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AGGREGATE PRODUCTION FUNCTIONS AND MEDIUM-RANGE GROWTH PROJECTIONS

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The conceptual basis for believing in the existence of a simple and stable relationship between a measure of aggregate inputs and a measure of aggregate output is uncertain at best. Yet an aggregate production function is a very convenient tool for theoretically exploring some of the determinants of economic growth, and it has served as a framework for some interesting empirical studies. Moreover, in an attempt to assess the growth prospects for an economy, to identify the variables that are likely to determine the growth rate, and to examine the policies affecting growth, the explicit or implicit use of an aggregate production function is almost indispensable.

In recent years economists have developed a variety of aggregate production functions. Several of these models are quite similar in basic conception but focus on different variables or make different assumptions about the interrelationships of the variables. Some represent amendments of earlier models. The different models yield somewhat dissimilar explanations of past growth and contingency forecasts with respect to future growth. The purpose of this paper is to try to place several of these models within a general framework so that their assumptions and implications can be compared and evaluated. First, the variables and relationships stressed by several different but related aggregate production models and the explanations they provide of the 1929–60 growth record of the United States will be examined. In the course of doing this a general production function will be developed which treats the different models as special cases. Then the analysis will turn to certain complementarity relationships between the variables that earlier formulations have tended to slight. As an application of some of the ideas, the concluding section will briefly examine a few of the quite different contingency forecasts provided by the different

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models and other aspects of the problem of forecasting economic growth.[1]

Before the different models are examined, it will be useful to set out the basic growth record they have to explain. Between 1929 and 1960 real deflated GNP grew at an average annual rate of approximately 2.9 per cent. However, the rate of growth of output varied considerably over the period. From 1929 to 1947 (roughly the period of the depression to the start of postwar normalcy), real GNP grew at an average annual rate of 2.5 per cent—the average of a significantly slower growth rate during the depression, a tremendous surge of output during the war, and a postwar slowdown. From 1947 to 1960 the growth rate averaged 3.5 per cent a year—somewhat faster through the mid-fifties and somewhat slower from the mid-fifties to 1960.

Between 1929 and 1960 there was also considerable variation in the amount of economic slack as measured by unemployed labor and underutilized capital. Since production functions of the sort examined in this paper are designed to deal with secular factors and are not well suited to deal with the effects upon productivity of changing degrees of slack, it is dangerous to examine year-to-year changes in GNP. Therefore, we will be concerned with the average growth rates over various periods where the end points are roughly comparable in terms of the unemployment rates. To adjust further for the differences in degree of slack at the various terminal dates used, the Okun cyclical adjustments will be used and all data normalized to a 4 per cent unemployment rate [16]. Potential GNP for a year is defined as what GNP would have been had the unemployment rate been at 4.0 per cent of the labor force. Potential labor input is defined as man-hours that would have been worked had the unemployment rate been 4.0 per cent. The Okun adjustment to derive potential labor input from actual labor input tries to take account of cyclical effects in average hours worked per week and labor force participation rates, as well as percentage employed. The adjustment to derive potential GNP from actual GNP involves the labor-input adjustment and a productivity adjustment related to the unemployment rate. If the Okun equations are used to adjust for unemployment being somewhat higher in 1960 than in 1929, the average rate of growth of real potential GNP over the period was approximately 3.1 per cent; 2.5 per cent from 1929 to 1947, and 4.0 per cent from 1947 to 1960 with growth slowing down in the latter part of this subperiod.

Although the different models tend to stress different variables and relations, all of them are based on the relationshlp between growth of output and growth of labor input and capital input. Table 1 shows the

[1] This paper will focus on short- and medium-run growth. The long-run, steady-state properties of the models will not be examined. This is not a paper on the "Golden Age." But for an interesting comparison of the long-range implications of several models, see E. Phelps, [17].

average rate of growth of potential GNP, gross capital stock, and potential labor input measured in man-hours for selected subperiods.

Perhaps the most interesting aspects shown by Table 1 are, first, only a small absolute variation in the rate of growth of labor input; second, considerable variation in the rate of growth of the capital stock with variations in the rate of growth of GNP positively associated with variations in the rate of growth of the capital stock; third, potential GNP growing at a faster pace than either capital input or labor input in each subperiod;[2] and, fourth, a more rapid growth of potential output after World War II than before. Whereas the division of the 1929–60 record into the subperiods of Table 1 is relatively arbitrary, almost any division shows these four phenomena. These are the facts that an aggregate analysis of economic growth must explain.

TABLE 1—THE HISTORICAL RECORD[a]
(percentage annual growth rates)

Year	Potential GNP	Potential man-hours	Gross capital stock
1929–60	3.1	.7	2.0
1929–47	2.5	.5	1.0
1947–60	4.0	.8	3.6
1947–54	4.4	.7	4.0
1954–60	3.5	.8	3.1

[a] The growth of potential output estimates is derived by using the Okun adjustments on the GNP series. The data for growth of potential man-hours are derived from the Knowles [13] series for actual man-hours, adjusted to get potential man-hours by the Okun equations. The capital stock series is from E. Denison [5]. Since producer's plant and equipment has grown at almost the same rate (in each subperiod) as total capital, I have not had to decide which concept is more relevant.

I. *A Comparative Analysis of Aggregate Production Functions*

The Cobb-Douglas Model and Its Extensions

The analysis of this section will be concerned with the Cobb-Douglas model and its extensions. The analysis could have been built around the more general constant elasticity of substitution model [2], but within a wide range which probably brackets the elasticity of substitution the conclusions are little different from those drawn from the simple Cobb-Douglas model. This fact, quite surprising to me at least, is proved in The RAND Memorandum from which this article is derived [15, ap-

[2] This statement does not hold true for earlier periods. During the 1909–29 period, potential GNP did not grow as rapidly as did the capital stock. Since much of Douglas' data [7] was for this period, this is one explanation of why he was able to get relatively good fits of Cobb-Douglas models without assuming any growth of total factor productivity. Kendrick [12] has called attention to this phenomenon. It is clear that growth of total factor productivity has been much greater since 1919 or 1929 than before.

pendix]. The analysis of this section also will be limited by the assumption that the effect of technical change is to shift the production function without altering its basic shape—that technical change is neutral. The reason for this limitation is that it is extremely difficult to measure the "nonneutrality" of technical change.[3] However, the effects of nonneutral technical change are examined in the preceding reference.

The Basic Cobb-Douglas Model. The basic Cobb-Douglas production function with constant returns to scale can be written:[4]

$$O_t = A_t L_t^{b} K_t^{1-b} \tag{1}$$

where O_t is potential GNP in year t, L_t and K_t are potential labor input and capital input, respectively, in year t, A_t is an index of total factor productivity, and b and $1-b$ are the elasticities of output with respect to labor and capital, respectively. In pure competitive equilibrium and assuming a number of other stringent conditions, the shares of capital and labor will be indicators of these elasticities.

Taking logarithms, differentiating, and assuming b to be a constant (the basic Cobb-Douglas, neutral technological change assumption), the rate of growth of output is:

$$\Delta O/O = \Delta A/A + b(\Delta L/L) + (1 - b)(\Delta K/K) \tag{2}$$

where $\Delta O/O$ is the relative rate of growth of GNP; $\Delta K/K$ and $\Delta L/L$ are the relative rates of growth of capital and labor inputs; and $\Delta A/A$ is the relative rate of growth of total factor productivity, or to put the matter more accurately, the part of the growth of output that cannot be explained by the growth of capital and labor using the simple Cobb-Douglas growth equation.[5] In a very real sense it is a measure of our ignorance.

Let us assume that the shares of labor and capital do provide an approximation to the output elasticity and roughly equal .75 and .25 over the period.[6] Further, assume that growth of man-hours and of gross capital stock are suitable measures of the growth of the services of these inputs.[7] Then $\Delta A/A$ can be calculated as a residual, $\Delta O/O - b(\Delta L/L) - (1-b)(\Delta K/K)$, and for $b=.75$ is presented in Table 2, column 1.

[3] But for one attempt, see [4].

[4] See Solow [25] for an example of the use of this kind of model, incorporating technological change.

[5] The relating of changes in GNP in one year to changes in the capital stock during that same year ignores problems of lags. However, since the analysis of this paper is concerned with average behavior of $\Delta O/O$ and $\Delta K/K$ over periods of several years, this problem is less serious.

[6] The sensitivity of the growth rate to different assumptions about output elasticities will be examined later. The $b=.75$ assumption perhaps should be considered an upper bound.

[7] Obviously there are some serious problems in using capital stock as a measure of capital input. Vernon Smith's suggestion [22] that stock is the relevant concept is comforting but not

Notice what a large share of the average annual growth rate the unexplained residual is made to explain (approximately two-thirds in all of the subperiods).[8] And notice that, still assuming b to be approximately .75, the postwar acceleration of growth must be, in large part, explained by an increase (by more than one-third) in the rate of growth of total factor productivity. Similarly, the retardation since 1954 must be explained by a slackening-off (but not to the prewar rate) of growth of total factor productivity.[9] These conclusions, however, are strongly dependent upon the assumption that $b=.75$.

Table 1 showed, first, that throughout the period potential GNP grew faster than either input, but capital grew faster than did labor;

TABLE 2—GROWTH OF TOTAL FACTOR PRODUCTIVITY[a]
(percentage annual growth rates)

Year	$\Delta A/A;\, b=.75$	$\Delta A/A;\, b=.5$	$\Delta O/O$
1929–60	2.1	1.7	3.1
1929–47	1.9	1.7	2.5
1947–60	2.5	1.8	4.0
1947–54	2.9	2.1	4.4
1954–60	2.1	1.6	3.5

[a] From Table 1 using equation (2).

second, that labor grew at a relatively constant rate; and, third, that the rate of growth of the capital stock varied considerably over the period, with simultaneous relatively rapid growth of both GNP and capital. Given these data, the greater the weight placed by the production function on the growth of capital—that is, the greater is $(1-b)$—the less weight must be placed on the average rate of growth of total factor productivity in explaining average growth of GNP. (If the assumption of constant returns to scale is relaxed and the coefficients can add up to greater than one, the importance of $\Delta A/A$ can be further reduced.)

completely convincing for this kind of analysis. Zvi Griliches, in an unpublished manuscript, has estimated that capital services have grown faster than gross stock by about half a percentage point a year.

[8] If Griliches is correct that capital services grew significantly faster than gross capital stock, then the unexplained residual is reduced somewhat, but it still is very large.

[9] Although almost any division of subperiods will show a postwar acceleration, it should be noted here that the selection of subperiods has influenced the interpretation of this phenomenon. Much of the increase in $\Delta A/A$ over its pre-1947 average was concentrated in two short periods of time: 1948–50 and 1957–59 [5]. In other postwar years $\Delta A/A$ was not significantly greater than its 1929–47 average. I do not know how this should be interpreted; in part, the explanation may lie in inadequate statistics. In any case, whether or not the increases in total factor productivity accounted for during these two periods are assumed to be, in fact, occurrences more evenly spread out in time, it is clear that $\Delta A/A$ has, on the average, been higher since 1947 than before 1947 and probably has slowed down somewhat in recent years.

The low weight assigned to capital elasticity in the preceding calculations ($1-b=.25$) resulted from the assumption (extraneous to the model) that factor shares are good estimates of factor elasticities. If this assumption is not adopted but rather b and $\Delta A/A$ are obtained by regression, the estimated $(1-b)$ is much larger—in the neighborhood of .5 or greater. And the average $\Delta A/A$ for the period is smaller, as is shown in Table 2. More important, changes in $\Delta A/A$ play a much smaller role in explaining changes in the growth rate over the subperiods, since the changes in $\Delta K/K$ now "explain" a larger share of the concurrent changes in $\Delta O/O$.

However, this result carries no great economic insight. The variation in $\Delta A/A$ calculated as a residual over the period represents the "errors" in a regression equation fitted for the entire period, and, if a regression is formally fitted, b will be calculated so as to minimize the variation of $\Delta A/A$. To put it another way, since $\Delta L/L$ is (empirically) relatively constant, and since $\Delta A/A$ is (by assumption of the regression) also a constant, the regression calculations will choose a $(1-b)$ best suited to "explain" changes in $\Delta O/O$ from changes in $\Delta K/K$. But if we have extraneous (to the model) information about b, the regression model must be constrained to take this information into account. If we continue to think strictly within the framework of the simple Cobb-Douglas model, the choice between the low $(1-b)$, and consequent large variation in $\Delta A/A$, explanation of past growth, and the high $(1-b)$, and consequent small variation of $\Delta A/A$, explanation depends on one's judgment on, first, the extent to which factor shares provide good estimates of factor elasticities (which in turn depends on which of the wide number of theories of income shares one holds), and, second, whether it is likely that the rate of technical change has in fact increased in the postwar period.[10]

In any case, it is extremely important that $\Delta A/A$ be explained. A growth theory that explains half of growth and much of the variation in growth by an unexplained residual (which is, after all, what "growth of total factor productivity" really is) is not much of a theory [6]. There are several different models that can be considered as providing amendments to the Cobb-Douglas model, which try to probe into the nature of $\Delta A/A$. One focuses on improvements in the quality of the capital stock. Another focuses on improvements in the quality of the labor force. By providing different descriptions of the determinants of $\Delta A/A$, these models further enrich the set of possible interpretations of the 1929–60 growth record.

[10] If it is not assumed that b was a constant over the period, but instead possibly varied from subperiod to subperiod, then the range of alternative explanations increases greatly. For such an analysis, see Brown and Popkin [4]. While this possibility is excluded by the assumptions of this section, the more general model developed in [15, appendix] does admit this possibility.

Capital Quality and the Embodiment Effect. One of the factors that has contributed to the growth of total factor productivity has been advances in technology which have improved the productivity of new capital goods. In his 1959 article, Robert Solow [23] suggested that the basic Cobb-Douglas model did not take account of the strong complementarity between technical change and investment. In his article, and in subsequent articles based on it [26], a distinction is made between "organizational" improvements which do not require new capital and "design" improvements which do. In contrast with the implicit assumption of the basic Cobb-Douglas model, Solow assumed that design improvements need to be embodied in new capital equipment. The effect of this need for "embodiment" is, as we shall see, to raise the sensitivity of the growth rate to changes in the rate of growth of capital.[11]

The Solow model can be written:

$$O_t = A'_t L_t^b J_t^{1-b}. \tag{3}$$

Although K in the basic Cobb-Douglas model is, in a sense, the number of machines (a proxy for the flow-of-machines services), J in a Solow model is a quality-weighted number of machines with new machines given greater weight than old machines, reflecting the newer technology embodied in them. A'_t is an index of economic efficiency, differentiated from A_t of equation (1) by the fact that A_t includes things that are incorporated in J_t as well as in A'_t of equation (3).[12]

The rate of growth is:

$$\Delta O/O = \Delta A'/A' + b(\Delta L/L) + (1 - b)(\Delta J/J). \tag{4}$$

Assuming that advancing technology permits the quality of new machines to improve at λ_k per cent a year, J can be written:

$$J_t = \sum_{o}^{t} K_{vt}(1 + \lambda_k)^v. \tag{5}$$

K_{vt} is the amount of capital built in year v (of vintage v) which is still in use at time t.[13]

[11] It should be noted here again that the analysis of this paper relates for the short and medium run, not the long run, steady state, or "Golden Age."

[12] In a later formulation of his model, Solow separates plant from equipment in his J measure.

[13] It is important to note that K_{vt} is gross capital of vintage v remaining at time t, and its measurement does not take into account any declines in value due to obsolescence. K_{vt} is smaller than K_{vv} if the physical productivity of machines declines through the years or if some machines are actually discarded. But the effect of obsolescence is not in the K_{vt} measure. Let C_{vt} be the measure of the value of capital of vintage v at year t which includes the effect of obsolescence. Then, if the productivity of capital improves at a rate λ_k a year,

$$C_{vt} = K_{vt}(1 + \lambda_k)^{-(t-v)},$$

or

$$K_{vt} = C_{vt}(1 + \lambda_k)^{t-v}.$$

To compare the growth implications of equations (2) and (4), it is useful to use the following approximation to equation (5):[14]

(5a) $$J_t = B(1+\lambda_k)^t K_t[1+\lambda_k(\bar{a}_o - \bar{a}_t)].$$

Here $\bar{a}_t$ and $\bar{a}_o$ are the average ages of capital at times t and o, respectively.[15] This simplification involves a single moment of the age distribution of capital and replaces an equation involving the full distribution of vintages. Empirical tests suggest that equation (5a) is a very good approximation indeed.[16]

For small values of λ_k and $\bar{a}_t$ not very different from $\bar{a}_o$, $\Delta J/J$ then can be approximated by:[17]

(6) $$\Delta J/J = \Delta K/K + \lambda_k - \lambda_k \Delta\bar{a}.$$

$\Delta\bar{a}$ is the change in the average age of capital. Equation (6) makes a good deal of economic sense and could have been arrived at directly. The first two terms of equation (6) give the rate of growth of the quality-adjusted capital stock when the age distribution of the capital stock is not changing through time. The third term provides an adjustment when the age distribution is changing. A given age distribution determines a given difference between average quality and the quality of new capital. If each old machine were one year older, the difference between average quality and new quality would be larger by λ_k. More generally change in the gap between average quality and the quality of new equipment is approximately equal to $-\lambda_k\Delta\bar{a}$.

Using equation (6) in equation (4):

And, using equation (5),

$$J_t = \sum_0^t C_{vt}(1+\lambda_k)^{t-v}(1+\lambda_k)^v = (1+\lambda_k)^t C_t,$$

and

$$\Delta J/J \approx \lambda_k + \Delta C_t/C_t.$$

Thus the Solow embodiment model reduces to the simple Cobb-Douglas if capital net of obsolescence is used in the equation.

[14] (1) $$J_t = (1+\lambda)^{-a_o}(1+\lambda)^t \sum K_{vt}(1+\lambda)^{v+a-t}$$

(2) $$J_t \approx B(\bar{a}_o)(1+\lambda)^t K_t \sum K_{vt}/K_t(1+\lambda v+\lambda\bar{a}_o-\lambda t)$$

where $B(\bar{a}_o)=(1+\lambda)^{-\bar{a}_o}$

(3) $$J_t = B(1+\lambda)^t K_t\left[1+\lambda\bar{a}_o - \sum \frac{K_{vt}}{K_t}\lambda(t-v)\right]$$

(4) $$J_t \approx B(1+\lambda)^t K_t[1+\lambda(\bar{a}_o-\bar{a}_t)]$$

[15] Knowles [13] also works with an "average age" model.

[16] Various series of J for different values of λ prepared by Richard Attiyeh were compared with series of J using the approximation formula. In all cases the series were almost identical.

[17] Thus $1+\lambda_k(\bar{a}_o-\bar{a}_t)\approx 1$. The assumption that $\bar{a}_t$ and $\bar{a}_o$ do not differ significantly is appropriate for examining short-run changes.

$$\Delta O/O = [\Delta A'/A' + (1-b)\lambda_k - (1-b)\lambda_k\Delta\bar{a}] + b\Delta L/L + (1-b)\Delta K/K. \tag{4a}$$

Save for the term $-(1-b)\lambda_k\Delta\bar{a}$, the growth equation for the embodied technical change model is similar to that for the simple Cobb-Douglas model.[18] If the average age of capital does not change, $\Delta A'/A' + (1-b)\lambda_k$ is the rate of growth of total factor productivity, that is, the $\Delta A/A$ of the original Cobb-Douglas model. $(1-b)\lambda_k$ is the part that needs to be embodied in new capital, and $\Delta A'/A'$ is the part that does not. The term involving changes in the average age, however, makes a great difference. Changes in $\bar{a}$ reflect changes in the difference between the average technology in use and the best available technology. Given $\Delta A'/A'$ and λ_k, the rate of growth of total factor productivity will be higher if the average age of capital is falling than if the average age is constant or increasing. We shall continue to call the bracketed set of terms "growth of total factor productivity."

TABLE 3—CHANGES IN THE AVERAGE AGE OF CAPITAL[a]

Year	$\bar{a}$ (average age)	Subperiod	Average $\Delta\bar{a}$ (average change in average age)
1929	16.5	1929–60	+.006
1947	20.0	1929–47	+.20
1954	18.0	1947–60	−.23
1960	17.0	1947–54	−.28
		1954–60	−.17

[a] The data are from Knowles [13].

Table 3 presents the average annual change in the average age of capital for the relevant subperiods of the 1929–60 period.

Zvi Griliches has suggested that one of the objectives of growth theory should be to reduce the unexplained residual; one of the ways that he has tried to do this is to consider explicitly improvements in the quality of capital.[19] In a way this simply passes back the problem to another stage—we still have to explain why the quality of capital increased. But it does permit us to understand better what is going on.

Let us assume for the moment (the assumption will be modified later) that all of total-factor-productivity growth is the result of design tech-

[18] Recalling an earlier footnote, notice that, if the capital stock net of obsolescence is used rather than gross capital stock, $J/J = \Delta C/C + \lambda_k$. There is no term involving $\Delta\bar{a}$ in the $\Delta J/J$ equation and, if C rather than K is used in the $\Delta 0/0$ equation, there again is no term involving Δa, since the average age of capital is caught in the net capital stock measure. In other words the "embodiment" effect is taken care of by the "net" capital measure.

[19] Z. Griliches [9] and also several unpublished manuscripts.

nical change that needs to be embodied, and hence all of the residual of the simple Cobb-Douglas model really is the result of the failure of the model to take into account improved capital quality. Then, for any value of b, in each of the subperiods $(1-b)\lambda_k-(1-b)\lambda_k\Delta\bar{a}$ must, by definition, be equal to $\Delta A/A$ as calculated from the simple Cobb-Douglas model. Thus, assuming all technical change to be embodied, $(1-b)\lambda_k$ of the Solow model can be calculated from the relationship $(1-b)\lambda_k=(\Delta A/A)/(1-\Delta\bar{a})$ where $\Delta A/A$ is derived from equation (2) and was presented in Table 2. Estimates of $(1-b)\lambda_k$ are presented in Table 4.

TABLE 4—THE COMPONENTS OF EMBODIED TECHNICAL CHANGE[a]
(percentage annual growth rate)

Periods	$(1-b)=.25$			$(1-b)=.50$		
	$\Delta A/A$	$(1-b)\lambda_k$	$-(1-b)\lambda_k\Delta\bar{a}$	$\Delta A/A$	$(1-b)\lambda_k$	$-(1-b)\lambda_k\Delta\bar{a}$
1929–60	2.1	2.1	0	1.7	1.7	0
1929–47	1.9	2.3	−.4	1.7	2.1	−.4
1947–60	2.5	2.0	.5	1.8	1.5	.3
1947–54	2.9	2.3	.6	2.1	1.7	.4
1954–60	2.1	1.8	.3	1.6	1.4	.2

[a] Data from Tables 2 and 3.

The implications of this extreme assumption are interesting. Notice that if all growth of total factor productivity were the result of technical change which needs to be embodied in new capital (improved quality of capital) and if $(1-b)$ is assumed to be equal to .25 (capital's share of GNP), a considerable part of the variation in the growth rates during the subperiods is explained by different rates of growth of capital and labor and different trends in the average age of capital; thus less weight need be placed on variation in λ_k. These factors are sufficient to explain the postwar acceleration of growth, if not the slowdown in recent years. There is no need to assume that "technical change" was faster in the immediate postwar era than in the prewar era. In a way embodiment explains too much because, with the full-embodiment model, it is necessary to assume that since 1954 the rate of technological progress has fallen sharply below its 1929–54 rate. But in general, assuming $(1-b)=.25$, and assuming a constant rate of technological progress, differences in the rate of growth of total factor productivity during the subperiods are relatively well explained by a widening gap between best practice and average practice before 1947 (reflected in a growing average age of capital) and a narrowing gap in the postwar period (reflecting in a falling average age of capital).

In the Solow model, has a mysteriously changing average age of capital replaced a mysteriously changing rate of growth of technology as the primary factor explaining changes in the growth rate? Not at all. Changes in the trend of the average age of capital are explained very nicely by changes in the rate of growth of the capital stock.

Assuming exponential depreciation (exclusive of obsolescence) at a rate δ a year, changes in the average age of capital can be approximated by the following expression:[20]

$$\Delta \bar{a} = 1 - (\Delta K/K + \delta)(\bar{a}_{t-1}). \tag{7}$$

Empirical checks suggest that equation (7) is a quite good approximation formula. Equation (7) makes good economic sense. $\Delta K/K+\delta$ is the rate of gross capital formation. If gross capital formation is zero, at the end of a year all old capital will be one year older, and there will be no new capital, so $\Delta\bar{a}=1$. At the end of a year $\Delta K/K+\delta$ is the ratio of new capital to total capital. The effect of new investment on the average age of capital is greater, the greater new investment is relative to the total capital stock, and the greater the average age of that capital stock.

[20] Let K_{ij} be the amount of capital built at time i that is still remaining at time j. Then:

$$\bar{a}_t = \frac{K_{t-1,t} + 2K_{t-2,t} + 3K_{t-3,t} + \cdots}{K_t}$$

$$\bar{a}_{t-1} = \frac{K_{t-2,t-1} + 2K_{t-3,t-1} + 3K_{t-4,t-1} \cdots}{K_{t-1}} .$$

But since

$$K_t - K_{t,t} = K_{t-1,t} + K_{t-2,t} + \cdots$$

$$\bar{a}_t = \frac{K_{t-2,t} + 2K_{t-3,t} + 3K_{t-3,t} + \cdots}{K_t} + \frac{K_t - K_{t,t}}{K_t} .$$

Assuming

$$K_{t-j,t} = (1 - \delta)K_{t-j,t-1}; \quad \text{and} \quad K_t = (1 + \Delta K/K)K_{t-1}$$

$$\bar{a}_t = \frac{(1-\delta)}{(1+\Delta K/K)}\bar{a}_{t-1} + \frac{K_{t-1}(1-\delta)}{K_{t-1}(1+\Delta K/K)}$$

$$\bar{a}_t - \bar{a}_{t-1} = \frac{1-\delta}{1+\Delta K/K} - \frac{(\Delta K/K + \delta)\bar{a}_{t-1}}{1+\Delta K/K} .$$

Thus for small $\Delta K/K$ and δ:

$$\Delta\bar{a} = 1 - (\Delta K/K + \delta)\bar{a}_{t-1}.$$

It might be noted that a more exact approximation is:

$$\Delta\bar{a} = 1 - (\Delta K/K + \delta)(\bar{a}_{t-1} + 1).$$

Using this equation for any $\Delta K/K$ and δ, it is possible to compute the long-run equilibrium average of capital:

$$\bar{a}_e = \frac{1}{\Delta K/K + \delta} - 1.$$

Thus, an increase in the rate of growth of capital would tend to increase the rate of reduction in the average age of capital (or reduce the rate of increase) or, in other words, in the medium run (if not in the very long run) [17] "embodiment" raises the sensitivity of the growth rate to the rate of growth of capital. What the simple Cobb-Douglas model misses is that the effect of capital growth on growth of potential *GNP* is determined not solely by $(1-b)$, but also by the rate at which "design improvements" are occurring and the gap between best practice and average practice. To put it another way, new investment not only leads to "more" capital—the magnitude of the effect determined by $(1-b)$—it leads to "more productive" capital—the magnitude of the effect determined by λ_k and $\bar{a}_t$.

Substituting equation (7) in equation (4a):

$$\text{(4b)} \quad \begin{aligned} \Delta O/O = \Delta A'/A' &+ (1-b)\lambda_k \bar{a}_{t-1}\delta + b\Delta L/L \\ &+ (1-b)(1+\lambda_k \bar{a}_{t-1})\Delta K/K. \end{aligned}$$

Notice that the coefficient in front of $\Delta K/K$ is no longer simply $(1-b)$. If $(1-b)\lambda_k = .02$ and $\bar{a}_t = 18$, then $(1-b)(1+\lambda_k\bar{a}_{t-1}) = .59$, more than double $(1-b) = .25$. The effect of embodiment, just as the effect of $(1-b)$ larger than capital's share of national income, is to increase the sensitivity of the rate of growth of *GNP* to the rate of growth of the capital stock.[21]

To recapitulate, the simple Cobb-Douglas model (with constant weights) yields two possible interpretations of the 1929–60 growth record. If $\Delta A/A$ is assumed to be constant, the growth record is explained by $(1-b)$ significantly greater than .25. If, on the other hand, $(1-b)$ is assumed to be approximately .25, changes in $\Delta A/A$ account for a major share of the changes in $\Delta K/K$. Under this second interpretation, the correlation between $\Delta A/A$ and $\Delta K/K$ needs to be explained.

The Solow model provides one possible explanation. $\Delta A/A$ is the result of technical change which needs to be embodied in new capital; growth of total factor productivity is really the result of improvement in the quality of new capital. Assuming $(1-b) = .25$, under this interpretation the variation in $\Delta A/A$ needed to explain changes in $\Delta O/O$ is quite well accounted for by variations in $\Delta K/K$, and there is little need to assume any dramatic change in the rate of growth of technological knowledge (λ_k) over the period.

This is a very interesting result. However, unfortunately it is pushing the point a bit to assume that all growth of total factor productivity is

[21] Statistically, it would be quite easy to confuse the effect of embodiment with the effect of a large static output elasticity $(1-b)$. And, if the statistician did not assume that the production function were homogeneous of degree one, embodiment could easily be mistaken for a high capital elasticity plus economies of scale (the coefficients of labor and capital adding to greater than one).

the result of such "design changes." We know, for example, that education (which presumably does not need to be embodied in new capital) has played an important role. To the extent that $\Delta A'/A'$ of equation (4) is not zero, then, assuming b to equal .75, changes in the average age of capital (caused by a varying rate of growth of the capital stock) are not sufficient of themselves to explain changes in the growth of total factor productivity among the subperiods. And the smaller λ_k, relative to $\Delta A'/A'$, the more the explanation of the acceleration of postwar growth must depend on either an increase in λ_k or $\Delta A'/A'$, or a $(1-b)$ substantially in excess of the share of capital.

Improved Labor Quality. Solow has focused attention on improvements in the quality of the capital stock. Edward Denison, following Theodore Schultz's lead, has drawn attention to improvements in the quality of labor input [5] [21]. Without too much forcing, the Denison model can be interpreted as introducing an average labor quality variable into the new-style Solow model. Thus

$$O_t = \overset{*}{A}_t(L_t q_t)^b J_t^{1-b}, \tag{8}$$

$$O_t = \overset{*}{A}_t Q_t^b J_t^{1-b}. \tag{8a}$$

Just as improvements in the quality of the capital stock are included in Solow's J, improvements in the quality of the labor force are included in Denison's $Q=Lq$. Just as A' was a narrower concept than A, so A^* does not include all that A' includes.[22] However, while it turns out that Solow's quality-of-capital measure can be nicely related to a single variable (the average age of capital can be related back to the rate of growth of the capital stock itself), as we shall see Denison's quality-of-labor measure is not so easily handled.

Defining $\lambda_L=\Delta q/q$ so that $\Delta Q/Q=\Delta L/L+\lambda_L$, the basic Denison growth equation, modified to incorporate the results drawn from the Solow model, can be written as follows:

$$\begin{aligned} \Delta O/O = \Delta A^*/A^* + b\lambda_L + (1-b)\lambda_k - (1-b)\lambda_k\Delta\bar{a} \\ + b\Delta L/L + (1-b)\Delta K/K. \end{aligned} \tag{9}$$

In this formulation $\Delta A^*/A^*+b\lambda_L+(1-b)\lambda_k-(1-b)\lambda_k\Delta\bar{a}$ is the rate of growth of total factor productivity, $\Delta A/A$ of equation (2); λ_L is the rate of improvement in the average quality of the work force. $\Delta A^*/A^*$ is improvements not directly "embodied" either in capital or labor (for example, improvements in the allocation of resources and better management practices).[23] Improvements that do not directly require em-

[22] For a somewhat different approach to the narrowing down of A, see Griliches [9].

[23] Since we are assuming that the static output elasticities add up to one, we are assuming away the possibilities of economies of scale as a source of growth of productivity. Of course, it is simple to relax this assumption.

bodiment in either capital or labor will be called "organizational."

It should be noted, however, that with a Cobb-Douglas type of function, improvements in the quality of *all* labor (or in average quality) can just as well be treated as disembodied change or as "organizational" change. While the breaking out of λ_L is an analytical convenience and permits us to understand the growth process better, as equation (9) is formulated, it is not correct to say that λ_L is productivity increase embodied in labor in the same sense that λ_k is productivity increase embodied in capital.

The reason that $\lambda_k \Delta \bar{a}$ enters the expression although there is no comparable term involving λ_L is that λ_k is defined in terms of the quality of *new* capital, and λ_L is defined in terms of the average quality of all labor. Improvements in basic educational standards, which principally affect the new entrants to the work force, could be treated like λ_k, in which case there would be a term equivalent to changes in the average age of the work force in the growth equation. And, in fact, when Denison calculates the effect of education on growth, he proceeds in roughly this way.

Thus in the Denison formulation the term $b\lambda_L$ has been added to the basic growth equation where λ_L is the rate of improvement in labor quality. Denison relates improvement in labor quality to three variables. $\lambda_L{}^E$ is improvement in labor quality due to improvement in educational attainment. He assumes that 60 per cent of the income differential associated with greater education is attributable to education. To suggest the orders of magnitude involved, his calculations are roughly consistent with the rule of thumb; each additional year of education increases labor quality by approximately 6 per cent.[24] Denison calculates that, considering both the increase in the average number of years of schooling per person in the work force and the increase in the length of the school year, improved average educational attainment increased labor quality at roughly one per cent a year over the period.

Denison then considers the changing age-sex composition of the work force. He concludes that the rate of improvement in composition, $\lambda_L{}^C$, proceeded at an average annual rate of .1 per cent.

Finally, Denison argues that, as the average work week declines, labor productivity per man-hour increases but with diminishing returns. He assumes that between 1929 and 1947 more than half of the decline in the average work week was offset by consequent improvements in labor productivity, and between 1947 and 1960 the offset was approximately one-third. Over the entire 1929–60 period, the rate of improvement in

[24] Each additional year of education is associated with, roughly, a 10 per cent increase in average income. Denison assumes that roughly 60 per cent of this is the result of the increased education.

average labor quality due to declining average hours of work, $\lambda_L{}^H$, averaged .3 per cent a year according to Denison.

Table 5 shows Denison's estimates of $\lambda_L{}^E$, $\lambda_L{}^C$, and $\lambda_L{}^H$. The table suggests that the rate of improvement in labor quality was roughly constant over the period. Assuming that $b=.75$, $b\lambda_L=1.0$ per cent a year or improvements in labor quality account for roughly half of the average

TABLE 5—COMPONENTS OF IMPROVED LABOR QUALITY[a]
(percentage annual growth rates)

Period	λ_L	$\lambda_L{}^E$	$\lambda_L{}^C$	λ_LQ
1929–60	1.4	1.0	.1	.3
1929–47	1.4	.9	.1	.4
1947–60	1.3	1.0	.1	.2
1947–54	1.3	1.0	.1	.2
1954–60	1.3	1.0	.1	.2

[a] Data from Denison [5].

TABLE 6—THE COMPONENTS OF $\Delta A/A$[a]
(percentage annual growth rates)

Period	$\Delta A/A$	$b\lambda_L$	$\Delta A/A-b\lambda_L=(1-b)\lambda_k(1-\Delta\bar{a})$	$(1-b)\lambda_k$
1929–47	1.9	1.0	0.9	1.1
1947–60	2.5	1.0	1.5	1.3
1947–54	2.9	1.0	1.9	1.5
1954–60	2.1	1.0	1.1	0.9

[a] Data from Tables 2, 4, and 5.

annual growth of total factor productivity of 2.0 per cent a year experienced over the 1929–60 period.

While the details of Denison's calculations are extremely bothersome, no one would argue either that improved labor quality has not been important or that technological change which requires new capital in order to be effective is the full story of productivity growth.[25] It can be shown (see Table 6) that if Denison's estimate of the contribution of improved labor quality is roughly correct, and if the rest of the explanation of growth of total factor productivity is improved capital quality resulting from technical change that needs to be embodied, then, assuming $(1-b)\approx 25$, these two factors split the credit roughly fifty-fifty. Under this explanation, however, it is impossible to assume that both

[25] Note that if $\Delta A/A\approx.02$ and $1-b=.25$, then, if all total-factor-productivity growth is technical change which needs to be embodied, $\lambda_k\approx.084$ ($b\lambda_k\approx.021$). It is interesting that Solow [26] never experiments with a λ_k this large. Naturally, therefore, his regressions yield a $(1-b)$ much greater than .25.

λ_L and λ_k have been constants. Taking Denison's estimate of a (constant) λ_L, the variation in $(1-b)\lambda_k$ of Table 6 is significantly greater than the variation of $(1-b)\lambda_k$ of Table 4, although less than the total variation in $\Delta A/A$ during the several periods.

Even if Denison's estimates of the rate of improvement in labor quality are high (and his estimate of the contribution of the shortened average work week certainly is suspect), design technical progress which needs to be embodied in new capital (in the Solow sense) probably was less than half of total-factor-productivity growth. And the lower is λ_k, the less sensitive is the rate of growth of GNP to the rate of growth of the capital stock. Denison has suggested (both in his book [5] and in a recently published article [5a]) that even this low λ_k overstates the sensitivity of the growth rate of output to the growth rate of capital, since much of new technology requires only marginal modification or addition to equipment, not totally new plant and equipment.

The conclusion must be that the embodiment effect, as Solow describes it, cannot have been large enough so that changes in the rate of growth of capital fully explain the variations in the growth rate of potential GNP that the United States has experienced during the 1929–60 period.[26] If the shares of capital and labor are assumed to provide tolerably good measures of the static output elasticities, the evidence suggests that $\Delta A^*/A^*+b\lambda_k+(1-b)\lambda_L$ increased sharply in the early postwar period and then declined somewhat after 1954. If we accept Denison's data, which suggest that λ_L has been relatively constant, the burden must fall on changing λ_k and $\Delta A^*/A^*$.

However, it is very interesting that $\Delta K/K$ and growth of total factor productivity have been so highly correlated. Although the Solow type of embodiment cannot fully explain this correlation, perhaps other factors can.

II. *The Cobb-Douglas Model in a More General Analysis of Economic Growth*

Solow's interaction effect describes one of the interactions among the variables of the Cobb-Douglas model, but there may be a number of other important relationships between the variables. They may be linked in the sense that changes in one determine the effect of changes

[26] Somehow many economists have come to view the requirement that new technology be embodied in new capital as in some sense a happy phenomenon. The reason for this seems to lie in the greater sensitivity of the growth rate to the investment rate that embodiment implies. But surely, the less the requirements for new technology to be embodied in new capital, the less costly is faster growth. Of course it might be replied that if growth itself were an objective regardless of cost, and if it were easier to influence $\Delta K/K$ than other variables that affect growth, then a strong embodiment provides a strong handle for policy. But surely this is a strange argument.

in another. Changes in one may stimulate changes in another or changes in certain underlying conditions not explicitly included in the model may have an effect on several of the variables of the model. This section will examine several obvious but important examples of these phenomena.

Sources of Interaction

Education, Technical Change, and Improved Allocation. Perhaps the greatest theoretical difficulty with Denison's method of examining the contribution of various factors to economic growth is that he does not deal explicitly with the very strong complementarity among the factors. In particular, it is quite clear that the effects upon GNP of the three principal contributors to growth of total factor productivity—technological change, improved educational standards and levels, and improved allocative efficiency—should not be viewed as independent.

Educated people, principally scientists and engineers, are a critical input to the research and development process; thus the rate at which technological understanding is increased is strongly related to the number of educated people applied to that purpose. The relatively high salaries these people receive are a direct reflection of their contribution to advancing technology. Were scientists and engineers the only inputs to the technological change process, then, in the absence of market imperfections, their salaries actually would be a good measure of technological change. Surely it is a mistake to measure the contribution of technological change to economic growth after subtracting the higher incomes that R&D scientists and engineers receive. Yet this is, in effect, what Denison's method does.

Although (as Denison points out) this direct and obvious linkage between educational input and technological change may not be of major quantitative importance, one might seriously propose the hypothesis that the need for and the return to educated people generally, not just research and development personnel, are in large part functions of the desired and actual rate of technological change. Industries and firms that have large research and development staffs also tend to have a relatively high percentage of scientists, engineers, and other trained people in other functions—management, sales, production [10]. This is scarcely surprising. New technological developments need to be evaluated by people in management who can understand them and who can understand the nature of the market for them. Information about new products needs to be communicated from the firm that develops them to the potential market by salesmen who can describe the product and its uses and can answer questions. In the early stages of production before the techniques become routinized, highly trained people are required to deal with the problems that invariably arise. In the absence of technological change, economic decision-making could be more routine. Just as R&D is essentially problem-solving, requiring highly trained people, the development of a new product or technique creates problems that require expertise in the form of experience and training, as well as imagination, for their solution. The relatively high remuneration of people who can deal imaginatively with these problems, just as the high salaries of R&D scientists and engineers, is in part, perhaps in major part, the reflection of the importance of technological change in economic growth. Should the pace of technological change diminish, the returns to higher education probably would also.

Nor can the importance of shifting allocation of labor and capital be evaluated independently of consideration of the effects of technological change. The fact that technological change proceeds at often dramatically different rates in different industries is one of the principal sources of economic disequilibrium in allocation of labor and capital. When technological change is relatively rapid in an industry faced by an elastic demand curve, it tends to increase the optimum share of the nation's capital and labor resources that should be allocated to that industry. A subsequent shift of labor and capital into that industry would increase national income. When technological change proceeds relatively rapidly in an industry facing an inelastic demand curve, that industry's optimum share of the economy's resources tends to decline. A subsequent shift of labor and capital out of that industry to others would be reflected in rising value of production. Clearly it would be a mistake to estimate the importance of technological change to economic growth by treating these subsequent reallocations as if they would have raised the value of output as much as they did, in the absence of technological change. For it is the pace and inequality among sectors of technological change which, in large part, determine the gains society can obtain by shifting resources from one industry to another.

Finally, to close the circle, one of the important lessons we have learned from experience with depressed areas and industries and with training and retraining programs is that basic literacy is almost a prerequisite for both learning of a new job and learning to do a new job. If a high level of education is essential to create technological change, a basic education is essential to permit people to adjust to it, and for the economy to gain maximum benefit from it. At lower levels as well as at higher levels, the returns to education are strongly affected by the pace of technological change.

New Plant and Equipment as a Source of Economic Flexibility. The preceding discussion has attempted to show that the sources of growth of total factor productivity cannot be viewed as independent. Thus it might be useful to look more broadly at the relationship between physical investment and growth of total factor productivity.

Solow has focused attention on the fact that often new capital is needed to embody new technology. Yet surely the advantage that new equipment has over old is not limited to a more up-to-date technology. As relative factor and product prices change, as demand and technology change, as the size of the market increases, new plant and equipment can be tailored to the changing economic situation. New plant and equipment, as well as education, plays a major role in providing the economy with flexibility.

As Johansen has suggested in his growth model, plant and equipment once built may be quite inflexible [11]. It is designed to work with a certain quantity of labor and to produce a certain quantity of output, as well as being designed around a certain technology.[27]

Denison and others have suggested that movement of labor from the farms to higher-productivity jobs has played a relatively important role in U.S. economic growth during the 1929–60 period. Massell has estimated that shifts of the relative allocation of capital and labor between industries account for approximately one quarter of total-factor-productivity growth in the postwar era [14]. As was noted earlier, these shifts in allocation, and the increase in the value of GNP resulting from those shifts, should not be considered independent of technological change. And it is clear that these shifts were at least partially dependent upon the creation of new capital. To the extent that old capital is inflexible, additional labor cannot be added to it without sharply diminishing returns. The productive movement of labor from one sector or industry to another is limited by the rate at which new plant is being built in the industry to which labor is moving. Thus both directly and indirectly, there is complementarity between the rate of growth of total factor productivity resulting from better allocating resources (labor and capital), and the rate of growth of the capital stock.[28] This is not embodiment in the Solow sense, but it has the same effect.

[27] If it were not for this inflexibility, we would not observe the phenomenon of obsolescence. If old equipment were flexible, if it could operate with widely varying quantities of labor and other variable inputs, then old paid-for capital equipment in good condition could be made competitive with new equipment simply by operating at very high capital-labor ratios (higher capital-labor ratios than for new equipment). But we observe that old equipment tends to operate at lower capital-labor ratios than new equipment; and as new and increasingly productive equipment comes into the market, the inability of older equipment to operate at higher capital-labor ratios, and thus to reduce variable unit costs, sooner or later makes that equipment obsolete even if it is in fine physical condition.

It also should be noted here that inflexibility of a Johansen sort may tend to make factor shares and output elasticities diverge [18].

[28] It is interesting that Massell's results suggest that shifting allocation of capital has been more important than shifting allocation of labor. If the analysis above is correct, it should be difficult to distinguish between the effects of the two; however, Massell's results tend to reinforce the belief that capital growth contributes to more efficient allocation.

The Johansen type of inflexibility also suggests that the extent to which the economy takes advantage of economies of scale is dependent upon new capital, not just more capital. To the extent that plant and equipment are indivisible, they must be built with a certain market size in mind. To take full advantage of increases in the size of the market, new plant and equipment must be better suited than existing plant for that larger-size market. Similarly, new plant and equipment must be built to take advantage of new opportunities for specialization.

The Johansen model therefore suggests that the complementarity between growth of total factor productivity and growth of the capital stock is more general than that suggested by the Solow model. In the Solow model new capital is needed to embody new technology. In the Johansen model new capital is needed to take maximum advantage of changing economic opportunities generally. An increase in the rate of growth of the capital stock should lead to an increase in the rate of growth of total factor productivity, not only by reducing the gap between average and best technology, but by creating a capital stock better suited to present demands, relative factor costs, and opportunities for economies of scale. The distance between the existing state of the economy and the production-possibility frontier which would exist in a world of perfectly flexible capital is strongly related to the rate of new investment.[29]

This point carries added significance when the uncertainties surrounding new technology are recognized. Embodiment, even in the strict Solow sense, is not a one-shot proposition. The early versions of a new product or process are likely to be quite primitive and plagued by unforeseen difficulties. Improvement and perfection is a sequential learning process, and the rate of learning is dependent not only upon the length of experience with a particular version of the technology but on the rate at which suggested improvements actually are tried out. To the extent that these improvements require embodiment, the rate of learning is strongly affected by the rate of new investment. This complementarity between learning and investment has been stressed by Arrow [1]. And to the extent that experience and experimentation in actual use are important aspects of the process of technological change, the Arrow effect implies that rapid rate of growth of capital not only keeps the technology in use close to the frontier but also helps to accelerate the rate at which the frontier advances.

It should be noted, however, that Denison's point with respect to the limited amount of new capital needed to embody new technology is also

[29] Perhaps it is this type of notion that Frankel [8] had in mind when he developed his aggregate production function in which an increase in the stock of capital increases the level of economic efficiency generally.

relevant to the more general complementarity relationships discussed above. Certainly not very much new capital is needed to take care of a moderate shift in labor allocation. Not very much new capital is needed to permit experimentation and experience with new techniques. It is therefore likely that, in the short run at least, the complementarity effects of new investment are subject to rather sharply diminishing returns. Yet there could still be a substantial amount of difference in effect between a very low rate of gross investment (such as we experienced during the 1930's) and a somewhat higher rate, such as the postwar average, if not between the very high rates of the early 1950's and the somewhat lower rates of the late 1950's.

Positive Feedback of Incentives. The complementarity between changes in the rate of capital growth and of total-factor-productivity growth also is the result of positive feedback of incentives. When the pace of design technical change increases, the profitability of new equipment relative to old increases, and an increase in investment is stimulated. Indeed, much of modern investment theory is hinged on the notion that investment booms are, in large part, the result of the development of new products and processes. Thus an increase in λ_k should stimulate an increase in $\Delta K/K$.

Schmookler's analysis suggests that the incentive mechanism also works the other way [20]. When the rate of investment is high, the potential market for inventions which require embodiment is high, thus an acceleration of the growth of the capital stock should stimulate an increase in λ_k. When Schmookler's hypothesis is linked with the Arrow hypothesis, incentive for technological advance is linked with opportunities for experimentation. This combined effect quite probably is very powerful.

The Effect of Prolonged Economic Slack. The causes for the correlation between growth of capital and growth of total factor productivity go deeper. Both almost certainly are depressed by prolonged economic slack of the sort experienced during the 1930's. It also is clear that both are stimulated when the economy comes out of a depression or recession. When jobs are plentiful, there is less pressure for protection from foreign competition, less incentive for featherbedding, less resistance to the adoption of new technology. It also is clear that the rate of growth of the capital stock tends to be higher when demand is pressing on capacity than when capacity is slack. While it is not inconceivable that the magnitude and design of the capital stock might be such that producers would be operating at preferred operating rates while unemployment is high, empirically it seems to be true that over the 1929–60 period, when unemployment has been high, capital has been operated at less than desired rates.

The effect of full employment on managerial incentives for innovation and on labor mobility is less clear.[30] However, although there is not enough evidence at present to predict whether the rate of capital formation and of total-factor-productivity growth would be greater in an economy that sustained a high level of employment or in one that fluctuated with moderate amplitude around a high level, given our present institutional and political structure, it seems probable that either of these possibilities would generate a faster average growth rate than would an economy with chronic high unemployment.[31]

It is reasonable to believe, then, that save for the World War II period and the Korean War period[32] stimulation for both capital growth and growth of total factor productivity should have tended to move (inversely) with the unemployment rate over the 1929–60 period. A glance at Tables 1 and 2 suggests that this, in fact, has been the case. It should be noted that these cyclical effects on growth of the capital stock and of productivity are additive to the effects of slack on growth of man-hours and productivity that Okun has analyzed.

Some Implications of Interaction

The analysis of interaction suggests that it is misleading to assume that the various factors resulting in the growth of total factor productivity are independent, and to estimate the contribution of technological change by examining the residual after having estimated the contribution of education, shifting allocation of resources, and so on. It also is misleading to assume that the contributions of the various factors leading to growth of total factor productivity are independent of the rate of growth of the capital stock.

The growth rate of potential output has varied considerably over the period, principally the result of related and interdependent changes in the rate of growth of the capital stock, the rate of technical progress, and improvements in economic efficiency generally. All these factors tend to move together. They tend to be encouraged by strong aggregate demand, discouraged by economic slack. A change in one tends to increase the importance of the others.

This suggests that a simple regression model that tries to estimate

[30] The data are reasonably clear that movement off the farms and to urban areas is greater in periods of high over-all employment. But the effect of full employment on labor mobility in the nonagricultural sectors is less clear.

[31] It may be significant that Kendrick, in comparing productivity growth in different industries, finds that industries which experienced only mild cyclical fluctuations tended to have a faster rate of growth of productivity than industries which experienced severe cyclical fluctuations.

[32] During these periods investment was limited by policy, and perhaps demand was so strong that incentives for innovation and for productive shifting of labor were dulled.

$\Delta A/A$, or λ_k, λ_L, $\Delta A^*/A^*$, and b for any period by relating $\Delta O/O$ to $\Delta K/K$ and $\Delta L/L$ may miss the point. The variables of the model are not independent, and λ_k and $\Delta A^*/A^*$ are undoubtedly not constants.

Assume that the conclusions above are roughly correct; that changes in $\Delta O/O$ are the result of interdependent changes in $\Delta K/K$, λ_k, and $\Delta A^*/A^*$, and that $\Delta L/L$ has been relatively constant compared with the other variables. Because changes in capital growth have been associated with changes in output growth, a simple linear regression would tend to give a high weight to $(1-b)$ in a model that does not include the embodiment effect. In a model that did not assume constant returns to scale, the effect of embodiment could be misinterpreted as economies of scale (the coefficients adding up to more than one) and a relatively high static capital elasticity.

But we know more about the process of economic growth than the factors and relationships treated formally by the aggregative growth models we use. For example, if we believe that factor shares tend to equal the elasticities of output with respect to the factors, we are not justified in estimating b and $(1-b)$ from a simple regression. If we believe that various components of total-factor-productivity growth are affected by growth of capital directly, or by the same variables that influence capital growth, we are not justified in assuming them to be constants, nor are we justified in estimating these variables by regression, ignoring this interaction effect. If we believe that the returns to education are related to the pace of technological change, we are not free to treat λ_k and λ_L as independent.

In sum, the aggregative production function may be a useful part of the framework for studying economic growth. But it is a mistake to try to introduce into the production function variables such as average years of education without an explicit theory that shows *how* that variable should be entered. Further, it is a mistake to ignore what we know about economic processes from other kinds of analysis. Regressions of growth of GNP with respect to growth of capital and labor should be constrained by what little we know of the relationship between output elasticities and factor shares. To accomplish this and also to take account of the other interactions discussed in this section probably will require that the production function be imbedded explicitly or in a richer, more complete model of economic growth [3].

III. *Medium-Range Growth Projections*

The preceding analysis has some rather interesting implications with respect to medium-range (say one decade) growth projections. While the range of possible forecasting models is very wide, in order to limit the discussion it will be convenient to focus the discussion upon a comparison of the growth projections for the 1960–70 period that Robert Solow has derived from his model, and the projections of Edward Denison. However, great liberties will be taken in interpreting both of these authors.

The General Framework

As shown in Section II, the amended Cobb-Douglas model can be written in three equivalent ways:

$$\Delta O/O = \Delta A/A + b\Delta L/L + (1-b)\Delta K/K \tag{10a}$$

$$\begin{aligned}\Delta O/O = {} & [\Delta A^*/A^* + b\lambda_L + (1-b)\lambda_k - (1-b)\lambda_k\Delta\bar{a}] \\ & + b\Delta L/L + (1-b)\Delta K/K\end{aligned} \tag{10b}$$

$$\begin{aligned}\Delta O/O = {} & [\Delta A^*/A^* + b\lambda_L + (1-b)\lambda_k\bar{a}_{t-1}\delta] + b\Delta L/L \\ & + (1-b)(1+\lambda_k\bar{a}_{t-1})\Delta K/K\end{aligned} \tag{10c}$$

These are the basic equations we shall use for comparing alternative growth projections. For some purposes one version is more convenient, for other purposes another version. Note that all three versions can be interpreted as relating growth of output to growth of labor, growth of capital, and growth of productivity of capital and labor. The difference between equation (10a) and the other two versions is that equation (10a) contains no explicit interaction relationship.

In this paper we are not particularly concerned with the difficult problem of projecting potential labor input since the production functions being examined shed no light on that topic. For this reason the projection of the Bureau of Labor Statistics for labor force growth of about 1.7 or 1.8 per cent a year over the 1960–70 period will be accepted without criticism, and, for want of a better hypothesis, it will be assumed that, abstracting from cyclical considerations, the average work week will decline at .5 or .6 per cent a year. Thus potential man-hours will be assumed to grow at 1.1 to 1.3 per cent a year. This rate of growth is more than half again greater than the growth of potential man-hours during the 1929–60 period. This is mainly the lagged result of the relatively high birth rates of the postwar era.

Note that if the unemployment rate decreases over the period, the growth of actual man-hours would exceed the growth of potential man-hours as a result of cyclical gains in the employment rate, in the length of the average work week, and in the rates of labor force participation. Likewise, growth of actual output would exceed the growth of potential output due both to the cyclical effect on growth of man-hours and to the cyclical effect on productivity. Although the analysis of this section will be of potential output and input, not actual output, it is important to note that if the unemployment rate is 4.0 per cent in 1970 (as compared with nearly 6.0 per cent in 1960), and if the Okun adjustments are correct, the annual rate of growth of actual GNP will average .6 percentage points greater than the rate of growth of potential GNP. Thus if potential grows at 4.0 per cent, and full employment is achieved in 1970, actual GNP will have grown at 4.6 per cent a year over the decade. And this is aside from any effects that decreasing economic slack would have on the rate of growth of potential GNP.

Once a rate of growth of labor input is assumed, the task of growth projection using the extended Cobb-Douglas model breaks naturally into two parts. The first is projection of a likely range for the rate of growth of the capital stock. The second is projection of a likely range of growth of total factor productivity. However, $\Delta A/A$ and $\Delta K/K$ can not be assumed independent.

TABLE 7—THE POSTWAR AND LONGER-RUN GROWTH RECORD[a]
(percentage annual growth rates)

Period	$\Delta O/O$	$\Delta K/K$	$\Delta A/A$; $b=.75$	$\Delta A/A$; $b=.5$
1929–60	3.1	2.0	2.1	1.7
1947–60	4.0	3.6	2.5	1.8

[a] Data from Tables 1 and 2.

TABLE 8—COMPARATIVE PROJECTIONS[a]
(percentage annual growth rates)

	$\Delta O/O$; $b=.75$		$\Delta O/O$; $b=.5$	
	$\Delta A/A$ 29–60	$\Delta A/A$ 47–60	$\Delta A/A$ 29–60	$\Delta A/A$ 47–60
$\Delta K/K$ 29–60	3.5	3.9	3.3	3.4
$\Delta K/K$ 47–60	3.9	4.3	4.1	4.2

[a] Data from Tables 1 and 2.

Let us use the 1929–60 experience as a guide to the range of possibilities. The relevant numbers from Tables 1 and 2 are reproduced in Table 7.

Assuming $\Delta L/L=.012$, let us examine how the projected potential GNP growth rate depends on whether the 1929–60 experience or the 1947–60 experience holds during the 1960's for $\Delta K/K$ and $\Delta A/A$ for $b=.75$ and $b=.5$, respectively.

Table 8 expresses the heart of the growth-projection problem. It matters a good deal whether the 1960–70 experience with respect to both growth of total factor productivity and growth of the capital stock resembles the postwar experience or the longer-run average experience.

Despite the fact that the simple Cobb-Douglas model (with factor shares providing estimates of output elasticities) places a low weight on growth of capital, the difference between the postwar rate of capital growth and the long-run average rate is so great that the difference matters greatly. Even in the low-capital-weight model ($1-b=.25$), the difference in the rate of growth of capital over the two comparison periods makes almost as great a difference in the growth projections for the 1960's as do differences in the rate of growth of total factor productivity. And in the model in which capital is heavily weighted ($1-b=.50$), they make almost all the difference.

As was pointed out in Section II, a model with a high correlation between rate of growth of total factor productivity and the rate of growth of the capital stock, but with a low static output elasticity with respect to capital, tends to yield the same conclusions as a low correlation, high static output elasticity model. It is not surprising, therefore, that the principal diagonals of the two matrices of Table 8 are almost identical. Solow's "embodied" technical change growth model brings out this point clearly.

Growth Projections with Complete Embodiment

As was shown in Section II, since embodiment results in a high correlation between rate of growth of capital and rate of growth of total factor productivity, the "embodied" technical change model is capable of explaining changes in the rate of growth of potential GNP over the 1929–60 period without recourse either to a postwar acceleration of technical change or to a static elasticity of output with respect to capital significantly in excess of the share of capital in national income.

At the polar extreme, then, if all total factor productivity growth were the result of embodied technical change, and $(1-b)$ were .25, then $(1-b)\lambda_k$ over the 1929–60 period must have been 2.0 per cent a year. With $\Delta A^*/A^*=\lambda_1=0$, equation (10c) can be written:

$$(11)\quad \Delta O/O = [(1-b)\lambda_k\bar{a}_{t-1}\delta] + b\Delta L/L + (1-b)(1+\lambda_k\bar{a}_{t-1})\Delta K/K.$$

With $\bar{a}_{t-1}=17$ (the average age of capital in 1960), $\delta=.035$, $\Delta L/L=.012$, and $(1-b)\lambda_k=.02$, the growth rate would be the following function of the rate of growth of the capital stock:[33]

$$(11a)\quad \Delta O/O = .022 + .59\Delta K/K \text{ [Projection with full embodiment].}$$

If $\Delta K/K=.02$ (the 1929–60 average), then $\Delta O/O=.034$. If $\Delta K/K=.036$ (the 1947–60 average), then $\Delta O/O=.044$.

[33] The constant term of equation (11a) is $(1-b)\lambda_k\bar{a}_{t-1}\delta+b\Delta L/L$. The coefficient before $\Delta K/K$ is, of course, $(1-b)(1+\lambda_k\bar{a}_{t-1})$.

Projections with Limited Embodiment

Equations relating the growth rate of potential GNP to the rate of growth of the capital stock also can be derived for the totally "disembodied" technological change extreme (equation (10a)). Assuming $b=.75$ and assuming $\Delta A/A=.02$ or .0025 (the 1929–60 and 1947–60 rates respectively):

$$\Delta O/O = \underset{(.034)}{(.029)} + .25\Delta K/K \text{ [Projection with no embodiment].} \tag{12}$$

The values of $\Delta O/O$ for $\Delta K/K=.02$ and .036, for $\Delta A/A=.02$ and .025, were given in Table 1.[34] Notice that the "embodied technical change projection" is more than twice as sensitive to the rate of growth of the capital stock as the more conventional model. Notice also that if equation (12) is used, the growth projector is forced to decide what rate of growth of total factor productivity to assume, for the experience of the 1929–60 period does not provide a unique answer.

This certainly is so for Denison. According to his model, $\Delta A/A$ varied considerably over the period. Indeed both λ_k and $\Delta A^*/A^*$ varied. Denison must make a choice here. The choice he actually makes is guided by several considerations.

Denison's analysis suggests several reasons why the rate of growth of total factor productivity should be less rapid during the 1960's than during the 1947–60 period. Increases in productivity resulting from declines in the work week should be much less important. There should be a decline in the rate of growth of average school years per worker. Productivity advance resulting from economies of scale should be smaller.

But, even assuming that this analysis is correct, and that the effect of the declining work week and of increasing educational standards will be weaker during the 1960's than in earlier decades, Denison still has some freedom left if he recognizes the fact that his "residual" was greater in the postwar era than in the prewar era. It is interesting that Denison ends up projecting a rate of technical change faster than the 1929–60 average, but significantly smaller than the 1947–60 average.[35] And, since he projects a decline in the effect of variables that contribute to growth of total factor productivity, he ends up actually projecting a slightly smaller rate of growth of total factor productivity during the

[34] The two constants of equation (12) result from the two different assumptions about $\Delta A/A$. The constants are $\Delta A/A+b\Delta L/L$.

[35] It should be noted that my use of 1947 as a dividing line between the postwar and earlier periods is critical here. Although Denison's projected rate of technological change is less than the 1947–60 average, it is roughly equal to the 1954–60 average rate, and to the 1950–60 average rate.

1960–70 period than during the 1929–60 period, not just than during the 1947–60 period.[36]

Denison's "best guess" projection is for a 3.3 per cent annual growth of potential GNP, with a growth of the capital stock of 2.5 per cent a year. Taking great liberties with Denison's formulation, assuming that $(1-b)\lambda_k = .005$ and that all of λ_k needs to be embodied (which Denison correctly would deny), the basic Denison growth equation can be written:[37]

$$(13) \quad \Delta O/O = .025 + .33\Delta K/K \text{ [Denison, with full embodiment of } \lambda_k\text{].}$$

This yields a 3.3 per cent growth rate of $\Delta K/K = .025$. If $\Delta K/K = .036$, as it has averaged since 1947, $\Delta O/O$ .036.

But the growth-projection equation that Denison finally reaches is far less important than the analysis on which he bases his projection. The assumptions of interest are, first, that several of the factors contributing to growth of total factor productivity will be less important during the 1960's than earlier and, second, that embodiment is quite unimportant.

Denison undoubtedly is correct in assigning a smaller weight to λ_k than does the pure embodied technical change model. However, due to the strong links between education and technical change, his attempt to treat the effects of technical change as a residual undoubtedly leads to an understatement of the importance of technical change. He also undoubtedly is correct in arguing that often technological change requires only a small amount of new capital. But if the analysis of Section II is correct, he probably also has underestimated the importance of other, if less direct, connections between growth of capital and growth of total factor productivity. (Of course these are not treated by the formal embodiment model either.) And, although he may be right that the longer-run experience of growth of total factor productivity is a better prediction for the 1960's than the postwar experience, the point is surely open to question. Thus his estimates both of the constant and the capital growth sensitivity terms may well be low.

The Major Uncertainties

While other projection models certainly could provide higher or lower forecasts, the comparison of the Solow and Denison projections suggests the range of uncertainty with respect to the medium-range

[36] This point of course is not dependent upon choice of subperiods.

[37] The constant in equation (13) is derived from Denison's projection of λ_L and $\Delta A^*/A^*$ on the assumptions that $b = .75$ (slightly smaller than Denison's assumption) and that $\Delta L/L = .012$. The coefficient before $\Delta K/K$ is derived from the assumption that $(1-b)\lambda_k = .005$. λ_k and $\Delta A^*/A^*$ were adjusted to add up to be consistent with Denison's projection for technological change plus changes in efficiency and economies of scale.

growth prospects of the U. S. economy and helps to identify the principal causes of the uncertainty.

To the extent that the experience of the 1929–60 period can shed light on the prospects for the future, during the 1960 period growth of potential GNP is likely to be somewhere between an annual rate of 3.2 per cent and 4.3 per cent. The high end of the range will be achieved if the rates of growth of the capital stock and of total factor productivity are at the postwar rate, the lower end if they are at the longer-run average rate.

Much of the uncertainty, therefore, relates to the interpretation of the acceleration of growth of capital and of total factor productivity in the postwar period. One interpretation is that the postwar spurt essentially represents a making up of ground lost during the Great Depression, that the slowdown during the past several years shows that the "making up" has been completed, and that the 1929–60 average represents an average of below-normal growth and above-normal growth. According to this interpretation we should not expect growth of the capital stock or of total factor productivity during the 1960–70 period to differ greatly from the 1929–60 average rates, and the recent slowdown suggests that to project the 1947–60 rates would be unrealistically optimistic.

Another interpretation is that the 1929–60 average is much too heavily influenced by the depression decade of the 1930's. While it probably is true that the very rapid rate of growth of capital and of total factor productivity during the 1947–54 period was, in large part, a make-up phenomenon, this cannot be said of the post-1954 period. It should be recalled that the rate of growth of the capital stock, since 1954, has been significantly greater than the 1929–60 average and, after adjusting for changing degrees of slack, the same is true of growth of total factor productivity. Further, it is quite clear that the average rate of growth of the capital stock since 1954 has been less than it would have been had fiscal and monetary policy been more effective in keeping aggregate demand pressing on aggregate potential. Growth of total factor productivity undoubtedly also has been depressed by economic slack.

Thus, according to this interpretation, the 1954–60 record (even more the 1929–60 record) significantly understates the growth of the capital stock and the growth of total factor productivity we may expect in an economy where aggregate demand is not permitted to lag significantly behind growth of economic potential.

Further, according to the more optimistic interpretation, the postwar increase in the rate of R&D spending has not been without effect. The latent rate of technical change—the rate at which productivity could be increased as a result of new technical knowledge were there sufficient demand to spur innovation and reduce resistance, and sufficient investment to embody the new technology and to permit high labor mobility —probably has been greater in the postwar era than before. Even with the economic sluggishness of the past five years, which certainly has made innovation more difficult, the rate of growth of potential total factor productivity has exceeded its long-run average.

From analysis of the Solow and Denison projections it is clear that this difference in interpretation lies at the root of the differences in the growth projections. Perhaps looking at the experience of the economy over a longer period of time than 1929–60 can shed some light on which interpretation is likely to be more nearly correct. Between 1909 and 1960 the rate of growth of the capital stock averaged approximately 2.5 per cent a year, and between 1909 and 1929 the average annual rate of growth of the capital stock was approximately 3.0 per cent. This suggests that the 1929–60 capital growth experience probably was heavily affected by the depression years. And to the extent that growth of capital and growth of total factor productivity are correlated, this suggests that the 1929–60 experience for $\Delta A/A$ also was heavily affected by the depression.

The experience of the past few years also suggests that since the war there has been a change and that it would be a mistake to extrapolate the longer run 1929–60 experience into the future. From 1959–62, potential GNP has grown at an annual rate of greater than 3.5 per cent. This is faster than the rate that Denison projects for the future. And yet, from 1959–62 the growth of labor input was significantly less than we can expect during the middle and late 1960's, and, as a result of persistent economic slack, the capital stock grew at an annual rate of only about 2.0 per cent. If we have in fact achieved an annual growth potential of approximately 3.5 per cent under these conditions, we certainly should do better as the rate of growth of the labor force increases, provided we can achieve and maintain adequate aggregate demand.

Although Solow's model undoubtedly overstates the correlation between growth of capital and growth of total factor productivity, Denison's model undoubtedly understates it. It is likely that during the 1960's the rate of growth of the capital stock and of total factor productivity will both be near the high postwar rates or neither will be. If this is correct, and if it also is correct that one of the major reasons for the postward acceleration has been the absence of deep recession and (save for the past several years) prolonged slack, our growth record during the 1960's may be more dependent on an ability to reduce economic slack than on any other measure. We shall have full employment and rapid growth of potential GNP together or we shall have neither.

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