Cities in Space

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For EC 3381 Urban Economics
January, 2012

Model

- **In Chapter 1, we have seen how the tradeoff between scale economies and transport cost determines the optimal production location. Here, we put this tradeoff in a continuous space to see that how the number/spacing of cities are determined by this tradeoff.**
- Looking at the role of cities, how they are spaced, etc....
- The geographic space is the real line $\mathbb{R}$.
- An infinite mass of consumers is uniformly distributed with a density of d.
- We can think of these consumers as farmers who would locate themselves uniformly if agricultural productivity were uniform all over the plane and if the farming technology were Leontief in land and labor.
- Or, think of them as smaller cities to be served by the production of big cities.

Model

- There is a continuum of commodities labeled $y \in [0, z]$, where z is exogenously given.

- Each consumer demands one unit of each $y \in [0, z]$. To produce goods $[0, z]$, total fixed cost is $\Phi(z)$, with $\Phi'(z) > 0$ and $\Phi''(z) > 0$.

- This can be justified by assuming a fixed cost $\phi(y)$ is required for each $y \in [0, z]$, $\phi'(.) > 0$, $\Phi(z) = \int_0^z \phi(y)\, dy$.

- The marginal cost is a constant c.

- To transport any good requires a cost of t per unit of distance.

- **The presence of fixed cost of production implies that scale economies exist.**

Social Planner's Problem

- We can ignore variable cost. So, only fixed cost $\Phi(z)$ and transport cost matters.

- Social planner decides how to place these cities.

- Given the real line $\mathbb{R}$ and the uniform distribution of consumers, cities must be evenly spaced throughout the entire real line.

- Given uniform distribution of consumers, evenly spacing z-cities must be optimal.

- Let this distance be ℓ. The transport cost for each good is

$$2\int_0^{\frac{\ell}{2}} tx dx = \frac{t\ell^2}{4}.$$

Social Planner's Problem

- The social planner's objective is to minimize per capita cost by choosing an optimal ℓ:

$$\min_{\ell} \frac{1}{d\ell}\left[\Phi\left(z\right)+\frac{zdt\ell^2}{4}\right].$$

- First-order condition

$$-\frac{\Phi\left(z\right)}{d\ell^2}+\frac{zt}{4}=0.$$

- Solution:

$$\ell^*=2\sqrt{\frac{\Phi\left(z\right)}{ztd}}.$$

- Check second-order condition.

- Comparative statics of z, d, t on ℓ^*.

- $1/\ell^*$ measures the number of cities per unit distance.

Possibilities of City Hierarchy?

- Assume the *hierarchy property*: at any location, if z is produced, then all $y \in [0, z]$ are also produced.
- Consider $z' < z$. Place the z'-city in the middle between any two neighboring z-cities.
- Decide whether there are savings on per capita cost by having these z'-cities? (Problem Set 2).
- Central place theory......