Chapter 2: Analyzing Urban Spatial Structure (Brueckner's textbook)

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Urban Spatial Structure

- Cities may have one or multiple centers. Start with the largest center, Central Business District (CBD).
- Spatial regularities
  - Building height diminishes with distance from CBD.
  - Housing price and land rent diminish
  - Housing size increases
  - Population density diminishes

Urban Spatial Structure

- In this and next chapters, we focus on a simple model that explain these regularities
- Like any model, we set up assumptions to simplify the analysis, to focus on the more important phenomenon, and to assume away less important details.
- Mostly through diagrammatic analysis in the textbook. But, we will also formalize in mathematical terms later.

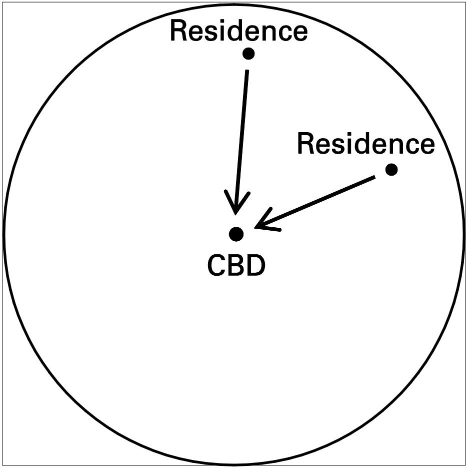
Monocentric City Model

- Alonso-Muth-Mills monocentric city model.
- Job and business concentrate in CBD, which is assumed to be a point.
- Focus on the housing and commuting part of the city life. . . . . .
- Will talk about employment sub-centers later, but the main results here hold. . . .
- Aside: a good model is such that the main result is relatively robust to adding details

Monocentric City Model

- Alonso-Muth-Mills monocentric city model.
- The city has a dense network of radial roads. . . .
- Households are identical. . . ..same preference, same income earned from work in CBD. . . . . . relaxation of this assumption has interesting implications. . . .next chapter.
- Each household consume two goods: housing and bread. “Bread” is just a name for everything else.

Figure 2.1 Radial commuting



Monocentric City Model

- Commuting Cost
  - Money cost
  - Time cost
- Focus on only money cost:
  - t per unit distance.
  - Income y. For a household live x distance away from the CBD, its disposable income $y - tx$.
  - Let the price of bread be normalized to 1.

Monocentric City Model

- Utility: $u(c, q)$, where c, q is the amount of bread and size of dwelling, respectively.
- Budget constraint
$$c + pq = y - tx.$$
- p is the price per square foot (or, whatever unit you want to choose).
- Each individuals households are renters. So, pq is the rent.

Consumer Locational Equilibrium

- Households are free to choose where to live.
- Let $c(x), q(x)$ be the quantities consumed for a household at x.
- In equilibrium, it must be that $u(c(x), q(x))$ the same for all x. Why?
- Implication: $p(x)$ must be decreasing in x.
- Intuition: compensating differential.
- Not obvious in algebra. But, clear with a graph with "well-behaved preference."

Figure 2.2 Consumer choice

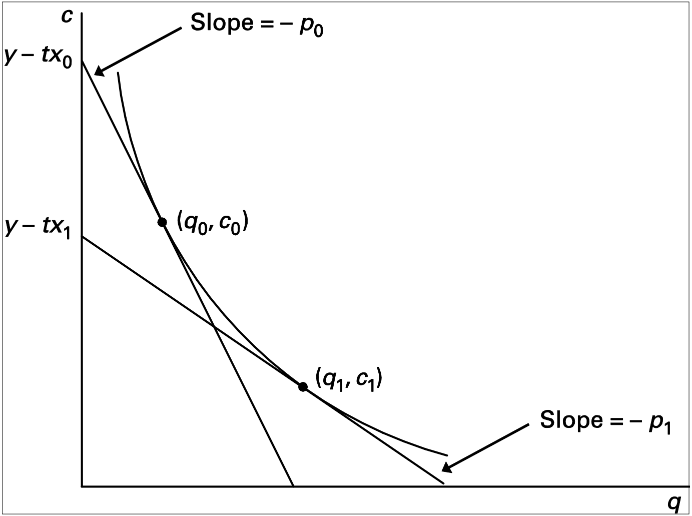
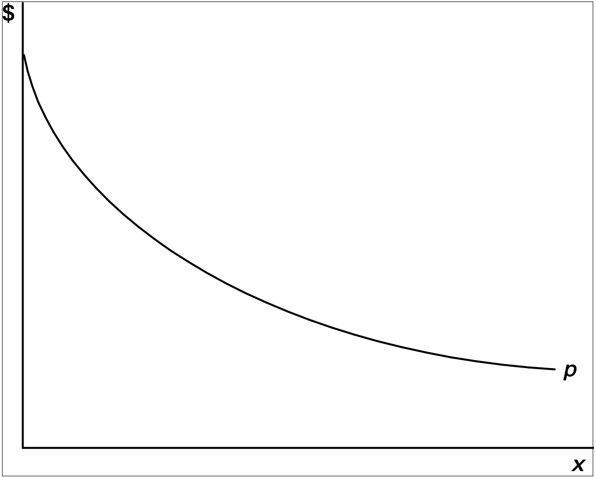


Figure 2.3 Housing-price curve



Consumer Locational Equilibrium

- Figure 2.2.
- $p(x)$ decreases in x.
- $q(x)$ increases in x.
- $c(x)$ decreases in x.
- 1 and 2 are the regularities mentioned.
- When generalizing to two income groups, 3 doesn't survive the generalization, but 1 and 2 are.

Consumer Locational Equilibrium

- Housing price gradient:
$$\frac{\partial p(x)}{\partial x} = -\frac{t}{q(x)}.$$

- We will derive this equation mathematically later.

- The equation implies a "convex" housing price gradient. See Figure 2.3.

- Intuition: for households living close to the downtown, $q(x)$ is small, the drop in $p(x)$ in x should be large to generate enough compensation to increasing commuting cost. Similarly, when $q(x)$ is large for large x's, the gradient $\frac{\partial p(x)}{\partial x}$ is smaller.

Housing Production

- Production function: $Q = H(N, l)$, where Q is the floor space, N is building material, and l is land.
- Assume
  - diminishing marginal product of material. See Figure 2.4. For a given l, increasing height is *increasingly* costly. (thicker beams, stronger foundations, more space devoted to elevators and stairways)
  - constant returns to scale in H and l. See Figure 2.5.
  - i : price of materials
  - $r(x)$: land rent at location x.

Figure 2.4 Making a building taller

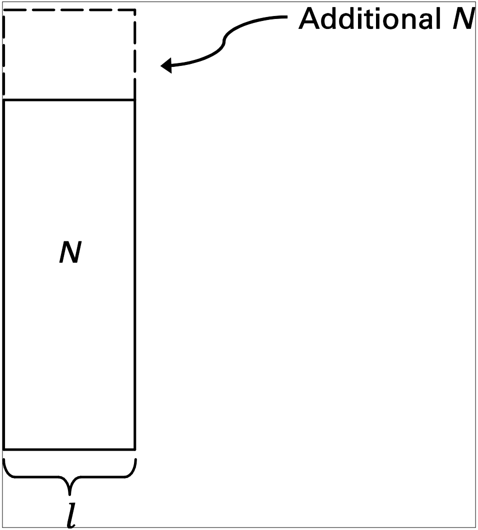
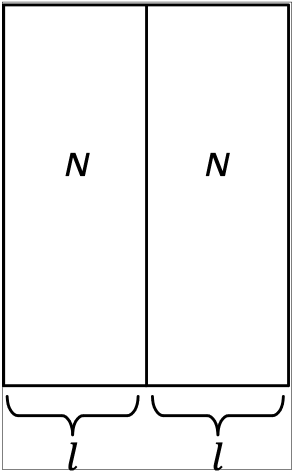


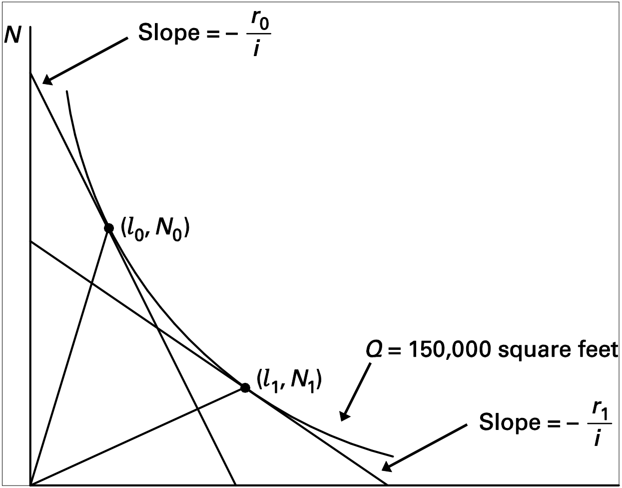
Figure 2.5 Constant returns to scale



Producer Locational Equilibrium

- Production cost $= iN + rl$.
- There are numerous developers, who take $p(x)$ as given at each x.
- Compensating differential in p implies compensating differential in r.
- At each x, the profit of developers must be zero. (equal profit across locations and free entry)
- Profit maximization $\Longrightarrow$ Cost minimization.
- Figure 2.7: for $x_1 > x_0$, $p(x_1) < p(x_0)$, $r(x_1) < r(x_0)$, $\frac{N(x_0)}{l(x_0)} > \frac{N(x_1)}{l(x_1)}$. That is, building height decreases in x.

Figure 2.7 Cost minimization by housing developer



Population density

- $\frac{N(x)}{l(x)}$ decreases in x.

- $q(x)$ increases in x.

- Population density $D(x)$: the number of people per unit land = number of dwellings per unit land: $\frac{N(x)}{l(x)} \frac{1}{q(x)}$.

- So, $D(x)$ decreases in x (see Figure 2.8). This is a very robust fact.

- Run regression (Figure 2.9)

$$\ln(D(x)) = \beta_0 + \beta_1 x + ...$$

- The results so far is summarized in Figure 2.10.

Figure 2.8 Population density

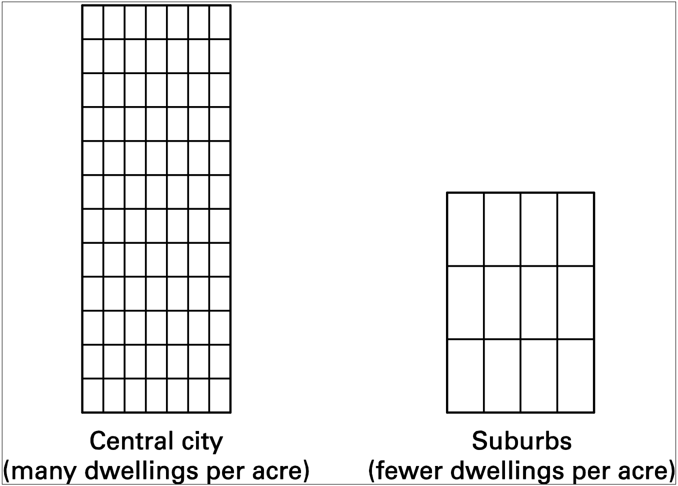


Figure 2.9 Population-density regression

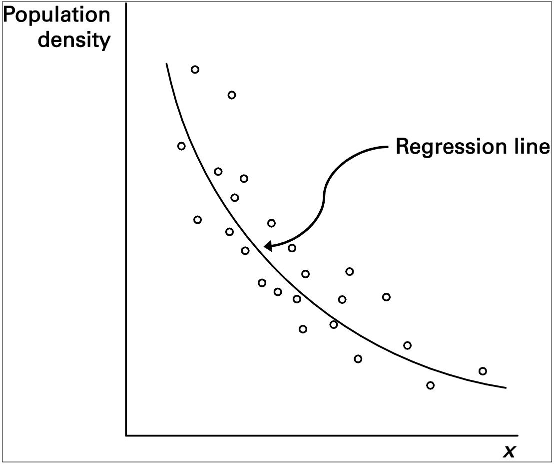
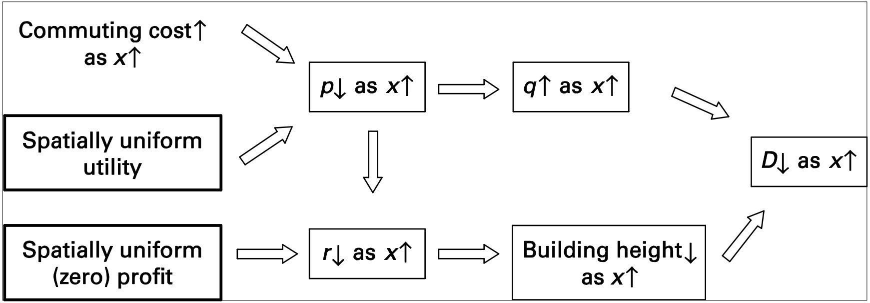


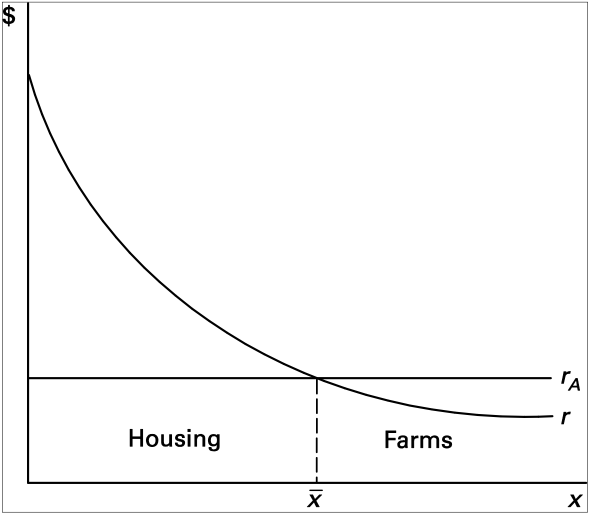
Figure 2.10 Logical structure of the model



Intercity Predictions

- So far, only analyzed diffenrential behaviors across; what about the aggregate? Population, city boundary?
- Assume an outside option of land, e.g., land revenue from agricultural production. Denote such value as r_A.
- The intersection between the land rent curve, $r(x)$, and r_A determines the city boundary $\bar{x}$. See Figure 2.11.
- Suppose an exogenously given city population L. The housing production must be able to meet the housing demand by L.
- Supply of housing = Demand of housing. Adjustments of $p(x)$ and $r(x)$ must be so that there is no excess demand (or supply) of housing. (Later will use a mathematical model to formalize this.)

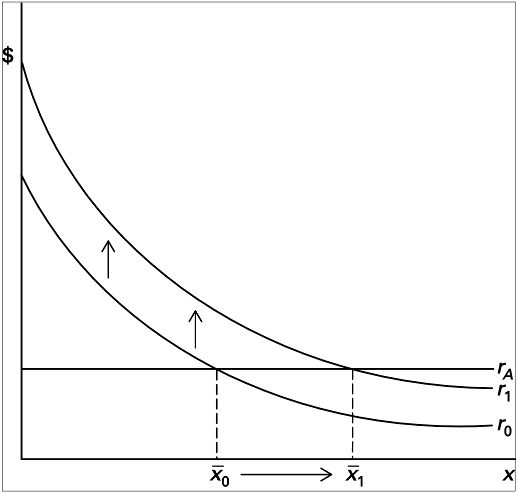
Figure 2.11 Determination of city's edge



Intercity Predictions

- Now that we can determine equilibrium, the model is complete.
- Comparative statics: L, r_A, t, y. Such results can be interpreted as intercity differences.
- Comparative statics of L: (suppose L increases; comparing big and small city, e.g., Peoria and Chicago)
  1. Housing market has excess demand.
  2. $p(x)$ increases at all x. (Can we prove this?)
  3. $q(x)$ decreases at all x. (substitution effect)
  4. $r(x)$ increases at all x.
  5. Taller buildings at all x because $r(x)/i$ increases.
  6. Point 3 and 5 give higher $D(x)$ at all x.
  7. Point 4 implies that $\bar{x}$ increases.
  8. Point 6 and 7 imply that city housing accomodates more people, eliminating the excess demand.

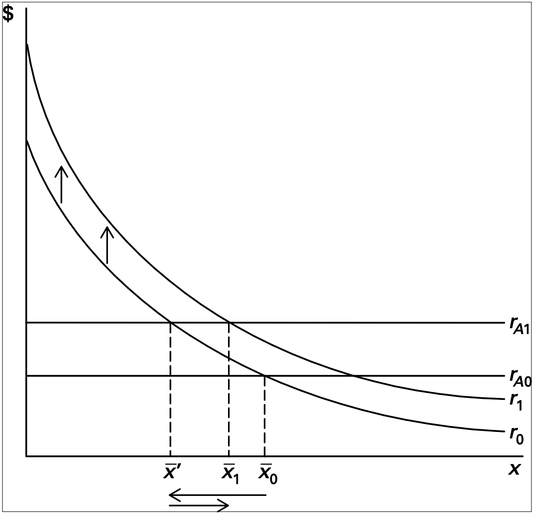
Figure 2.12 Effect of a higher L



Intercity Predictions

- Comparative statics of r_A: (suppose r_A increases; comparing Peoria, IL and Tuscon, AZ)
  1. Shortage in land for housing.
  2. $p(x)$ increases at all x within $\bar{x}'$.
  3. $q(x)$ decreases at all x within $\bar{x}'$. (substitution effect)
  4. $r(x)$ increases at all x within $\bar{x}'$.
  5. Taller buildings at all x because $r(x)/i$ increases.
  6. Point 3 and 5 give higher $D(x)$ at all x.
  7. Point 4 implies that $\bar{x}$ increases to $\bar{x}_1$.(See Figure 2.13).
  8. Point 6 and 7 imply that city housing increases to accomodates more people, eliminating the excess demand.
  9. $\bar{x}_1 < \bar{x}_0$ because now city is denser (point 6).

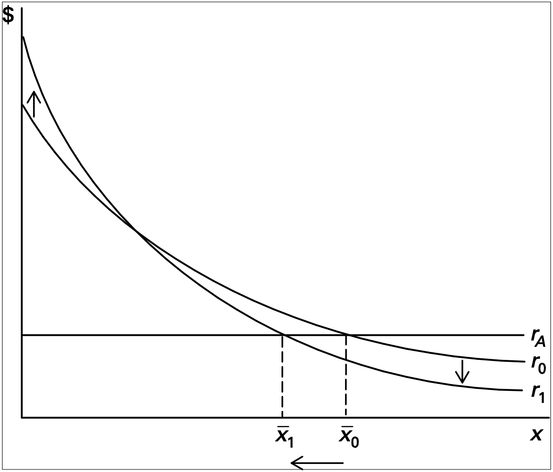
Figure 2.13 Effect of a higher rA



Intercity Predictions

- Comparative statics of t: (suppose t increases; comparing high and low gasoline costs, e.g., US and Europe)
  1. Income effect: decreases in both c and q, but the effect is stronger for large x. Given prices $p(x)$ at each x, $u(x)$ drops more for larger x, thus creating incentives for people to move toward the CBD. (See Figure 2.2).
  2. $p(x)$ rises at x close to the CBD but falls at x at suburbs.
  3. $q(x)$ decreases at x close to the CBD (substitution effect + income effect). But, the effect on $q(x)$ for large x is ambiguous, because the two effects conflicts.
  4. $r(x)$ follows the same pattern as $p(x)$.
  5. Taller buildings at x close to the CBD, shorter builiding at x large.
  6. Point 3 and 5 give higher $D(x)$ at x close to the CBD. Changes in $D(x)$ for large x are ambiguous.
  7. Point 4 implies that $\bar{x}$ increases.
  8. Point 6 and 7 imply that city housing accomodates more people, eliminating the excess demand.

Figure 2.14 Effect of a higher t



Migration

- Comparative statics of y: (suppose y increases; comparing high and low income cities; think about productivities.....)
- "Mathematical analysis" show that the effects are exactly opposite of that of t. Can we show this?
- Migration between cities: So far, we deal with "closed city model," in which each city is examined in isolation and no migration is allowed (and cities are not adjacent to each other).

Open City Model

- Open city model: allow migration.
- E.g., cities with large y are better off than cities with small y. (productivity/wage is location-specific).
- That attracts migration from the low-y cities to high-y cities.
- The population in large-y cities keep increasing until the increase in $p(x)\ \bar{x}$ (hence the drop in $q(x)$ and increase in commuting costs) offsets the utility differential.