# Chapter 3: Modifications of the Urban Model (Brueckner's textbook)

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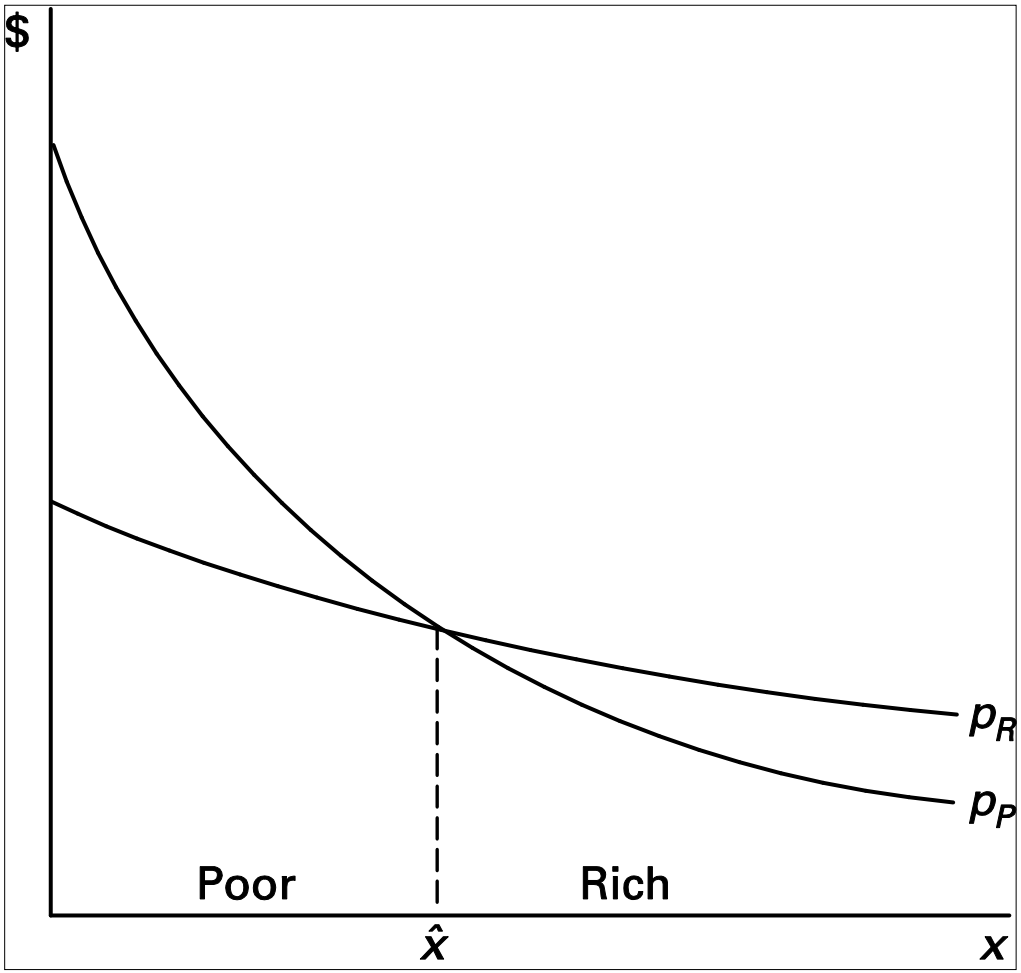
# Urban Spatial Structure

- In this chapter, we relax several assumptions (one at a time) made in Ch. 2 to see the implications on urban spatial structure.
  - Income difference
  - Adds a freeway to the city's transportation system (more than radial roads). We skip this section.
  - More job sites.
  - Durability of housing
  - Developing country and rural-urban migration.

# A City with Two Income Groups

- Two groups: an income of $y_R/y_P$ for each individual in the *rich/poor* group, and $y_R > y_P$.
- In the US, richer households tend to live in suburbs, while the poorer live close to city center. Is this true in the monocentric city model?
- Yes, and indeed, the pattern of housing prices is given in Figure 3.1. But, why?
- $p_P$ is the housing price curve of the poors, i.e., this depicts the willingness to pay (bid) that makes every individual with income $y_P$ indifferent. Similarly interpretation for $p_R$.

# Figure 3.1 Locations of rich and poor



# A City with Two Income Groups

- Recall housing price gradient:

$$\frac{\partial p(x)}{\partial x} = -\frac{t}{q(x)}.$$

- In general, $q_R(x) > q_P(x)$ because group $R$ has higher income. So, rich's housing price gradient is flatter because.

$$\frac{t}{q_R(x)} < \frac{t}{q_P(x)}$$

- But, the statement "$q_R(x) > q_P(x)$ because group $R$ has higher income" is not precise, because dwelling size also depends on housing price $p(x)$.

# A City with Two Income Groups

- The two curves must intersect. Why?
- At the point of intersection, called $\hat{x}$, $p_R(\hat{x}) = p_P(\hat{x})$. This implies that $q_R(\hat{x}) > q_P(\hat{x})$. Thus, at least at the intersection, rich's housing price gradient is flatter.
- The last two points imply that the entire housing price curve for the rich is flatter than that for the poor, because they only intersect once (why?), as illustrated in Figure 3.1.

# A City with Two Income Groups

- Does this make sense?
- Poor here is simply those with small $y$.
- What if commuting cost also includes time cost?
  - Suppose time is split between working and commuting (ignore leisure for simplicity), and wages are such that $w_R > w_P$. Let the amount of time each individual has be normalized to 1. Suppose the time spent on commuting one mile is $\delta$. Then, disposable income for the rich is $(1 - \delta x) w_R - tx = w_R - (\delta w_R + t) x$. So, the commuting cost for the rich is in fact $m_R = \delta w_R + t$. Similarly, $m_P = \delta w_P + t$. So, compare the housing price graident
$$\frac{m_R}{q_R(x)} < \frac{m_P}{q_P(x)}?$$
  - The pattern become ambiguous. So, no decisive prediction from monocentric city model.

# A City with Two Income Groups

- Other explanations for the pattern found in the US?
- Commuting mode....poor relies on public transit. (Glaeser, Kahn, and Rappaport 2008)
- Older housing in city center....*rich prefers new housing*...(Brueckner and Rosenthal 2009)
- But, this pattern is not everywhere true...think of Paris, or even Singapore?, if the rich values amenity and amenity is abundant at the center...such as Paris...(Brueckner, Thisse, and Zenou 1999).

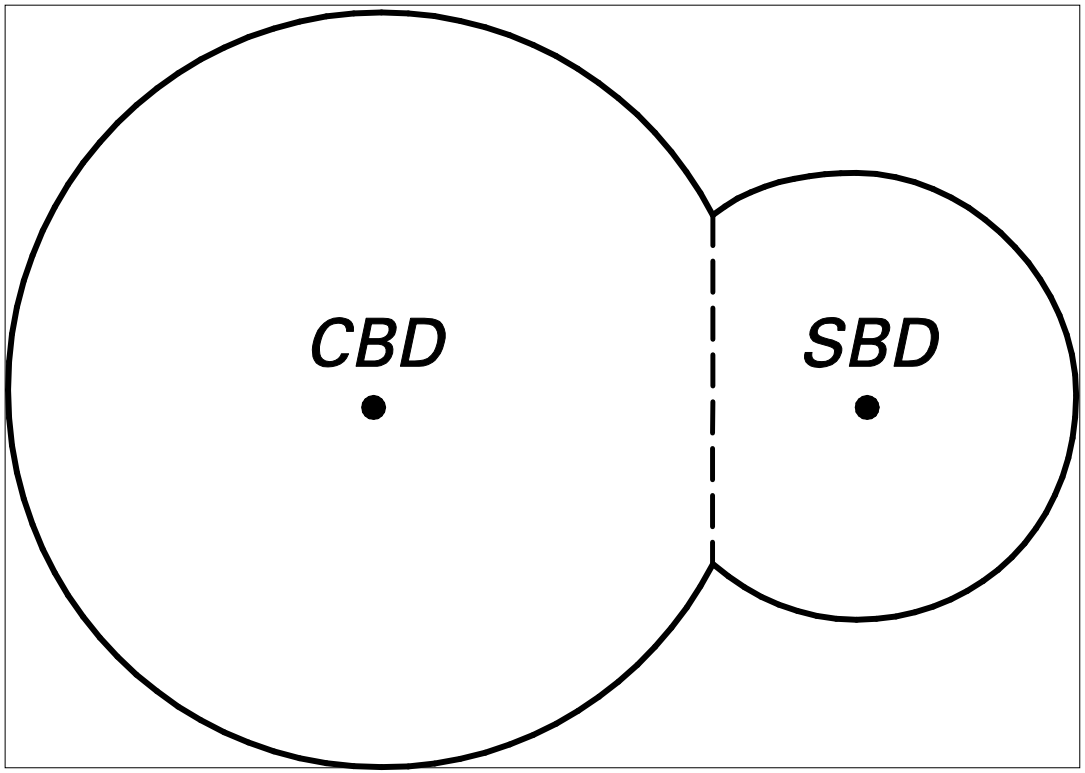
# Adding Employment Outside the CBD

- Case 1: additional jobs may be widely dispersed throughout the city, or
- Case 2: additional jobs concentrated at secondary employment center..
- Case 1: Suppose a worker living at $x^*$ from CBD, and an employer's location is $x^{**}$ distance from the CBD and on the worker's route to the CBD.
  - This employer would be able to offer a job and lure him by paying him $y - tx^{**}$.
  - $y - tx^{**} - t\left(x^* - x^{**}\right) = y - tx^*$, leaving the worker indifferent....in practice...just need to be a little higher.
  - Then, the disposal income at any individual taking jobs outside the CBD is the same as if he/she takes it from the CBD. All Ch. 2 predictions go through.

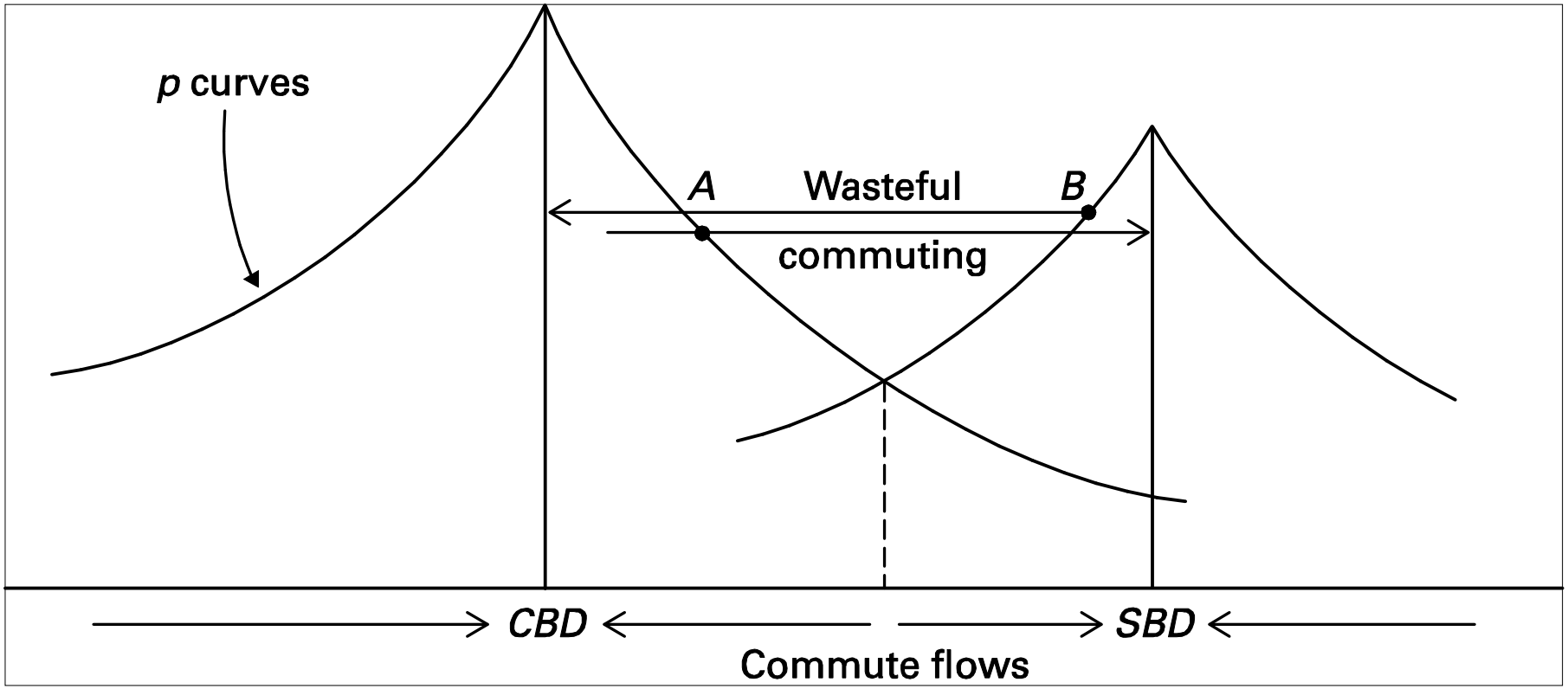
# Adding Employment Outside the CBD

- Case 2: Secondary Business District (SBD).
  - The analysis of the "polycentric city" is as straightforward as in Figure 3.3 and 3.4.
  - The pricing curve from the CBD will be higher if CBD offers a higher income than what is offered in SBD. This will result in the boundary being further away from CBD than from SBD. (What is Singapore's SBD?)
  - One thing it doesn't predict is the wasteful commuting that is often observed.
- It is puzzling? (Why not switch?)
- Maybe not so puzzling:
  - idiosyncratic preference on local amenity
  - multiple worker households.....if the data only report the head's work place, it may miss out the fact that the location might be determined by the spouse's work location, which is closer.

# Figure 3.3 Subcenter



# Figure 3.4 Wasteful commuting



# Durable Housing Capital

- Housing is durable. In Ch. 2, the model was a "one-shot": lacking the time dimension.
- In reality, the building heights pattern is not all that smooth as predicted by the basic model.
- Now, suppose that housing is durable.

# Durable Housing Capital

- Suppose the city grows outward over time by a fixed distance each year, adding "rings" as a tree does.
- Each ring is referred to as a "block," and years are denoted by $T$.
- The city comes into being at $T = 0$, thus adding its first block at $T = 0$.
- New buildings are constructed in a block when it is added to the city, and they are held for a fixed amount of time before being replaced.
- Say, the life span of a building is set at 3 years. The building constructed at $T = 0$ is replaced at $T = 3$, that constructed at $T = 1$ is replaced at $T = 4$, etc. See the age pattern in Table 3.1.

# Table 3.1 Building-age contours

| $T = 2$ | | $T = 3$ | | $T = 8$ | |
|---|---|---|---|---|---|
| Block | Age | Block | Age | Block | Age |
| 0 | 2 | 0 | 0 | 0 | 2 |
| 1 | 1 | 1 | 2 | 1 | 1 |
| 2 | 0 | 2 | 1 | 2 | 0 |
| | | 3 | 0 | 3 | 2 |
| | | | | 4 | 1 |
| | | | | 5 | 0 |
| | | | | 6 | 2 |
| | | | | 7 | 1 |
| | | | | 8 | 0 |

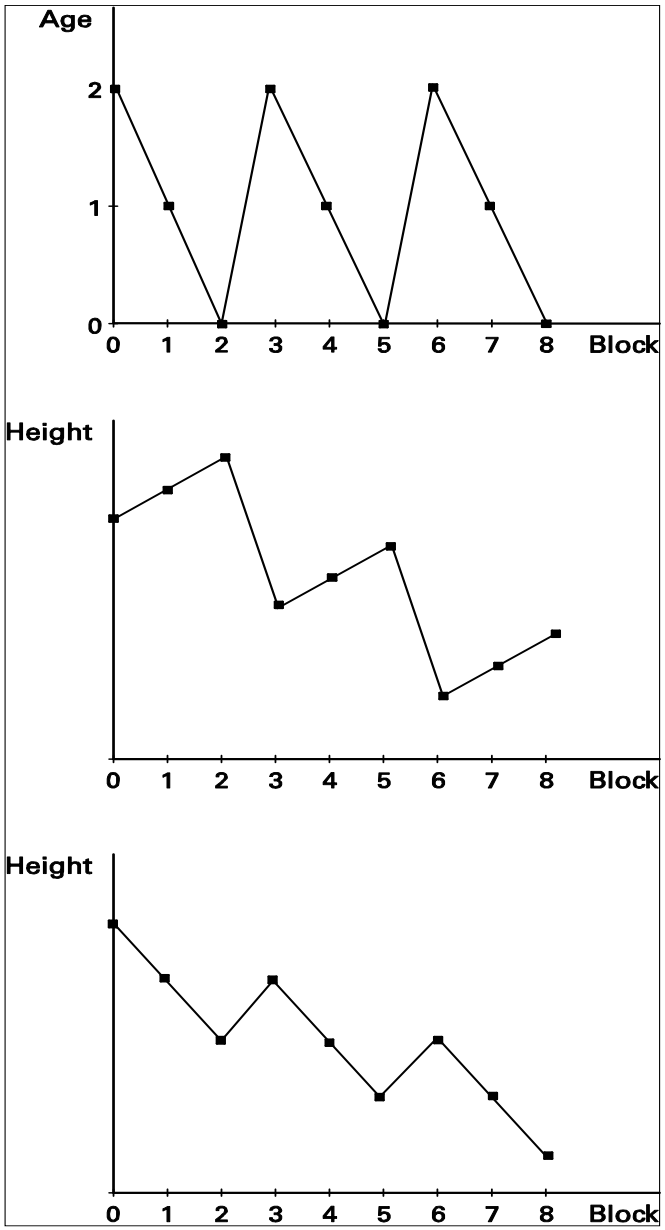
# Durable Housing Capital

- For buildings constructed at a given date, those farther away from the CBD will be shorter. (partial effect of distance as from the basic model).
- Suppose that for buildings constructed at a given location, those constructed later are taller. (partial effect of age.) Why? Is it necessary?
- Consider the spatial pattern of building heights at $T = 8$. Look at block $0, 1$, and 2. Block-0 buildings are older than those at block-1, but block-1 is farther away from the CBD. Two partial effects conflicts. Generally ambiguos.
- If the age effect dominates, then we may see a pattern in the middle panel of Fig. 3.5.
- From block 2 to 3, both effects work in the same direction, resulting in a sharp decrease in building height. This patter repeats.
- Note it is necessary that block 5 is shorter than block 2. (Why?) Despite a "sawtooth" pattern, the general trend of basic model still holds.

# Durable Housing Capital

- What if age effect such that later constructed buildings are shorter.
- Then, over block 0 to 2, age effect and distance work in the same direction.
- Between block 2 and 3, building height may increase or decrease. If age effect dominates, then block 3 is taller. The pattern repeats, as seen in the bottom panel of Fig. 3.5.

# Figure 3.5 Patterns of building age and building height



# Cities in Developing Countries

- In Ch. 2, talked about open cities where people migrate across cities.
- Now, consider rural-urban migration, which is still ongoing in developing countries.
  - In China, the current urbanization rate is still less than 60%, while in US and Japan, this rate has well exceeded 80%.
- Singapore is a city state, but even the immigration from other South-east or South Asian countries can be considered using this angle.
- Suppose a single open city (open meaning population $L$ is endogenous) and a very large pool of rural population whose per capita income is $y_A$.
- From the basic model in Ch. 2, city edge $\bar{x}$ strictly increases in $L$. Denote this dependence as $\bar{x} = \bar{x}(L)$.

# Cities in Developing Countries

- A rural resident only needs to compare with the urban resident living at the city edge. (Why?)
  - Better yet, the comparison can be done by comparing disposable income, because the housing price they face is the same. (Why?)
- Suppose rural resident has no need to commute. Given the city population $L$,
  - if $y - t\bar{x}(L) > y_A$, rural residents migrate to the city, increasing $L$.
  - if $y - t\bar{x}(L) < y_A$, some urban residents move to the country side, decreasing $L$.
  - Equilibrium attains if $y - t\bar{x}(L) = y_A$.
- Effect of $y_A$. Low $y_A$ induces large rural-urban migration.

# Cities in Developing Countries

- Effect of $y$ and $t$? As $y$ affects $\bar{x}$ directly and also affect $\bar{x}$ through its effect on $L$ indirectly. So, the analysis is complicated. Simiarly, for $t$. Such analysis indicates that equilibrium $L$ strictly increases in $y$ and $t$.

- Go with a simpler model:

  - Suppose fixed dwelling size and fixed height at all locations in the city.
  - Population density is then fixed at $\theta$. Let $\mu = 1/\theta$. Then, $\mu$ is actually the land use per person. (Note the typo in the textbook.)
  - Equilibrium in housing:
  $$\mu L = \pi \bar{x}^2.$$
  - So, in equilibrium,
  $$y - t\sqrt{\frac{\mu L}{\pi}} = y_A.$$
  - So, if $y$ increases, then equilibrium $L$ must also increase. Similar for $t$.

# Cities in Developing Countries

- High unemployment rate in cities in developing countries. Then, why move to cities?
- Harris and Todaro (1969). It is the expected income that matters.
- Suppose the number of jobs in the city is fixed at $J$.
- The probability of landing a job is $J/L$. (Suppose everybody has equal chance, for simplicity).

# Cities in Developing Countries

- Suppose unemployment results in zero income. (The result won't change if we assume a significantly low unemployment benefit).

- Then, in equilibrium,

$$\frac{yJ}{L} - t\sqrt{\frac{\mu L}{\pi}} = y_A.$$

  - Larger $L$ reduces disposal income in the city because of longer commute, and this Harris-Todaro effect reduces even further by reducing the chance of landing a job.

  - When $y$ increases, $t$ decreases or $y_A$ decreases, equilibrium $L$ rises. Same prediction as without this Harris-Todaro effect.

  - When $J$ increases, $L$ increases, i.e., more rural-urban migration.