Econ 4621—1/22/04

Moncentric Model of the City

- Explicit model of the internal structure of a city

- Developed in 1960s by Alonzo and Mills

- Monocentric structure is at odds with current structure of cities.

But...

- Can use the model to understand what changed

- Model remains useful abstraction for understanding the interplay of important forces

- Can use the model as a vehicle for understanding *qualitative* impacts of policy changes (e.g. *urban growth boundaries*)

- The building blocks of the model are very much alive in cutting edge research (e.g. Lucas and Rossi-Hansberg, *Econometrica*, 2002)

Model (corresponds to residential model in text, p. 177)

- Land is line, CBD is at zero.

- H is number of individuals. All are identical

- All individuals work in the CBD

- u a location, a distance u from CBD.

- w is wage

- t is commuting cost per unit distance, *dollars* per mile (so not in time units)

- $U(x, L)$ is utility function

—x is widgets, non-land consumption

—L is land consumption

—Assume utility function has usual properties (e.g. quasi-concave....)

- $\overline{R}$ is agricultural land value (in dollars)

- p is price of a widget in dollars (exogenous)

Equilibrium (rough, more detail later)

- $R(u)$: price of land at distance u from CBD

- $D(u)$: population density at u

- Supply equal demand for land

Consumer Problem

- Consumer takes as given $R(u)$ and p

- Chooses u, x, and L to maximize utility

$$\begin{aligned} \max_{(u,x,L)} U(x,L) \quad &: \\ \text{such that } px + R(u)L + tu &= w \end{aligned}$$

- Two steps.

(i) pick u

(ii) given u, pick x and L

- Goal of derivation. We get the following equation:

$$R'(u) = -\frac{t}{L(u)}$$

or

$$\begin{aligned} R'(u) * L(u) &= -t \\ MB &= MC \end{aligned}$$

Budget Constraint

- $px + R(u)L = w - tu$

- Graph

- Result: marginal rate of substitution condition (MRS)

$$\frac{U_x}{U_L} = \frac{p}{R}$$

- $U^*(u)$ Maximized utility given location u

$$U^*(u) = U(x^*(u), L^*(u))$$

- In equilibrum, $U^*(u)$ must be constant in u. (Individuals must be indifferent where to live.)

$$\frac{dU^*(u)}{du} = U_x \frac{dx^*}{du} + U_L \frac{dL^*}{du} = 0$$

- Using MRS condition,

$$p\frac{dx^*}{du} + R\frac{dL^*}{du} = 0.$$

- Next take the budget constraint

$$px + R(u)L + tu = w$$

and differentiate with respect to u

$$p\frac{dx^*}{du} + R'(u)L + R\frac{dL^*}{du} + t = 0$$

yields

$$R'(u)L + t = 0$$

or

$$R'(u) = -\frac{t}{L}.$$

which is what we wanted to show.