Salop's Spatial Competition

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Model

- Salop (1979, Bell Journal of Economics)
- The point of this model is provide a market mechanism that shows the tradeoff between fixed cost and transportation cost.
- The original model is aimed to explain product differentiation. But, it naturally has a geographic/spatial interpretation.
- Product differentiation: Hotelling (1929, Economic Journal).

Model

- Circular space. The length of the circle is normalized to 1.
- Consumers are uniformly distributed with a density of d.
- One good. Consumers have fixed demand for this good. The demand is normalized to 1 unit per person.
- There is an infinite pool of potential *entrants*. To enter, a fixed cost ϕ needs to be paid.

Model

- Suppose that n entrants enter. Then, they are evenly spaced on the circle. The distance between any two neighboring firms $= 1/n$.
- The marginal cost is a constant c.
- Each consumer travels to the location of nearest firm to purchase the good. Transport cost: t per unit of distance.

Timing

- Stage 1: firms decide whether to enter or not. In this stage, n is determined.
- Stage 2: Given evenly spaced n firms, firms set prices, consumers travel and purchase, and markets clear. The price equilibrium is determined by Nash equilibrium.
- Backward induction.

Pricing Stage

- Given n evenly spaced firms, we can solve the pricing decisions by looking at a Hotelling duopoly problem in which two firms, A and B, are located at the two end points of $[0, 1/n]$.

- Customer $x \in [0, 1/n]$ chooses between the two firms in buying a non-customized product, i.e.,$\min\{tx + p_A, t(1/n - x) + p_B\}$. Then, a customer of type $\tilde{x}$ who is indifferent between the two choices is given by

$$\tilde{x} = \frac{1}{2n} - \frac{p_A - p_B}{2t}. \tag{1}$$

- For firm A, it has a demand given by $\tilde{x}(p_A; p_B)$, and firm B has demand $1/n - \tilde{x}(p_B; p_A)$. Here, ";" means "given."

Pricing Stage

- Each firm maximizes its profit given its opponent's price

$$\begin{aligned} \pi_A(p_A; p_B) &= d\tilde{x}p_A = \left[\frac{1}{2n} - \frac{p_A - p_B}{2t}\right] p_A \\ \pi_B(p_B; p_A) &= d\left[1/n - \tilde{x}\right] p_B = \left[\frac{1}{2n} - \frac{p_B - p_A}{2t}\right] p_B \end{aligned}$$

- Solution: from first-order conditions, we get $\widehat{p}_A\left(p_B\right) = t/\left(2n\right) + p_B/2$ and $\widehat{p}_B\left(p_A\right) = t/\left(2n\right) + p_A/2$.

- Check second-order condition.

- Nash equilibrium: $\left(p_A, p_B\right) = \left(t/n, t/n\right)$. So, every firm charge the same price at $p^* = t/n$. And, $\tilde{x} = 1/\left(2n\right)$.

Entry stage

- The profit formula above is only for "one-side." So, the actual profit is $2d\tilde{x}p^* - \phi = td/n^2 - \phi$.
- Zero profit: (ignoring the integer contraints (the solution is an approximation))

$$\begin{aligned} \frac{td}{n^2} &= \phi. \\ \text{distance} &= \frac{1}{n} = \sqrt{\frac{\phi}{td}}. \end{aligned}$$

- Social planner's problem in the previous model. The distance between cities is determined by the tradeoff between transport cost and fixed cost. The larger the fixed cost, the optimal distance is larger. The larger the transport cost, the optimal distance is smaller. (Or, put differently, increasing distance incurs less fixed cost and more transport cost). Here, firms are like cities. Get the same direction (distance is $1/n$).
- Is the market solution here optimal? (left as an exercise)