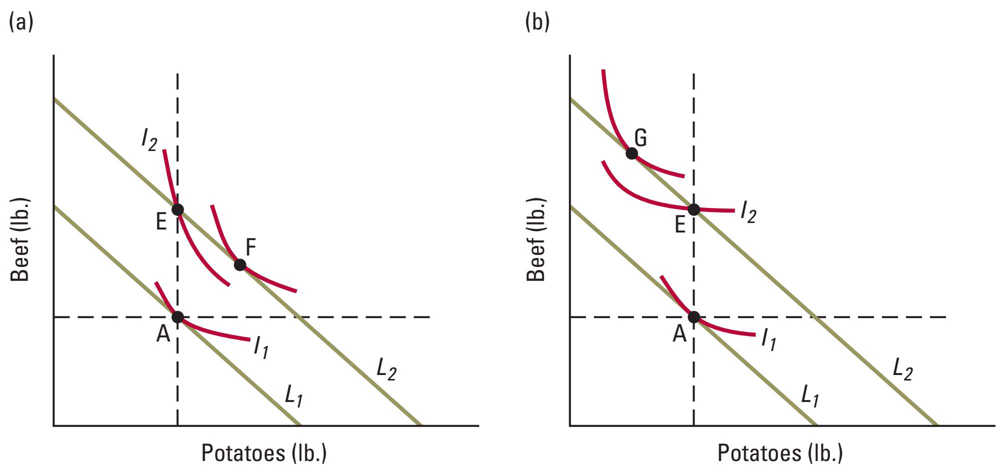
Now let's suppose that adding beef reduces the MRS_{PB}, making the indifference curve flatter (see Figure 5C.1(b)). In that case, I_2 must be flatter than the budget line at bundle E. That means the tangency condition is satisfied at some bundle to the left of E, such as G. Here, potato consumption falls with income, so potatoes are inferior.

Ordinarily, would we expect the consumer's MRS_{PB} to rise or fall when we add beef? Remember that MRS_{PB} tells us the rate at which we must add beef when we deprive the consumer of potatoes. It's natural to think that the extra pleasure the consumer gains from each additional pound of beef should fall as beef becomes more plentiful. That suggests that when the consumer has lots of beef, more beef will be required to compensate for the loss of a potato than when beef is scarce. If that's true, then MRS_{PB} rises as we add beef, and potato consumption rises with income. Economists refer to this case as "normal" because MRS_{PB} changes in what appears to be the most natural direction.

There is, however, nothing "abnormal" about inferior goods. Though adding beef may indeed reduce the extra pleasure the consumer gains from each additional pound

Figure 5C.1

The Features of Preferences That Determine whether Potatoes Are Normal or Inferior. Initially, the consumer's budget line is L_1 and he chooses bundle A. The indifference curve I_1 is tangent to L_1 at A. An increase in income shifts the budget line outward to L_2. If adding beef increases the MRS_{PB} [figure (a)], then the indifference curve I_2 is steeper than the budget line at bundle E. The best choice, bundle F, must then lie to the right of bundle A, which means potatoes are normal. If adding beef reduces the MRS_{PB} [figure (b)], then the indifference curve I_2 is flatter than the budget line at bundle E. The best choice, bundle G, must then lie to the left of bundle A, which means potatoes are inferior.



of beef, it may also reduce the extra pleasure the consumer gains from each additional potato. Remember our discussion of Figure 5.18. If the consumer eats potatoes mostly to stave off hunger, then an extra potato isn't nearly as important when beef is plentiful as it is when beef is scarce. If this effect is sufficiently strong, then MRS_{PB} may fall as we add beef, in which case potatoes are inferior.

Add-On 5D

VOLUME-SENSITIVE PRICING

Throughout most of Chapter 5, we assumed that each good is available in unlimited quantities at a single price. Application 5.4 was an exception—with tiered rates, the price paid for electricity depends on the amount purchased. This is an example of **volume-sensitive** pricing.

Pricing is **volume-sensitive** if the price paid for a good depends on the amount purchased.

In practice, volume-sensitive pricing is reasonably common. Does this mean we need to modify the theory of consumer behavior? Not at all. Even when the price of a good is tied to volume, we can still determine the consumer's best choice by applying the no-overlap rule.

Volume Penalties and Rationing

Sometimes, a good's price per unit *rises* with the amount purchased. This is called a **volume penalty**. Application 5.4 is an example.

For a **volume penalty**, a good's price per unit rises with the amount purchased.

We've already drawn a budget constraint for a situation involving a volume penalty (see Figure 5.23). Now let's see how the budget constraint changes as we vary the size of the penalty. Let's assume for the purpose of illustration that a consumer allocates his income between electricity and food. His income is \$110 per week and food costs \$1 per pound. If electricity is available in unlimited amounts at a price of \$0.10 per kwh, the consumer's budget constraint is the dark green line connecting points A and B in Figure 5D.1. The slope of this line is −0.1.

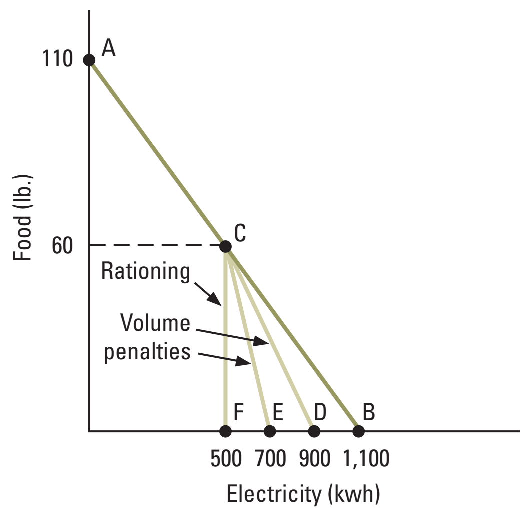


Figure 5D.1

Budget Constraints with Volume Penalties. When electricity costs \$0.10 per kwh, other goods cost \$1 per unit, and income is \$110, the budget constraint is the dark green line connecting bundles A and B. With a volume penalty on usage over 500 kwh, the portion of the budget constraint that runs between bundle C and the horizontal axis rotates toward the origin. Rationing is similar to a very large volume penalty.

Now suppose the power company imposes a volume penalty, charging \$0.15 per kwh for electricity consumption over 500 kilowatts. In that case, the consumer's budget constraint consists of the line segment connecting points A and C (the slope of which is -0.1), and the light green line segment connecting points C and D (the slope of which is -0.15).

When the volume penalty increases, the portion of the budget constraint running between C and the horizontal axis rotates toward the origin. If, for example, the power company charges \$0.30 per kwh for electricity consumption over 500 kilowatts, the consumer's budget constraint consists of the line segment connecting points A and C (the slope of which is -0.1), and the light green line segment connecting points C and E (the slope of which is -0.3). For very large volume penalties, the portion of the consumer's budget line that runs between C and the horizontal axis is nearly vertical. The imposition of a very large quantity penalty is therefore similar to rationing (which we discussed in Section 5.1). This makes sense: if extra electricity beyond 500 kwh were available for a price of a million dollars per kwh, then for all practical purposes, 500 kwh would be a fixed limit.

WORKED-OUT PROBLEM 5D.1

The Problem Owen can spend \$10 on electricity and food. The price of food is \$1 per pound and the price of electricity is \$0.50 per kwh up to 8 kwh. Beyond 8 kwh, additional electricity costs \$2 per kwh. Owen's MRS for electricity with food is F/E, where E stands for kwh of electricity and F stands for pounds of food. Draw Owen's budget constraint. How much electricity will he purchase?

The Solution Figure 5D.2 shows Owen's budget constraint. His best choice must be either (a) bundle G, (b) a point of tangency on the line connecting bundles A and G, or (c) a point of tangency on the line connecting bundles G and D.[1]

Could his best choice be a point of tangency on the line segment connecting bundles A and G? Let's look for a point of tangency on the line connecting bundles A and B. If it lies to the left of G, it's a best choice; if it lies to the right of G, it's not.

At a point of tangency, the price ratio must equal MRS_{EF}. The price ratio, P_E/P_F, is $^1/_2$ and the MRS_{EF} is F/E, so the bundle must satisfy $F/E = {}^1/_2$. That implies $F = E/2$. Since the bundle must also lie on the line connecting A and B, we know that $0.5E + F = 10$. Putting these formulas together gives us $E = 10$. The point of tangency is therefore bundle H, which lies to the right of G. We conclude that, on the line segment connecting bundles A and G, there is no point of tangency. Therefore, Owen's best choice does not lie on this segment.

Could Owen's best choice be a point of tangency on the line segment connecting bundles G and D? Let's look for a point of tangency on the line connecting C and D. If it lies to the right of G, it's a best choice; if it lies to the left of G, it's not.

At a point of tangency, the price ratio must equal MRS_{EF}. The price ratio, P_E/P_F, is 2 and the MRS_{EF} is F/E; so the bundle must satisfy $F/E = 2$. That implies $F = 2E$.

[1] Because of the form of Owen's MRS, his best choice cannot be a corner solution. When he spends all his income on electricity, his indifference curve is horizontal; when he spends all his money on food, his indifference curve is vertical. Neither bundle is a best choice because, in either case, his indifference curve crosses the budget constraint.

Since the bundle must also involve a total expenditure of \$10, we know that $(0.5 \times 8) + 2(E - 8) + F = 10$.[2] Putting these formulas together gives us $E = 5.5$. The point of tangency is therefore bundle J, which lies to the left of G. We conclude that, on the line segment connecting bundles G and D, there is no point of tangency. Therefore, Owen's best choice does not lie on this segment.

Does bundle G satisfy the no-overlap condition? At that point, Owen's MRS_{EF} is $^6/_8 = ^3/_4$. Since $^3/_4$ is greater than $^1/_2$ and less than 2, his indifference curve is steeper than the line connecting bundles A and G and flatter than the line connecting bundles G and D. Since each of Owen's indifference curves has a declining MRS, point G satisfies the no-overlap condition, and therefore it is Owen's best choice.

IN-TEXT EXERCISE 5D.1 **As in worked-out problem 5D.1, Owen can spend \$10 on electricity and food. The price of food is \$1 per pound and the price of electricity is \$1 per kwh. Electricity is rationed; no consumer can purchase more than 7 kwh. Owen's MRS for electricity with food is *F*/*E*. Draw his budget constraint. How much electricity will he purchase? How would your answer change if the electricity ration was 4 kwh instead of 7 kwh?**

Volume Discounts

At some point, almost everyone has taken advantage of an opportunity to pay a lower price by purchasing a larger volume. Buying pizza is cheaper by the pie than by the slice, soda is cheaper by the case than by the can, and beer is cheaper by the keg than by the

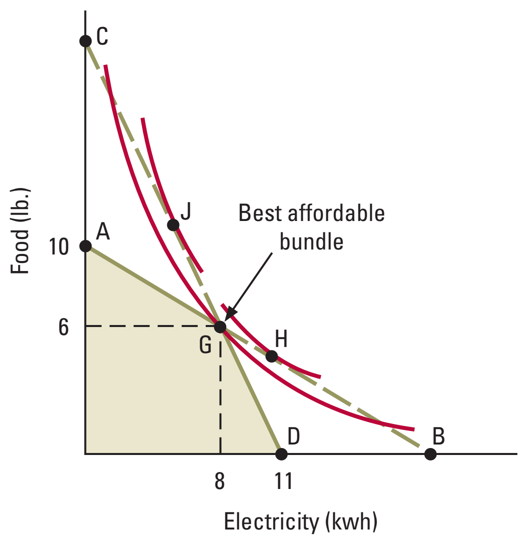
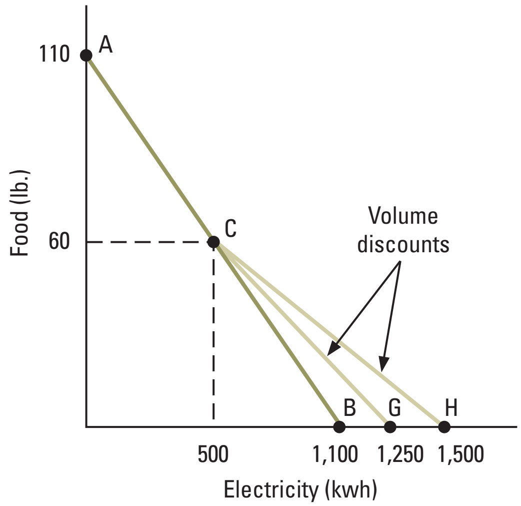


Figure 5D.2
Choices with a Volume Penalty for Electricity Consumption, Worked-Out Problem 5D.1. Owen's best affordable choice is bundle G, where his indifference curve is steeper than the line connecting A and G but shallower than the line connecting G and D.

[2]The first term is the cost of the first 8 kwh of electricity, the second is the cost of any additional kwh of electricity. The third term is the cost of food.

Figure 5D.3

Budget Constraints with Volume Discounts. When electricity costs $0.10 per kwh, food costs $1 per pound, and income is $110, the budget constraint is the dark green line connecting bundles A and B. With a volume discount on usage over 500 kwh, the portion of the budget constraint that runs between bundle C and the horizontal axis rotates away from the origin.



For a **volume discount**, a good's price per unit falls with the amount purchased.

bottle. Airlines reward "frequent fliers" with free flights, and many hotels offer special rates to guests who stay more than a few nights. Each of these examples illustrates a **volume discount**. We'll explain why firms might offer such discounts in Chapter 18.

Figure 5D.3 shows how a volume discount affects the shape of the budget constraint for a consumer who allocates his income between electricity and food. Once again, the budget line connecting bundles A and B is our starting point—it assumes that the consumer's income is $110 per week, food costs $1 per pound, and electricity costs $0.10 per kwh.

Now suppose the power company offers a volume discount, charging $0.08 per kwh for electricity consumption over 500 kilowatts.[3] In that case, the consumer's budget constraint consists of the line segment connecting points A and C (the slope of which is −0.1), and the light green line segment connecting points C and G (the slope of which is −0.08). When the volume discount increases, the portion of the budget constraint running between C and the horizontal axis rotates away from the origin. If, for example, the power company charges $0.06 per kwh for electricity consumption over 500 kilowatts, the consumer's budget constraint consists of the line segment connecting points A and C (the slope of which is −0.1), and the light green line segment connecting points C and H (the slope of which is −0.06).

You've probably come across companies that operate discount clubs. After paying a membership fee to join, customers can purchase certain goods at lower prices. As the following example shows, this is one way to create a volume discount.

[3]There are jurisdictions in which power companies charge lower prices at higher volumes, but this is much less common than volume penalties.

Application 5D.1

A Frequent Reader's Club

Books-a-Million, Inc., started out in 1917 as a street corner newsstand in Florence, Alabama. By 2006, it ranked as the third largest book retailer in the United States, with more than 200 stores located primarily in the southeastern states, as well as newsstand, wholesale, and Internet operations.

For an annual fee of $10, customers can join the Books-a-Million Millionaire's Club. Members receive a 10 percent discount on all Books-a-Million purchases. How does this affect customers' budget constraints?

Figure 5D.4 shows the affordable consumption bundles for a consumer who purchases books from Books-a-Million. The horizontal axis indicates the number of books, and the vertical axis indicates units of other goods (which we lump into a single category). We'll assume that the consumer's income is $30,000 per year, books cost $20 each (without a discount), and other goods cost $1 per unit. To focus on the relevant portion of the consumer's budget constraint, we've drawn the horizontal axis intersecting the vertical axis at 29,900 units of other goods, rather than at zero.

If the customer does not join the discount club, his budget constraint is the straight line labeled L_1 running through bundles A and E (both the solid and broken segments). The slope of this line is -20 (the consumer gives up 20 units of other goods per book). If the customer joins the discount club, his budget constraint is the straight line labeled L_2 running through bundles C and G (both the solid and broken segments). This line starts at C, rather than at A, because the customer pays the $10 membership fee whether or not he buys any books. Its slope is -18, rather than -20, because the customer receives a 10 percent discount on book purchases. Note that the total cost of 5 books is exactly the same—$100—whether or not the customer joins the discount club. That is why the two budget lines, L_1 and L_2, intersect at bundle B.

Since the customer can decide whether to join the discount club, he can afford any bundle on or below *either* L_1 or L_2. Assuming that the More-Is-Better Principle holds, he will select a bundle on one of the two solid green segments.

The possibility of joining the discount club provides the consumer with the same opportunities as a volume discount. To understand this point, suppose that Books-a-Million were to sell books at $20 apiece and offer a volume discount of 10 percent for annual purchases in excess of $100. The customer's budget constraint would then be identical to the one pictured in Figure 5D.4.[4]

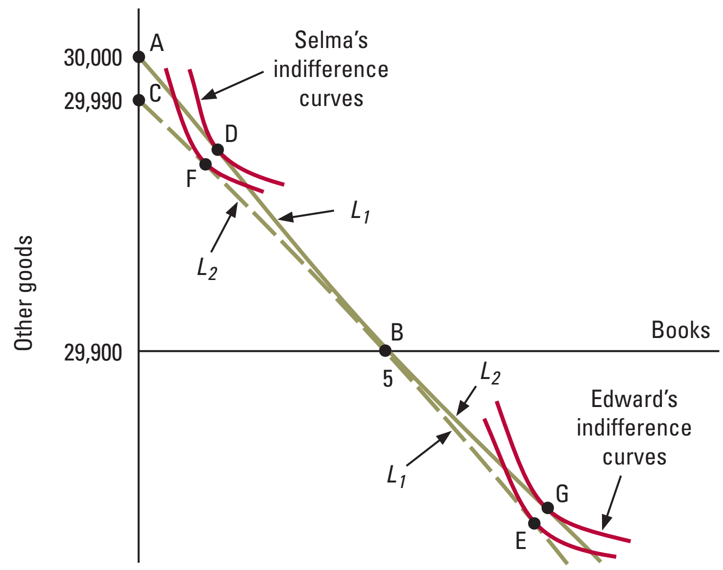
Figure 5D.4 also includes indifference curves for two customers. Selma and Edward. Compared to Edward, Selma places less value on books relative to other goods. Which bundle will each customer select?

Without a discount, Selma would choose bundle D and Edward would choose bundle E. As members of the discount club, Selma would choose bundle F and Edward would choose bundle G. Since Selma prefers bundle D to bundle F, she will not join the club. Since Edward prefers bundle G to bundle E, he will join. Edward will benefit from the quantity discount, while Selma will not.

[4]In practice, the effect of a volume discount may be a bit different, since customers may have difficulty predicting their annual purchases. Some may join the club and then purchase fewer than five books; others may choose not to join and then purchase more than five books.

Figure 5D.4

Membership in a Discount Book Club. If a consumer does not join the discount book club, he chooses from bundles on the line L_1. If he does join the club, he chooses from bundles on the line L_2. Edward joins the club while Selma does not. Edward purchases bundle D; Selma purchases bundle G.



ADDITIONAL EXERCISES

Exercise 5D.1: Volume discounts are much more common than volume penalties. Why? (Hint: What would you do if you went to a store intending to purchase two identical items and discovered that the store charges $100 for one and $250 for two?) Why, then, is it possible to impose volume penalties for some goods, such as electricity? Do sellers sometimes have difficulties with volume discounts? (Hint: What if you and three friends all want to buy the same object. One costs $100, but the store sells four or more for $350. What would you do? Can you think of any examples of this type of situation?)

Exercise 5.D2: Consumers buy sugar and other goods. Other goods cost $1 per unit. The price of sugar is 20 cents per ounce, but it is rationed. Each consumer is permitted to buy no more than 30 ounces. Paul has $20. Draw his budget constraint. Now suppose sugar is available on the black market for 50 cents per ounce. Show how Paul's budget constraint changes.

Exercise 5D.3: Colin can buy wireless telephone service at $6 per hour up to 5 hours and at $4 per hour for additional time. He also buys food at $1 per pound. His marginal rate of substitution for wireless service with food is $MRS_{WF} = \frac{F}{W}$, where F is pounds of food and W is the number of wireless hours. Suppose Colin's income is $48. What does he buy? (Hint: his budget constraint consists of two line segments, much like in Figure 5D.2. Find the best choice on each line by setting the marginal rate of substitution equal to the price ratio, and then determine whether he can actually buy these bundles. Graph these choices, and figure out which one he chooses.)

Exercise 5D.4: During the early to mid 1990s, AT&T's True USA® calling plan provided a different type of volume discount. AT&T billed customers at a standard rate for each minute of long-distance telephone usage. If a customer spent at least $10 and no more than $25, AT&T subtracted 10 percent *of the entire amount* (not just the amount over $10). Customers who spent at least $25 received a 20 percent discount on the entire bill. Imagine that a consumer can spend $40 total, and the price of food is $1 per pound. Draw the budget line for long-distance minutes versus food and identify the affordable consumption bundles. Draw families of indifference curves for which the consumer's spending on long distance would be: less than $10, exactly $10, between $10 and $25, exactly $25, and more than $25.