Chapter Fourteen

Consumer’s Surplus

Monetary Measures of Gains-to-Trade

- You can buy as much gasoline as you wish at \$1 per gallon once you enter the gasoline market.
- Q: What is the most you would pay to enter the market?

Monetary Measures of Gains-to-Trade

- A: You would pay up to the *dollar value* of the gains-to-trade you would enjoy once in the market.
- How can such gains-to-trade be measured?

Monetary Measures of Gains-to-Trade

- Three such measures are:
  - Consumer’s Surplus
  - Equivalent Variation, and
  - Compensating Variation.
- Only in one special circumstance do these three measures coincide.

Consumer's Surplus

- Suppose gasoline can be bought only in lumps of one gallon.
- Use r_1 to denote the most a single consumer would pay for a 1st gallon -- call this her reservation price for the 1st gallon.
- r_1 is the dollar equivalent of the marginal utility of the 1st gallon.

Consumer's Surplus

- Now that she has one gallon, use r_2 to denote the most she would pay for a 2nd gallon -- this is her reservation price for the 2nd gallon.
- r_2 is the dollar equivalent of the marginal utility of the 2nd gallon.

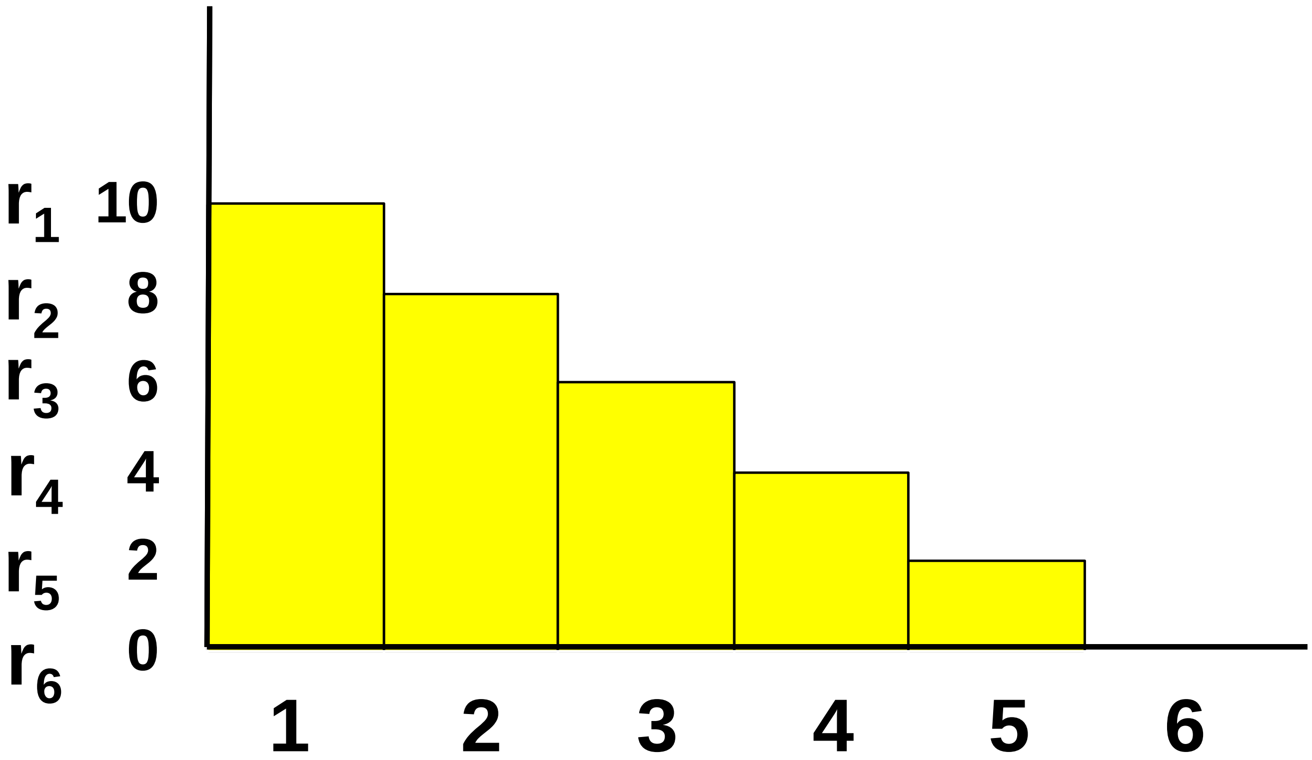
Consumer's Surplus

- Generally, if she already has n-1 gallons of gasoline then r_n denotes the most she will pay for an nth gallon.
- r_n is the dollar equivalent of the marginal utility of the nth gallon.

Consumer's Surplus

- $r_1 + \ldots + r_n$ will therefore be the dollar equivalent of the total change to utility from acquiring n gallons of gasoline at a price of \$0.
- So $r_1 + \ldots + r_n - p_G n$ will be the dollar equivalent of the total change to utility from acquiring n gallons of gasoline at a price of $\$p_G$ each.

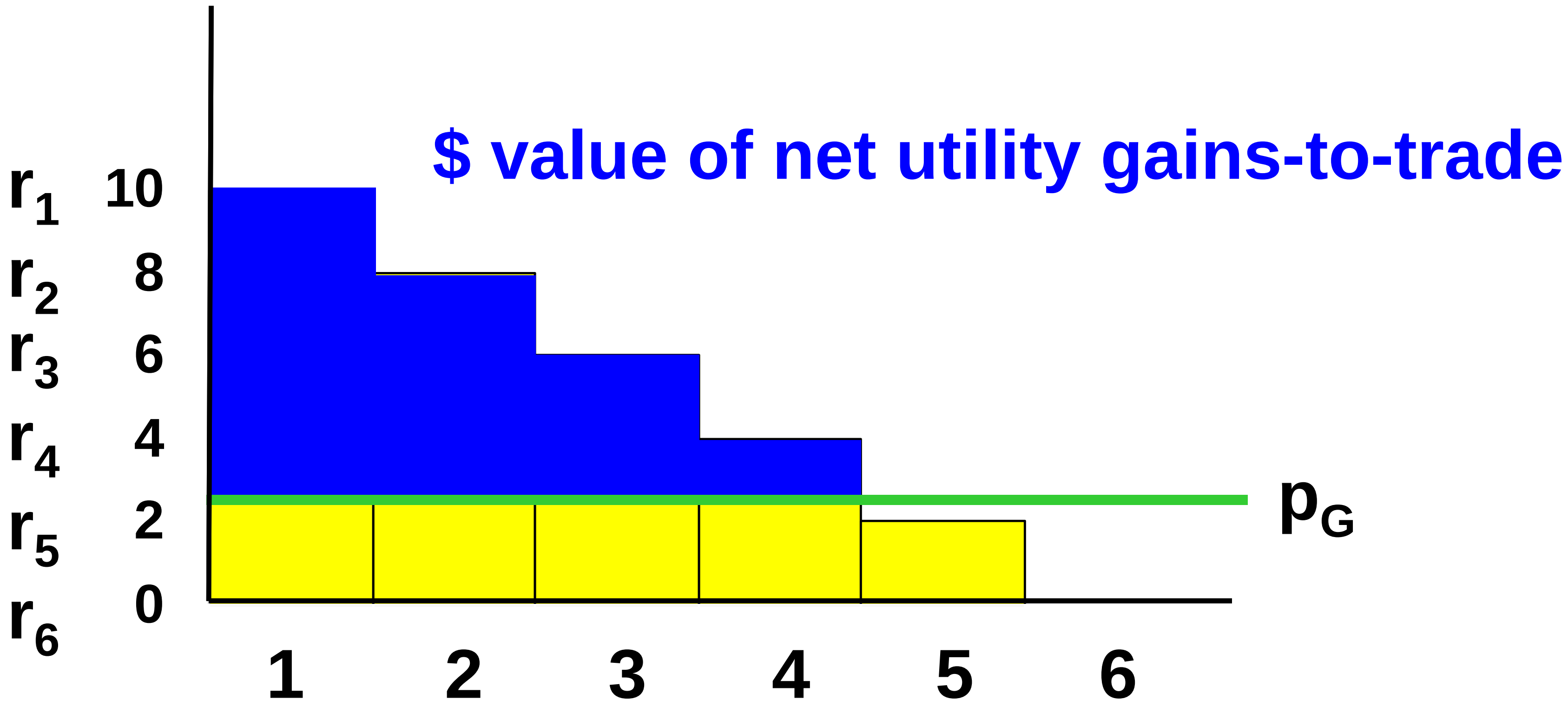
Gross Surplus



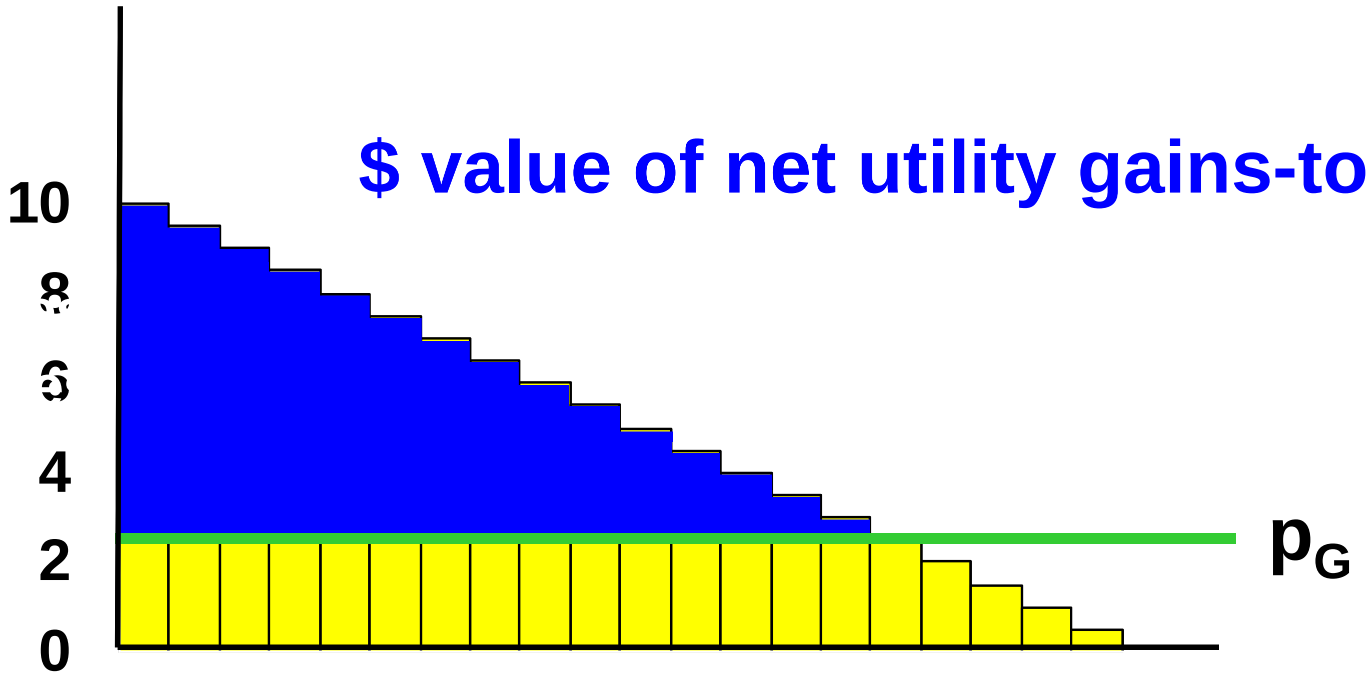
Net Surplus

- What is the monetary value of our consumer's gain-to-trading in the gasoline market at a price of \$$p_G$?

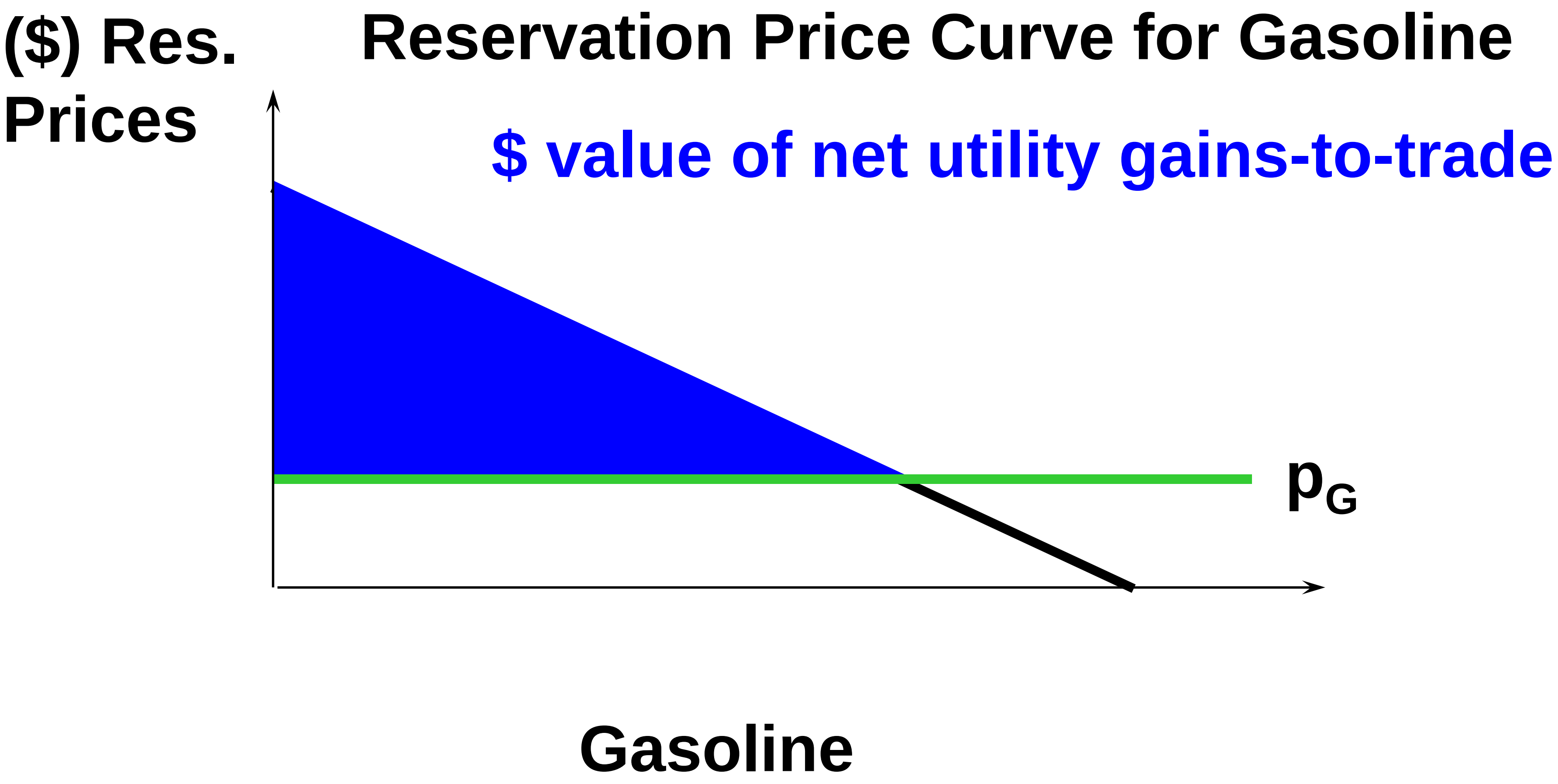
Consumer's Surplus



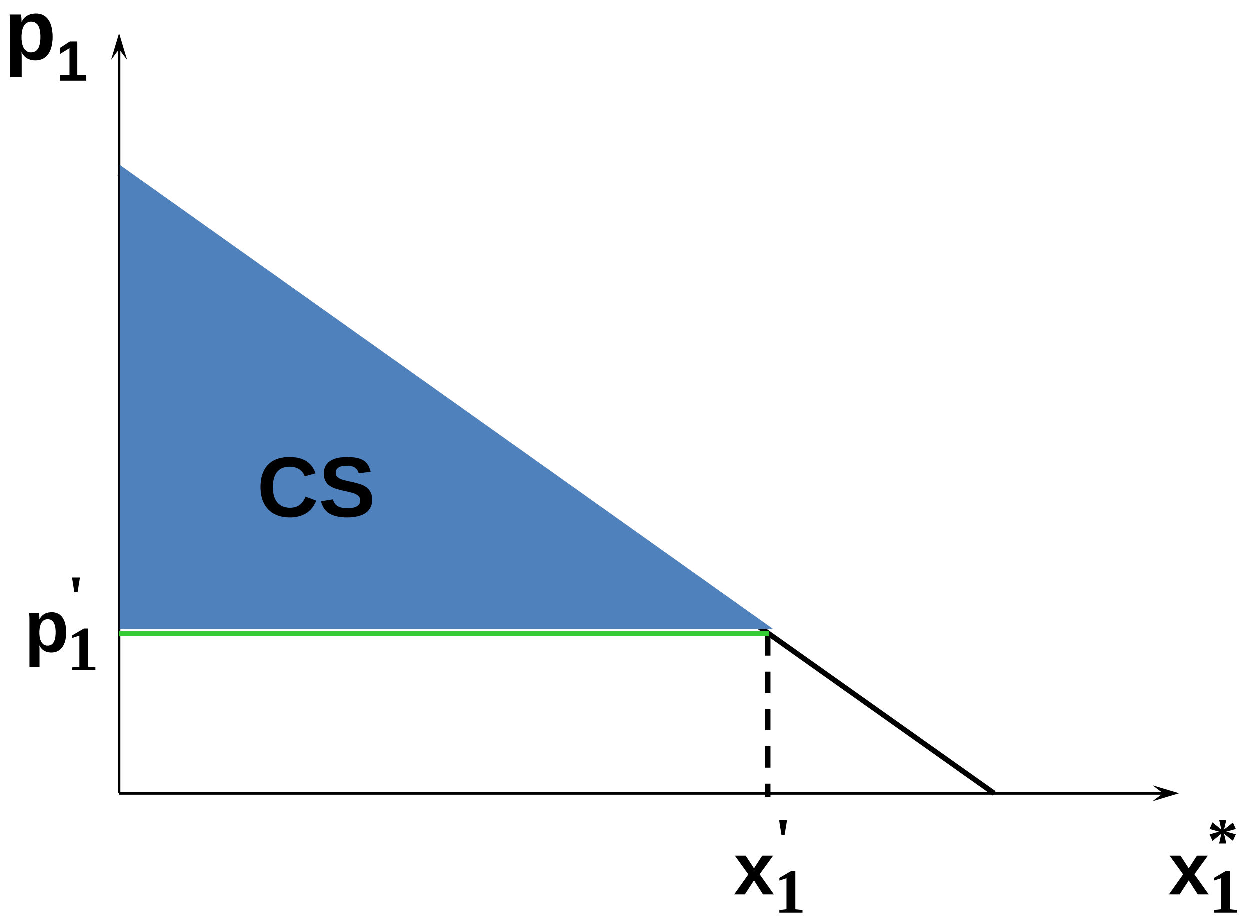
Consumer's Surplus



Consumer's Surplus



Consumer's Surplus under Ordinary Demand



Consumer's Surplus

- Consumer's Surplus is an exact dollar measure of utility gained from consuming commodity 1 when the consumer's utility function is quasilinear in commodity 2.
- Otherwise Consumer's Surplus is an approximation.

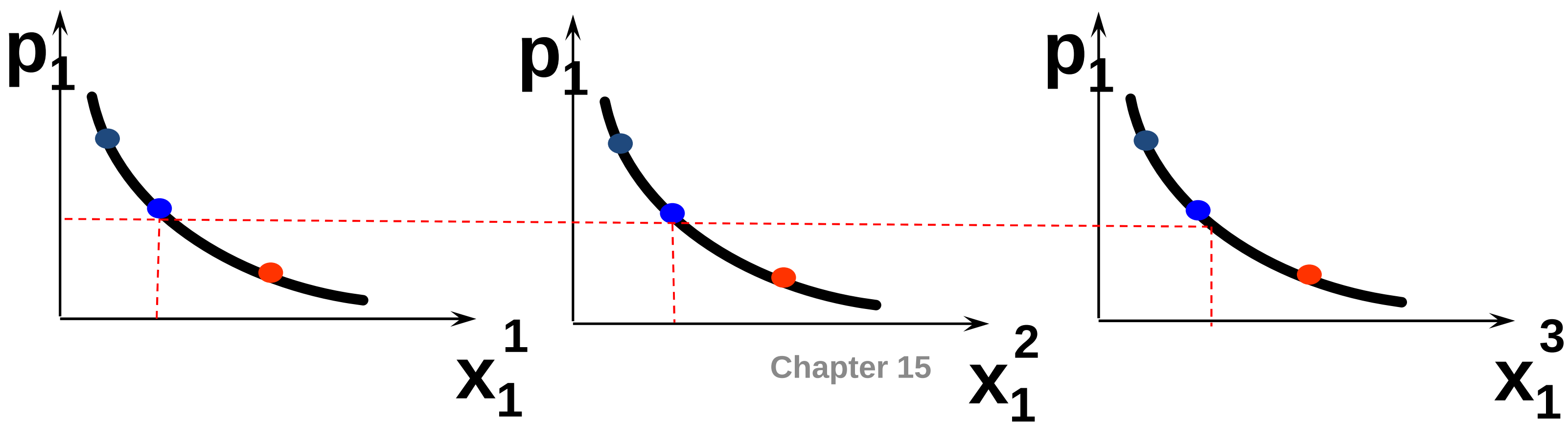
Chapter Fifteen

Market Demand

From Individual to Market Demand Functions

- Think of an economy containing n consumers, denoted by i = 1, … ,n.
- Consumer i’s ordinary demand function for commodity j is

$$x_j^{*i}(p_1,p_2,m^i)$$



From Individual to Market Demand Functions

- When all consumers are price-takers, the market demand function for commodity j is

$$X_j(p_1, p_2, m^1, \cdots, m^n) = \sum_{i=1}^{n} x_j^{*i}(p_1, p_2, m^i).$$

- If all consumers are identical then

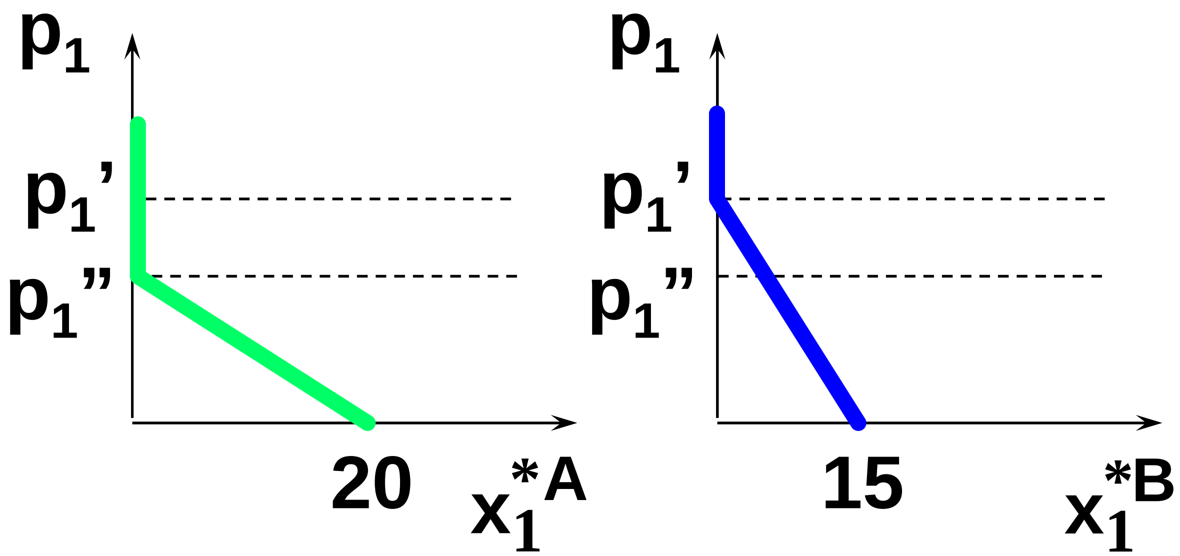
$$X_j(p_1, p_2, M) = n \times x_j^*(p_1, p_2, m)$$

where M = nm.

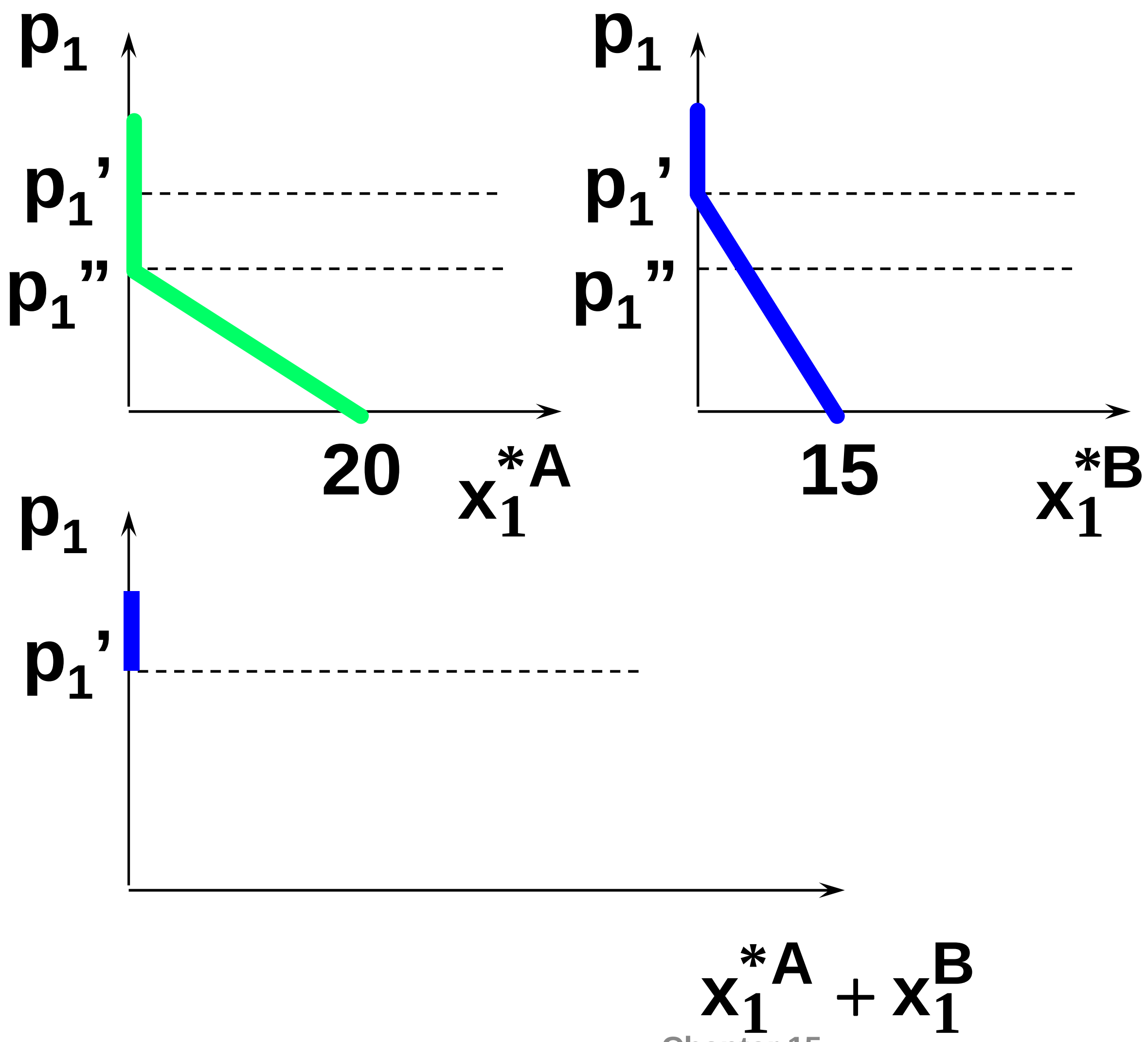
From Individual to Market Demand Functions

- The market demand curve is the “horizontal sum” of the individual consumers’ demand curves.
- E.g. suppose there are only two consumers; i = A,B.

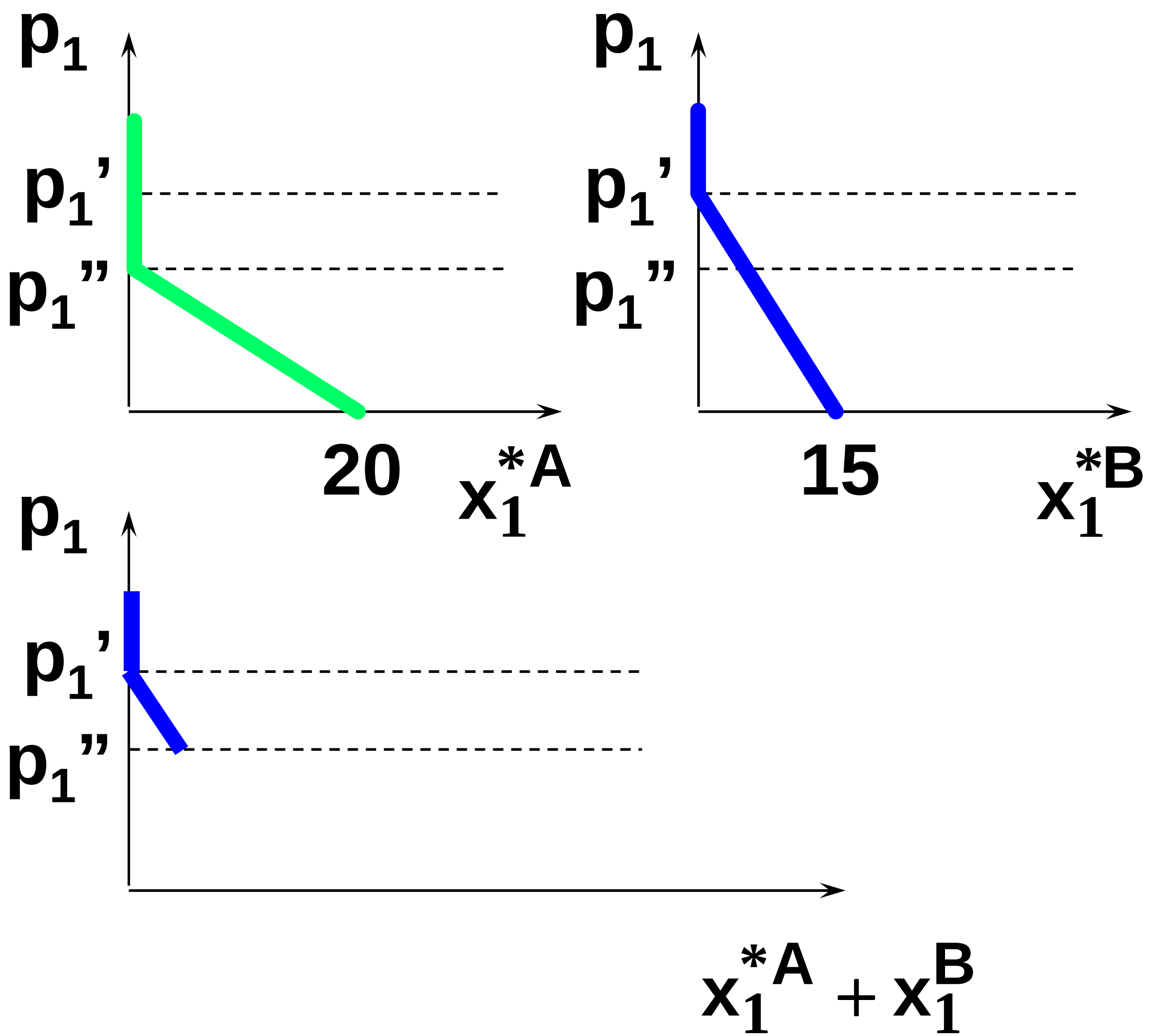
From Individual to Market Demand Functions



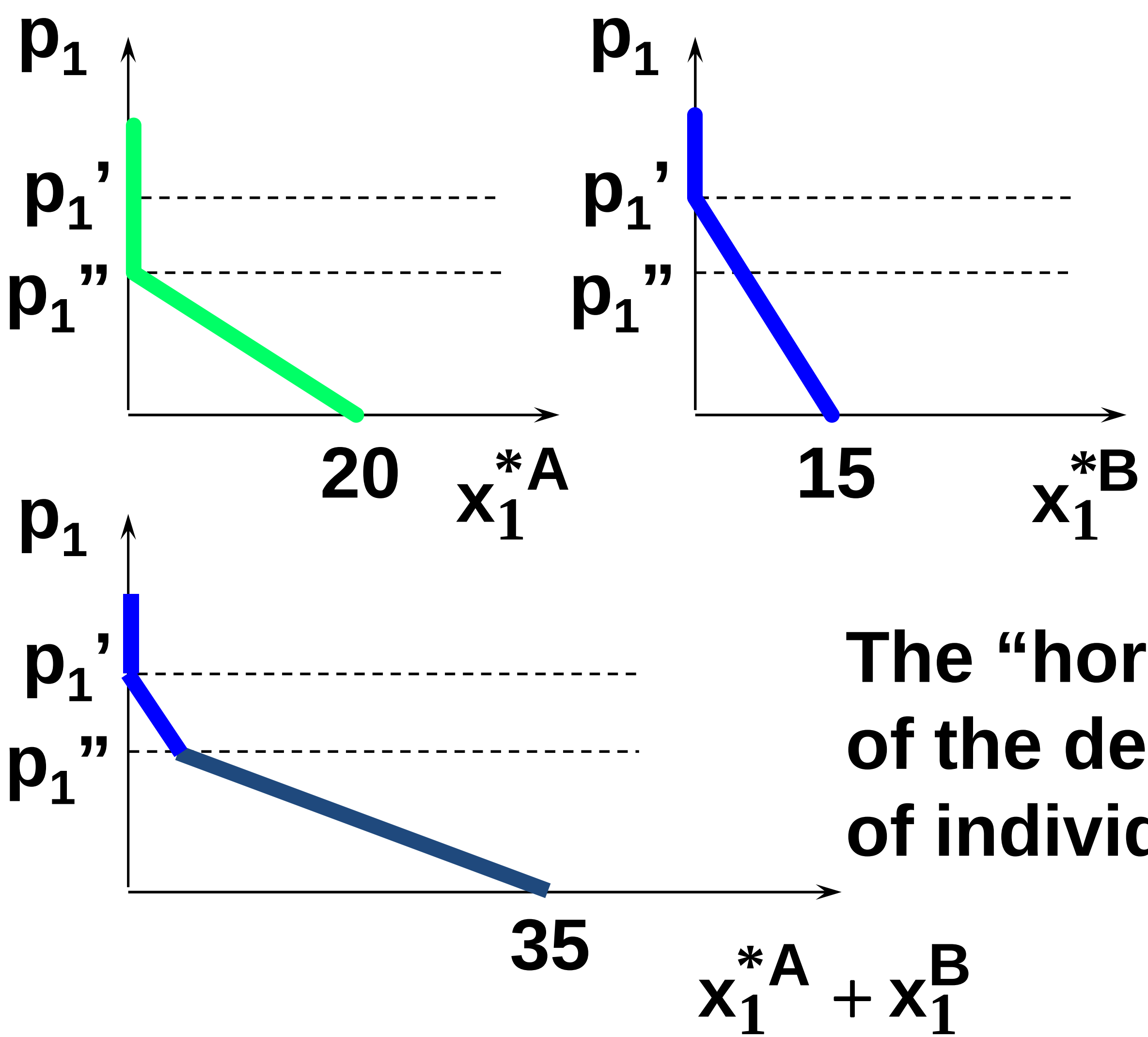
From Individual to Market Demand Functions



From Individual to Market Demand Functions



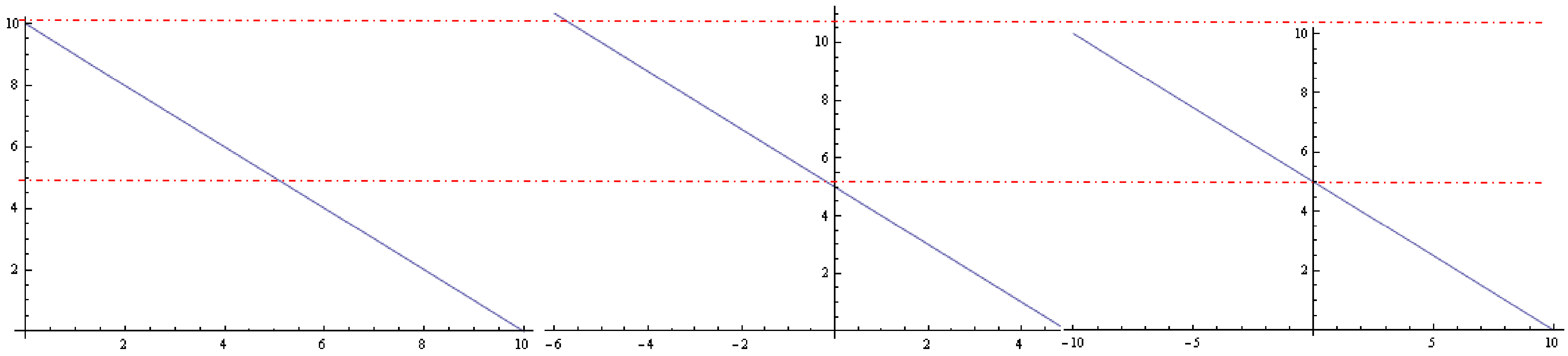
From Individual to Market Demand Functions



The "horizontal sum" of the demand curves of individuals A and B.

A numerical example

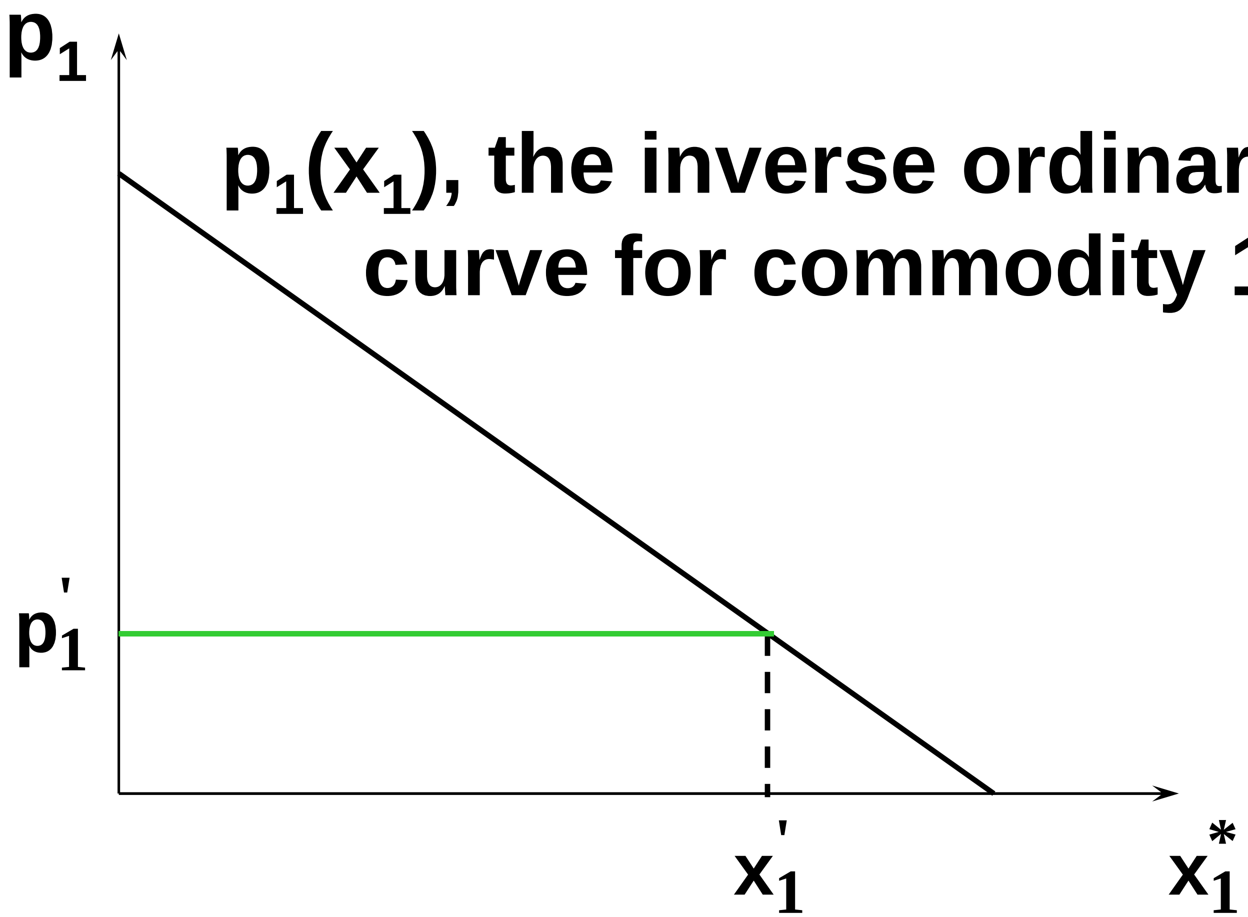
$$x_1^1(p) = 10 - p, \;\; x_1^2(p) = 5 - p, \;\; x_1^3(p) = 10 - 2p$$



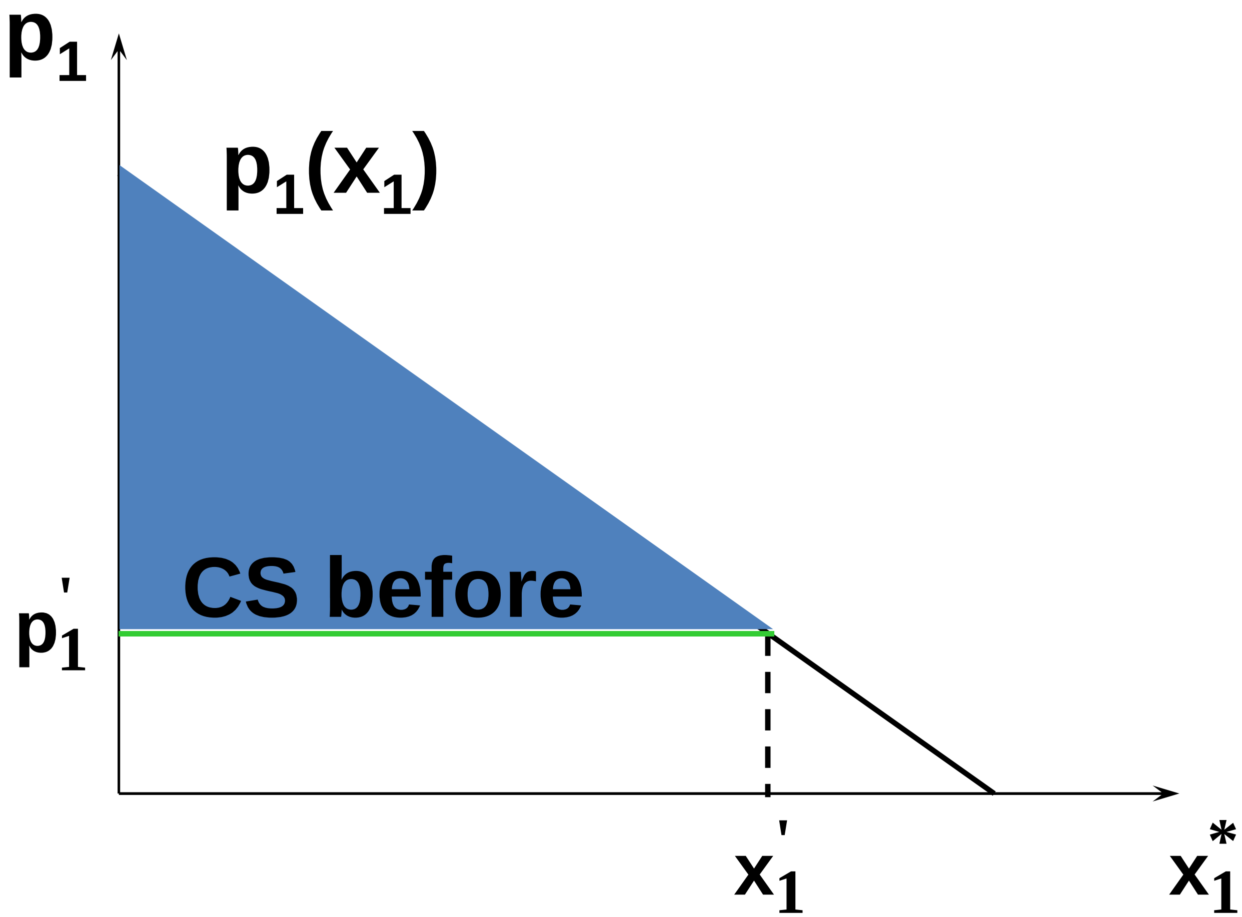
$$x_1(p) = 25 - 4p \quad for \;\; 0 \le p \le 5$$

$$= 10 - p \quad for \;\; 5 \le p \le 10$$

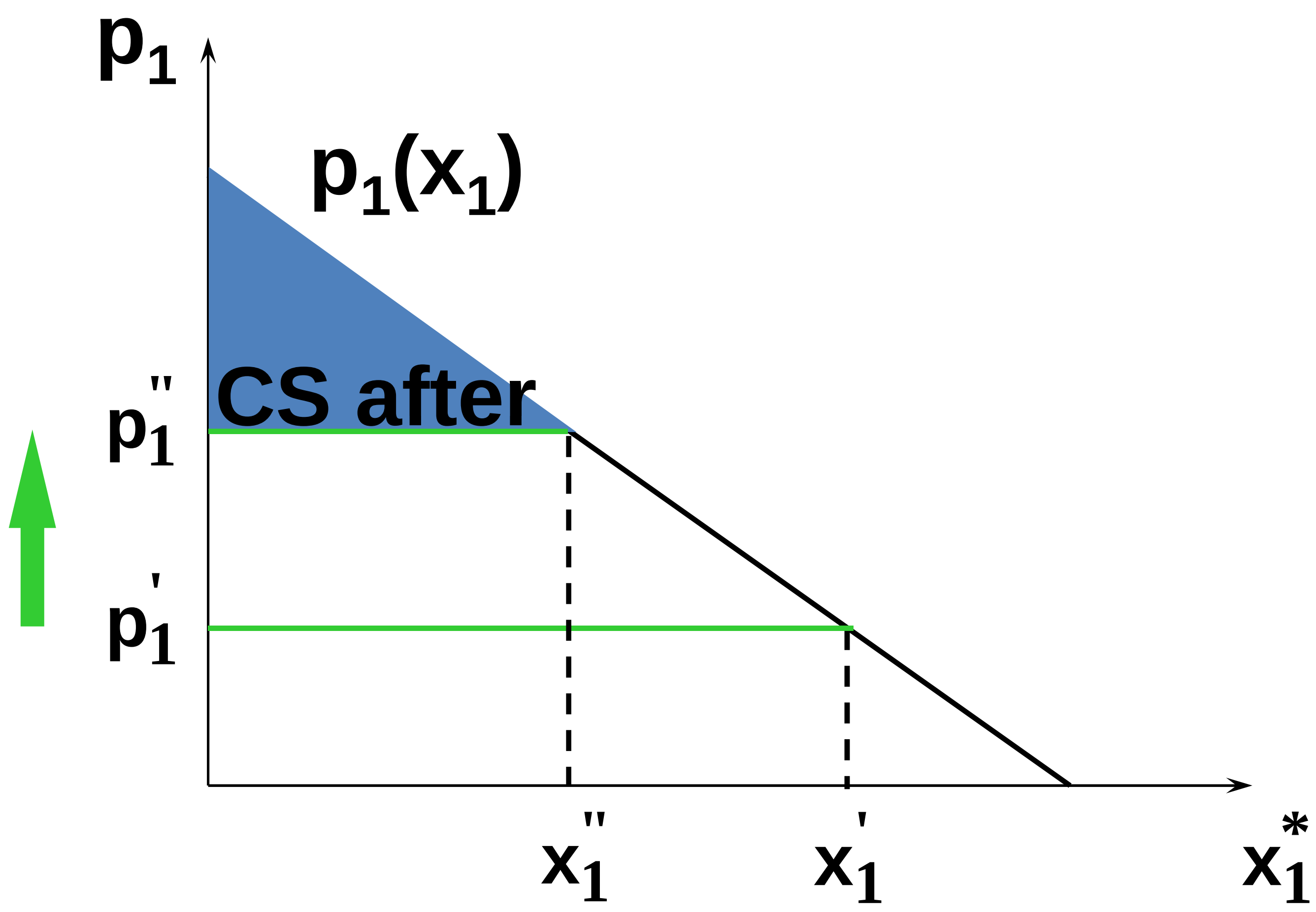
Consumer's Surplus



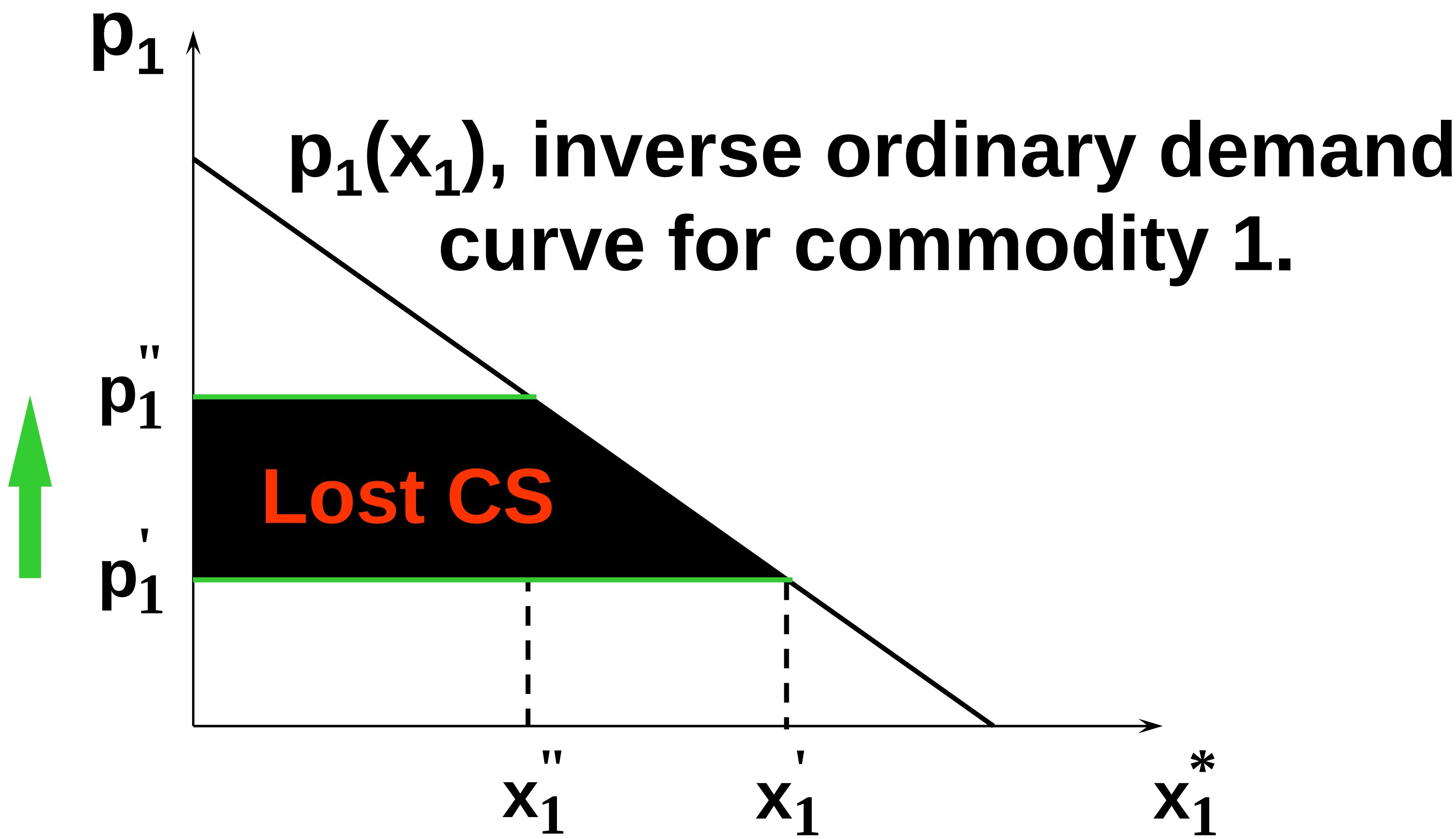
Consumer's Surplus



Change in Consumer's Surplus Due to Price Change



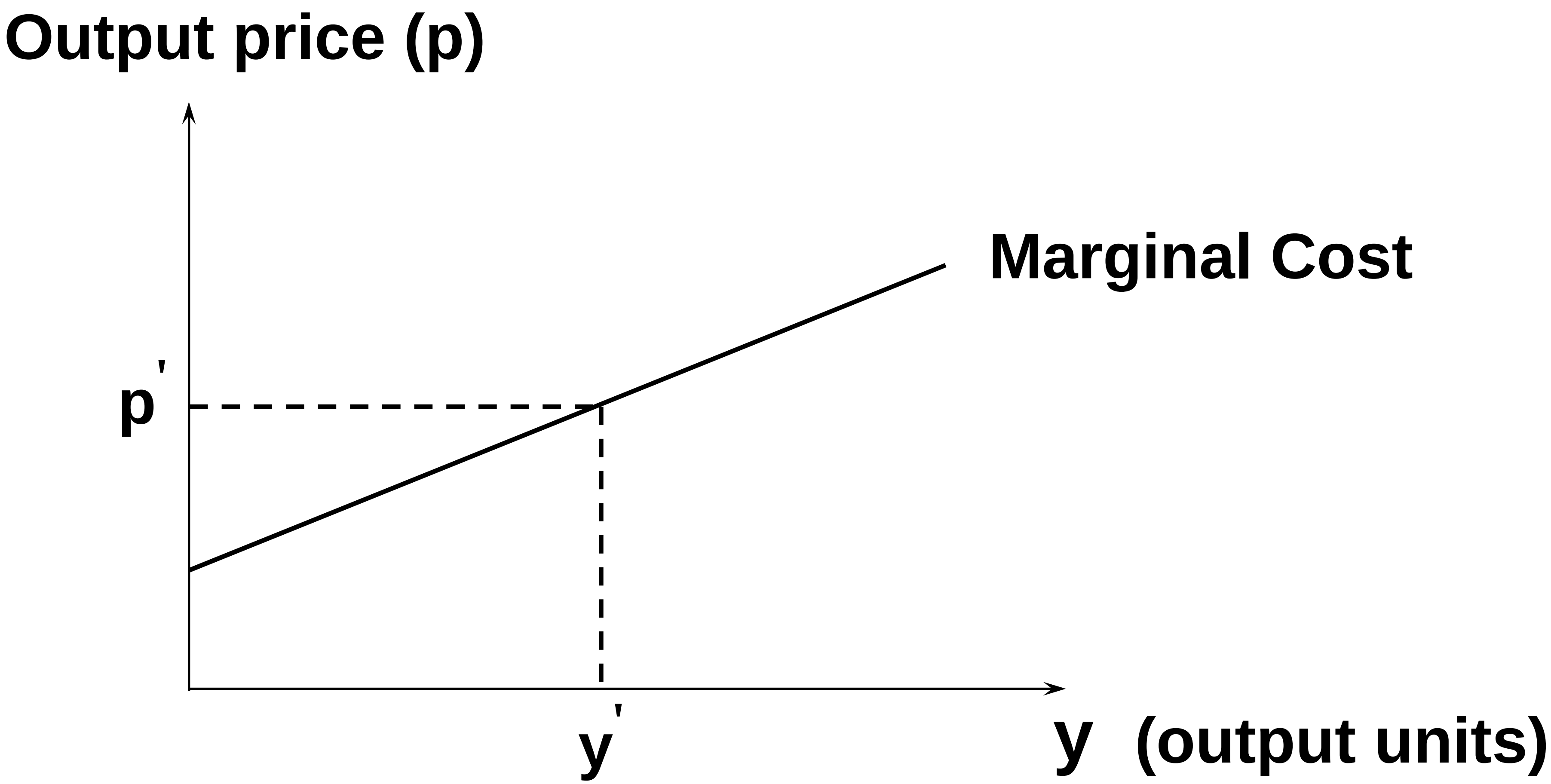
Change in Consumer's Surplus Due to Price Change



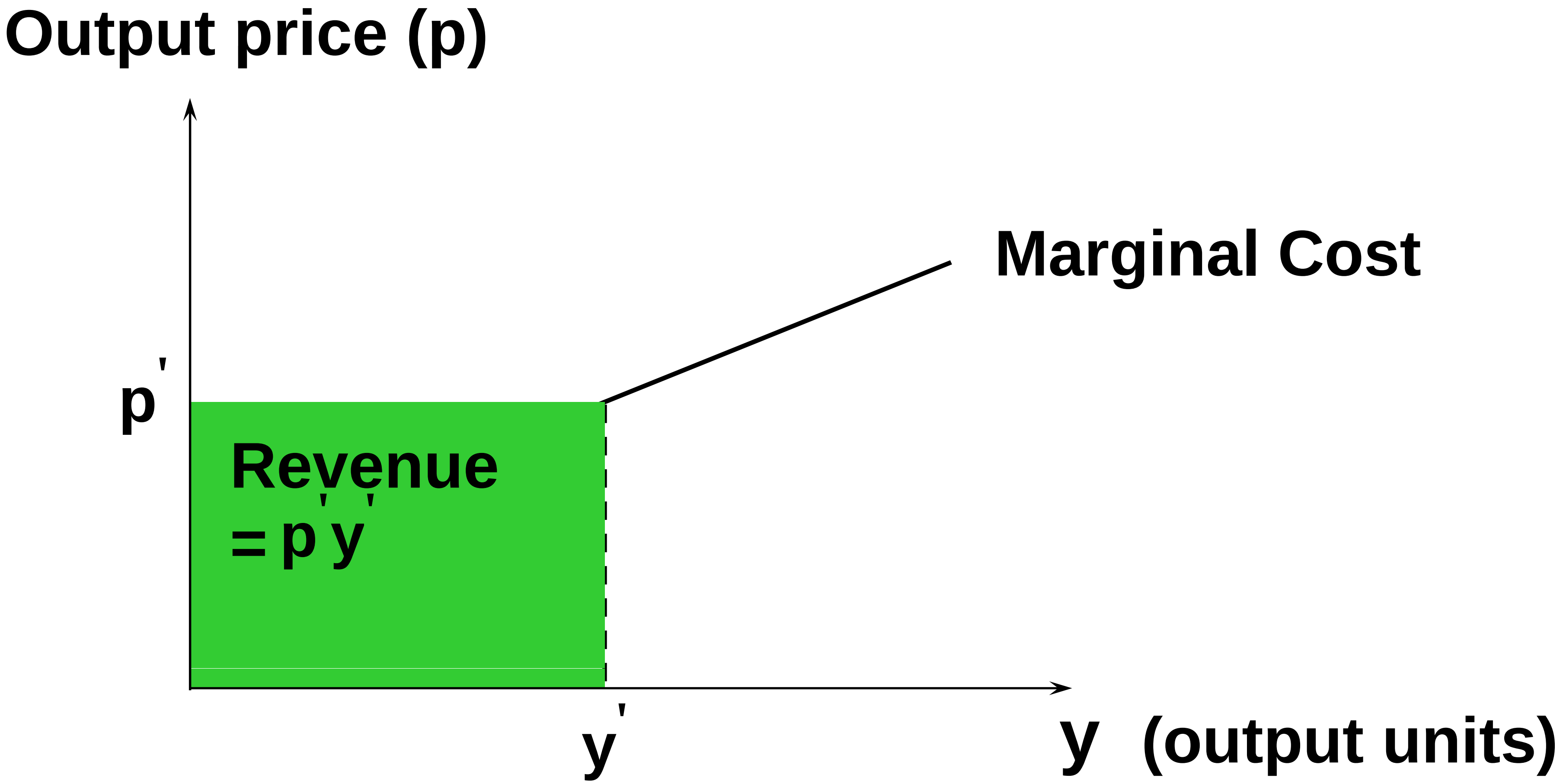
Producer's Surplus

- Changes in a firm's welfare can be measured in dollars much as for a consumer.

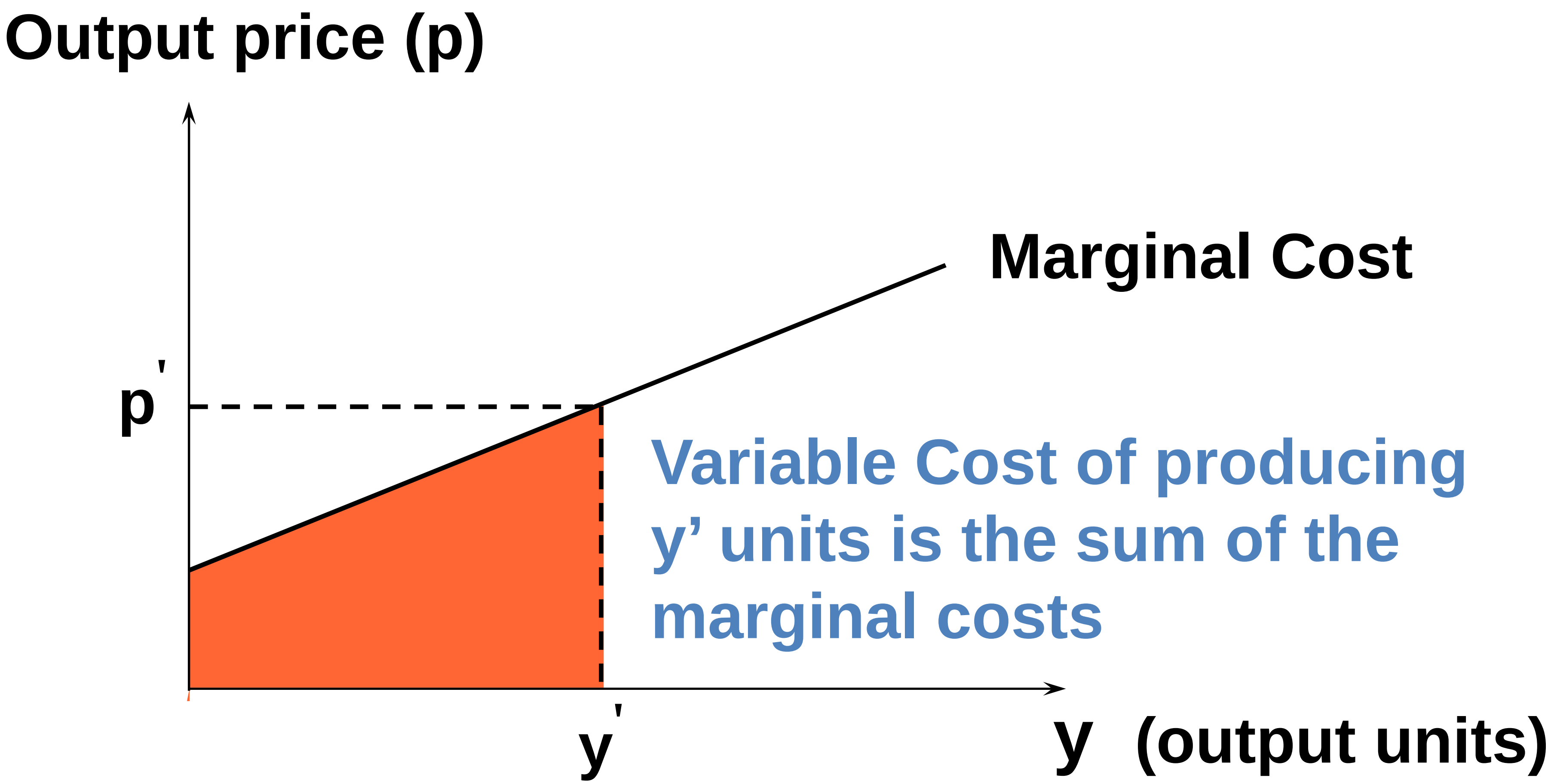
Producer's Surplus



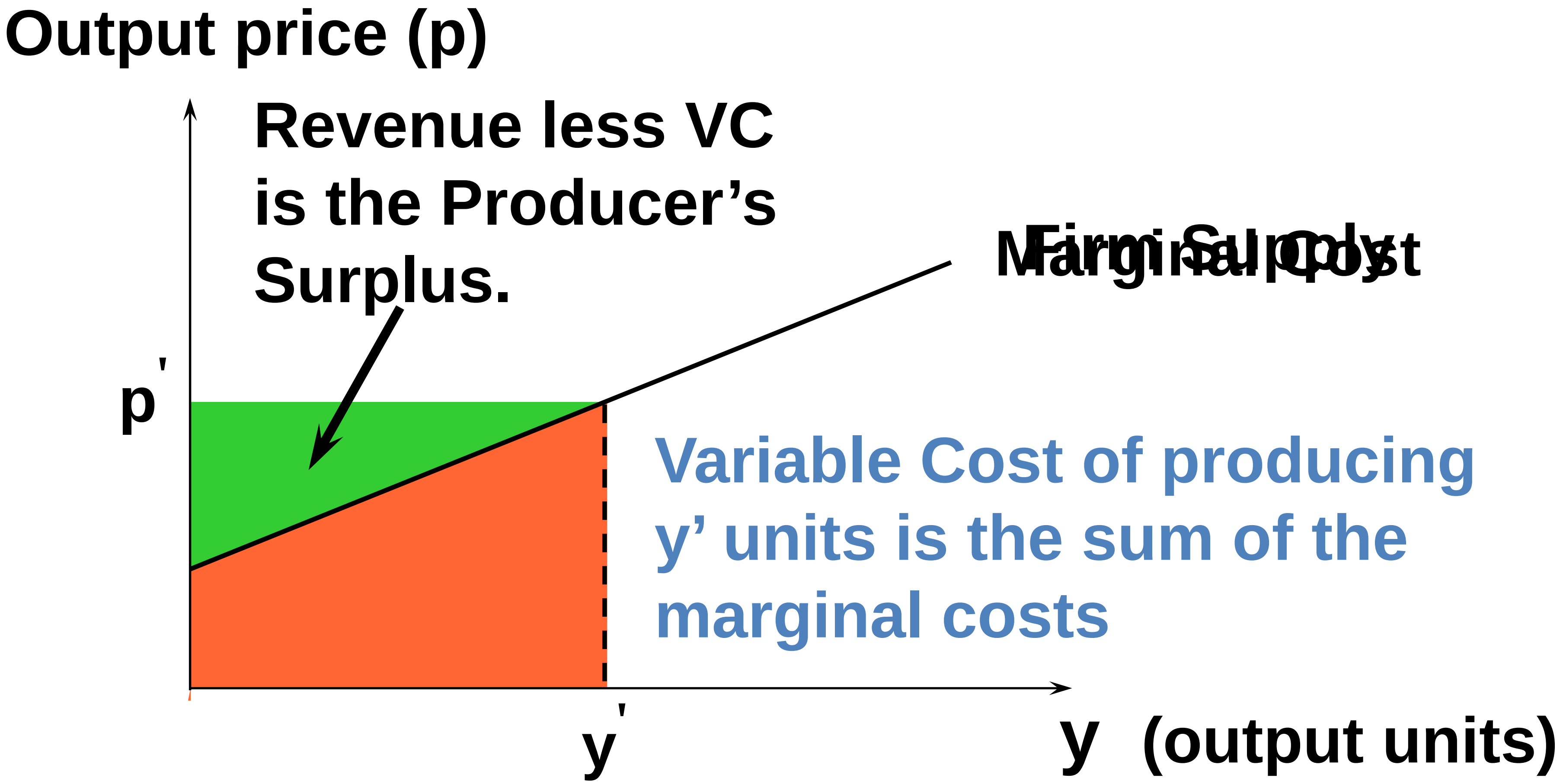
Producer’s Surplus



Producer's Surplus



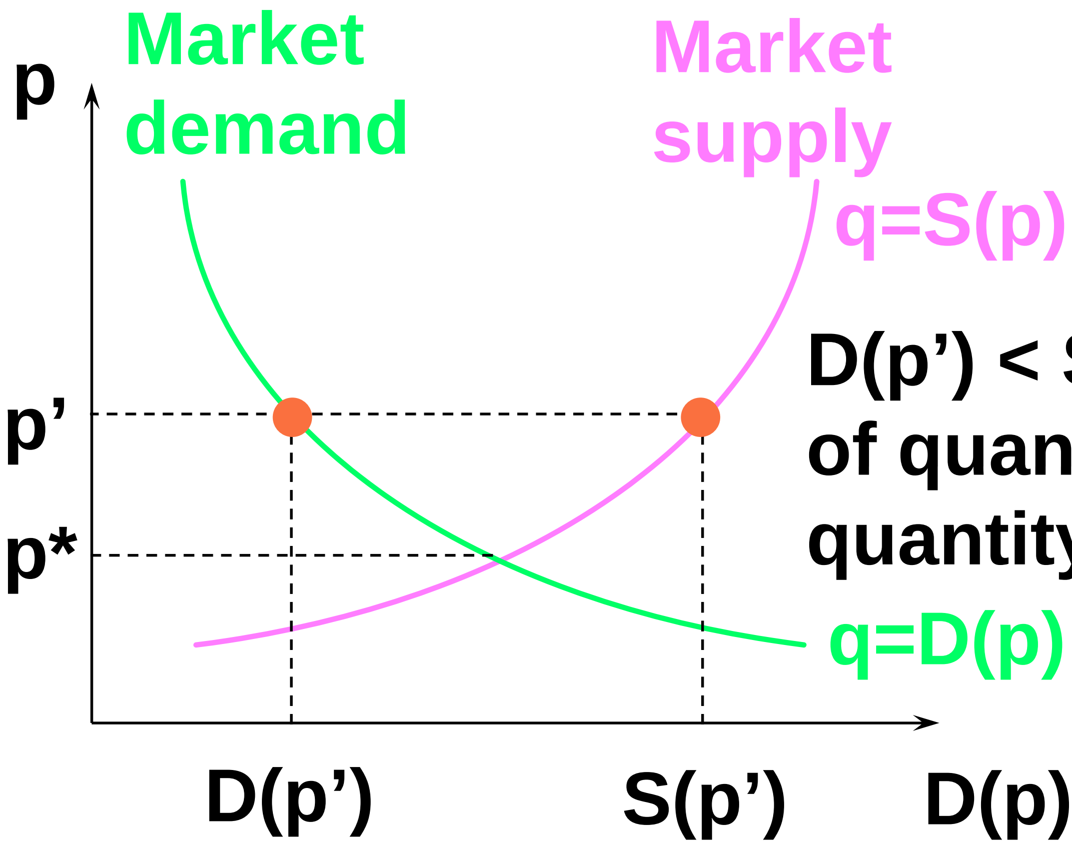
Producer’s Surplus



Market Supply

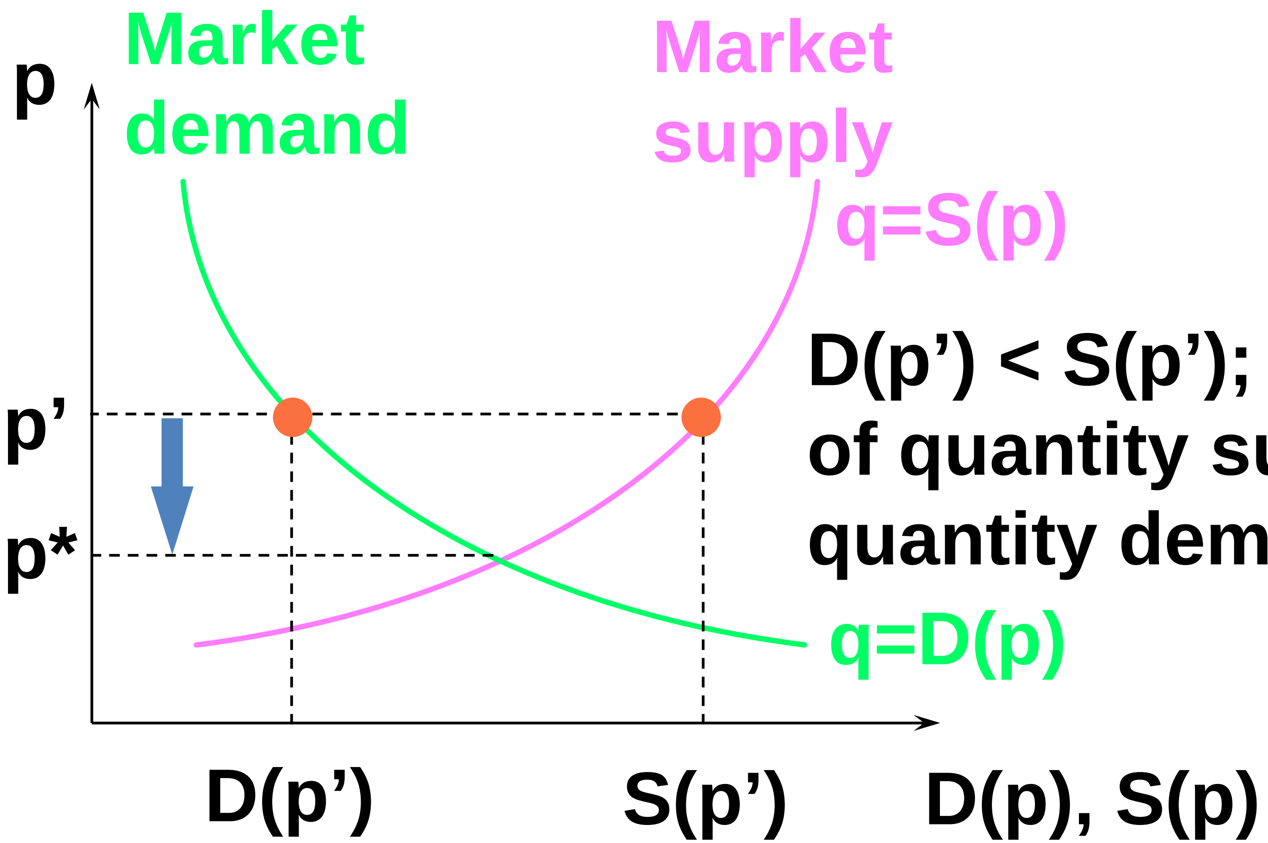
- Summing firm supply curves horizontally gives market supply

Excess Supply



D(p') < S(p'); an excess of quantity supplied over quantity demanded.

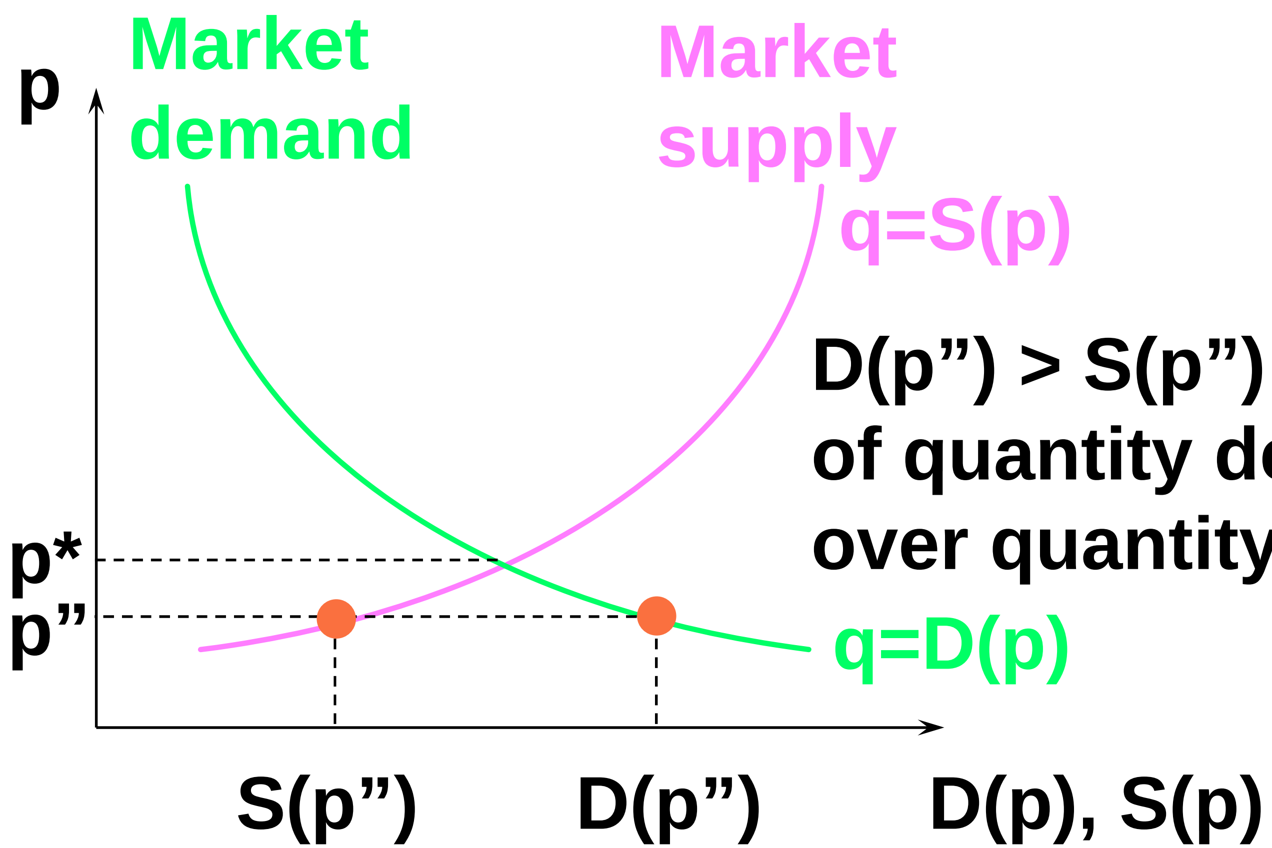
Downward price-pressure with Excess Supply



D(p') < S(p'); an excess of quantity supplied over quantity demanded.

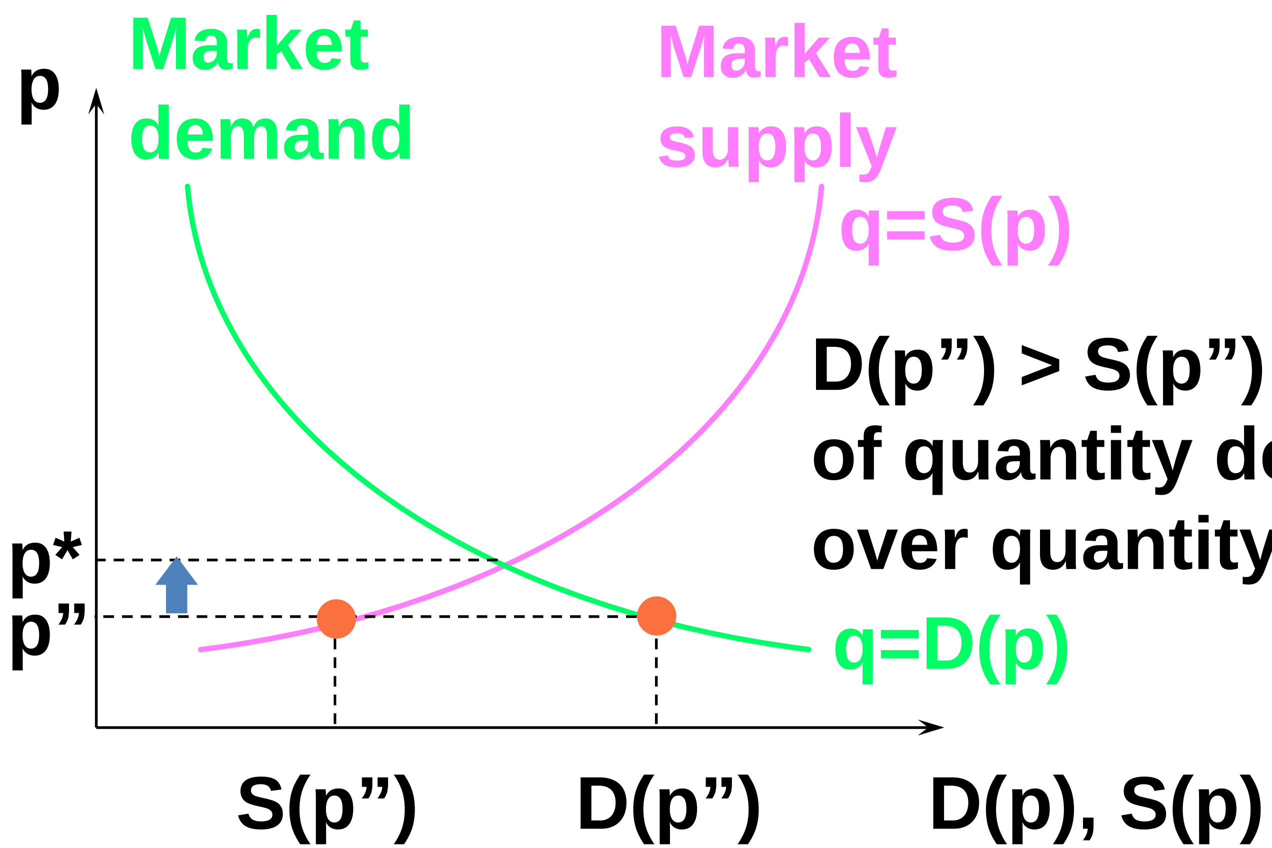
Market price must fall towards p*.

Excess Demand



D(p”) > S(p”); an excess of quantity demanded over quantity supplied.

Upward price-pressure with Excess Demand



Market price must rise towards p*.

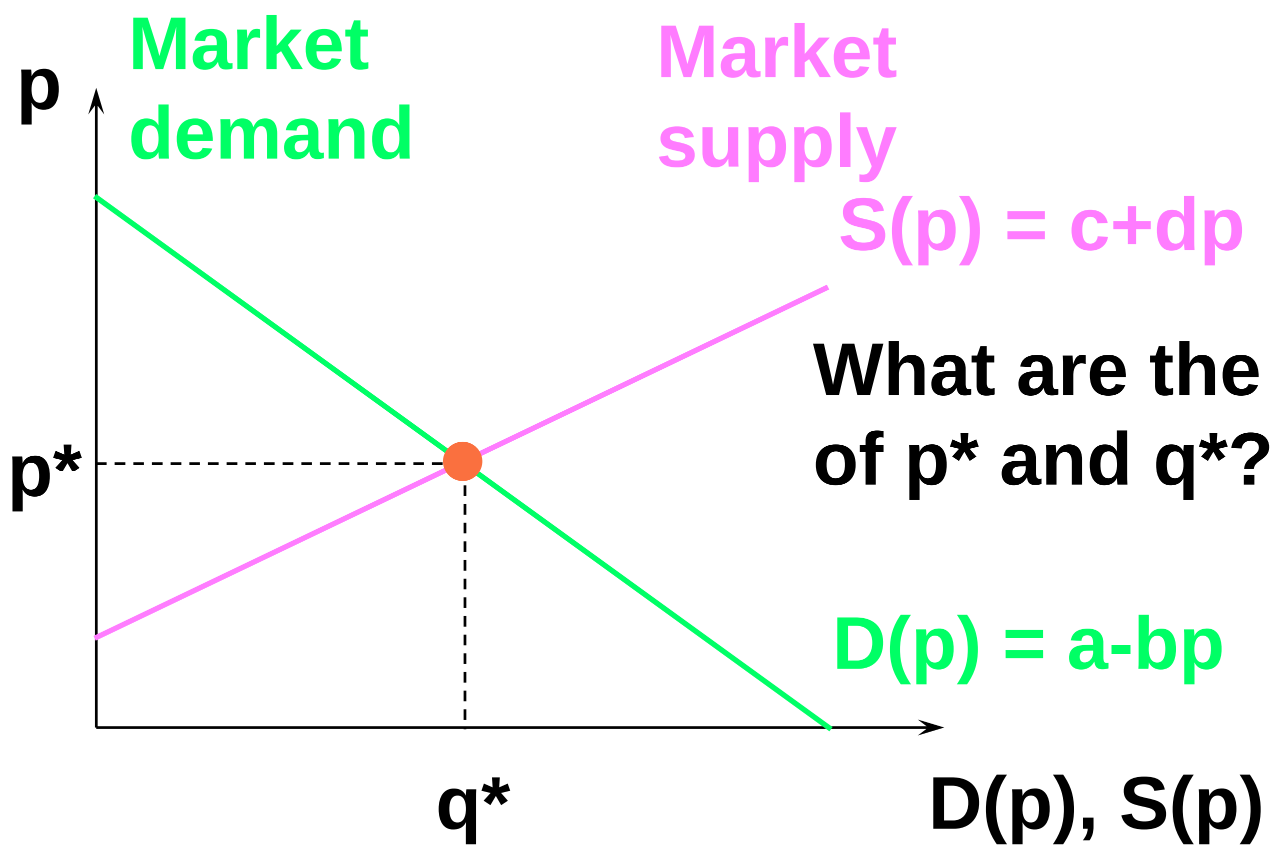
Calculation of Market Equilibrium

- An example of calculating a market equilibrium when the market demand and supply curves are linear.

$$D(p) = a - bp$$

$$S(p) = c + dp$$

Market Equilibrium in a Graph



What are the values of p* and q*?

Market Equilibrium

$$D(p) = a - bp$$

$$S(p) = c + dp$$

At the equilibrium price p*, D(p*) = S(p*).
That is,

$$a - bp^* = c + dp^*$$

which gives

$$p^* = \frac{a - c}{b + d}$$

and

$$q^* = D(p^*) = S(p^*) = \frac{ad + bc}{b + d}.$$

Market Equilibrium

- Can we calculate the market equilibrium using the inverse market demand and supply curves?

$$D(p) = a - bp \qquad S(p) = c + dp$$

Quantity Taxes

- A quantity tax levied at a rate of \$t is a tax of \$t paid on each unit traded.
- If the tax is levied on sellers then it is an excise tax.
- If the tax is levied on buyers then it is a sales tax.

Quantity Taxes (\$$t$ per unit)

Posted price at shelf = p

Excise tax: Seller pays

Final Price at checkout counter = p

Sales tax: Buyer pays

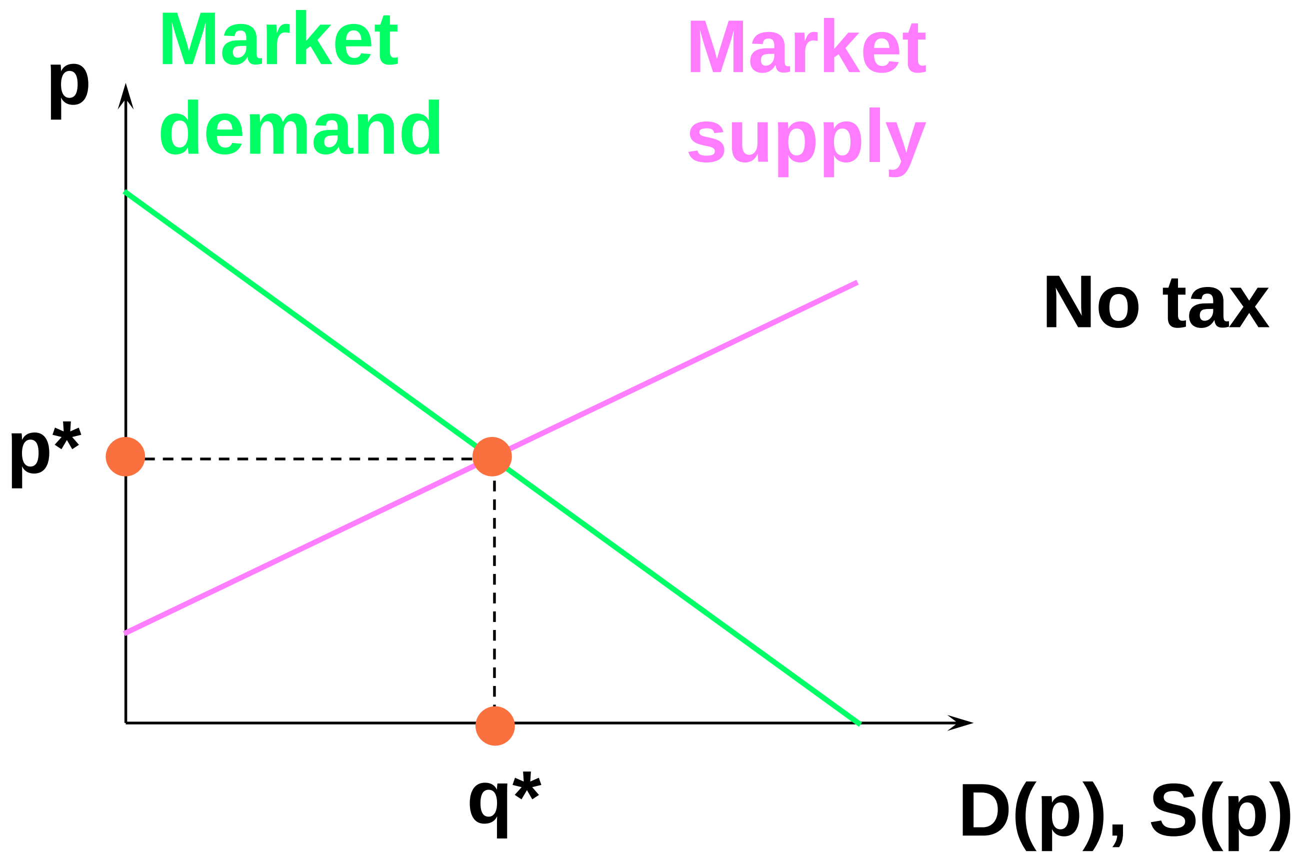
Final Price at checkout counter = $p + t$

Quantity Taxes

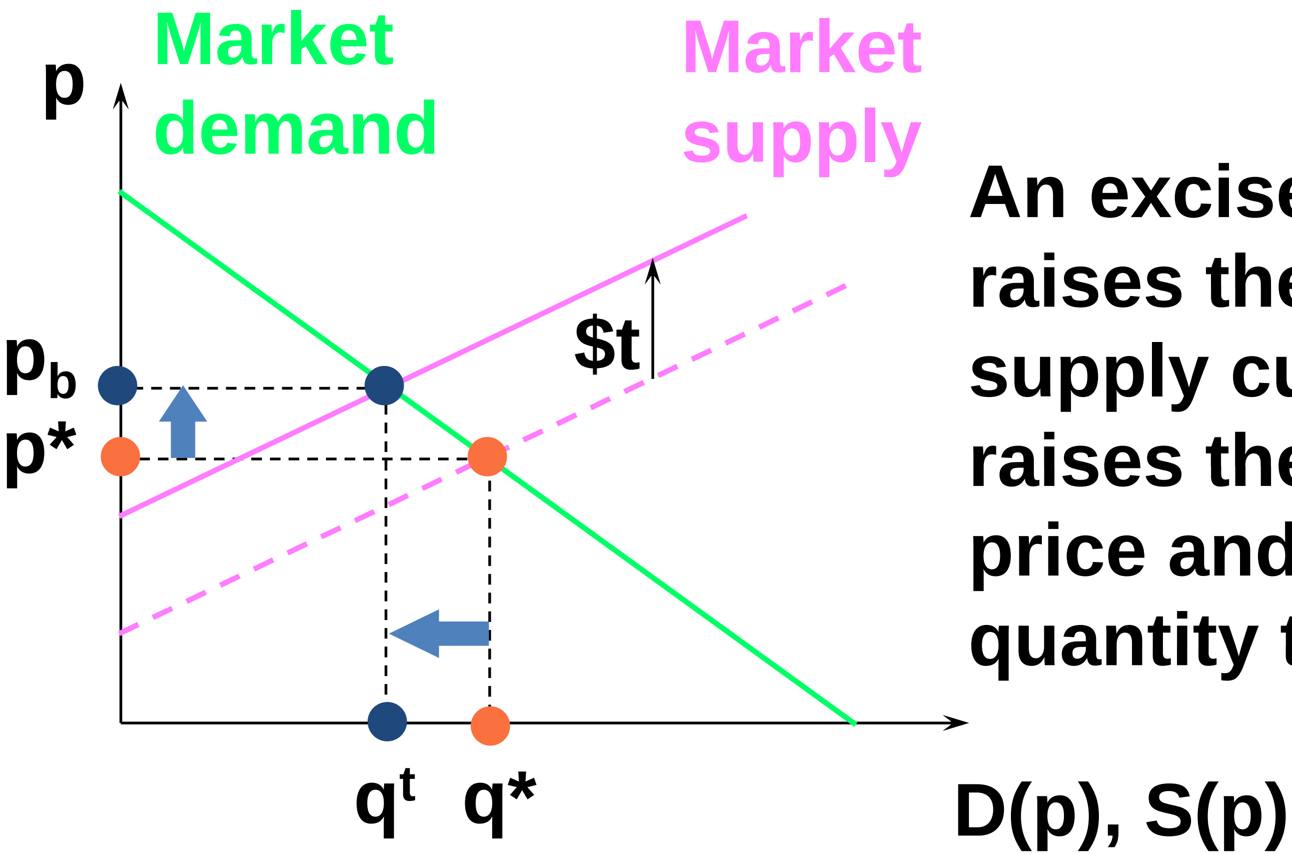
$$p^*_{before} \rightarrow p^*_{after}$$

- What is the effect of a quantity tax on a market's equilibrium?
- How are prices affected?
- How is the quantity traded affected?
- Who pays the tax?
- How are gains-to-trade altered?

Quantity Taxes & Market Equilibrium

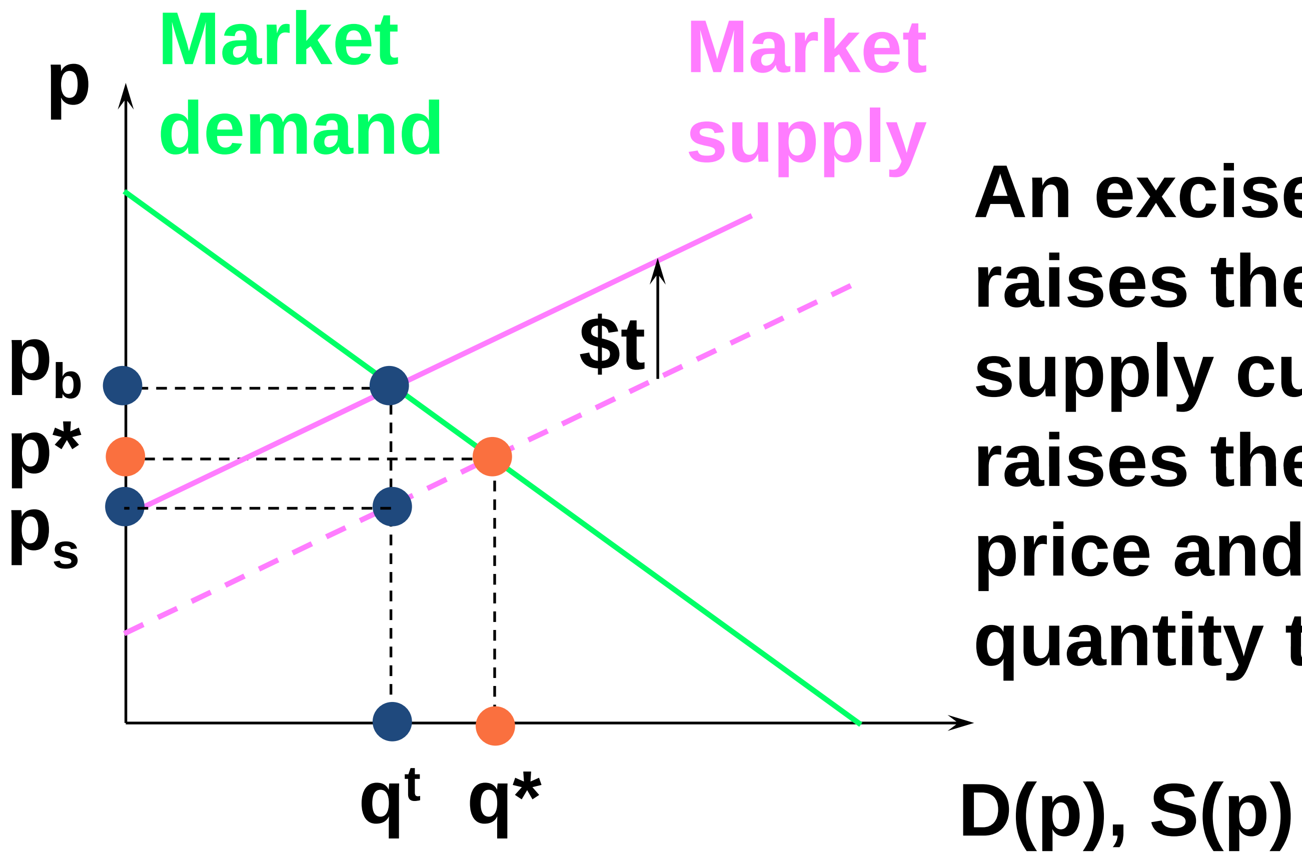


Quantity Taxes & Market Equilibrium



An excise tax raises the market supply curve by $t, raises the buyers' price and lowers the quantity traded.

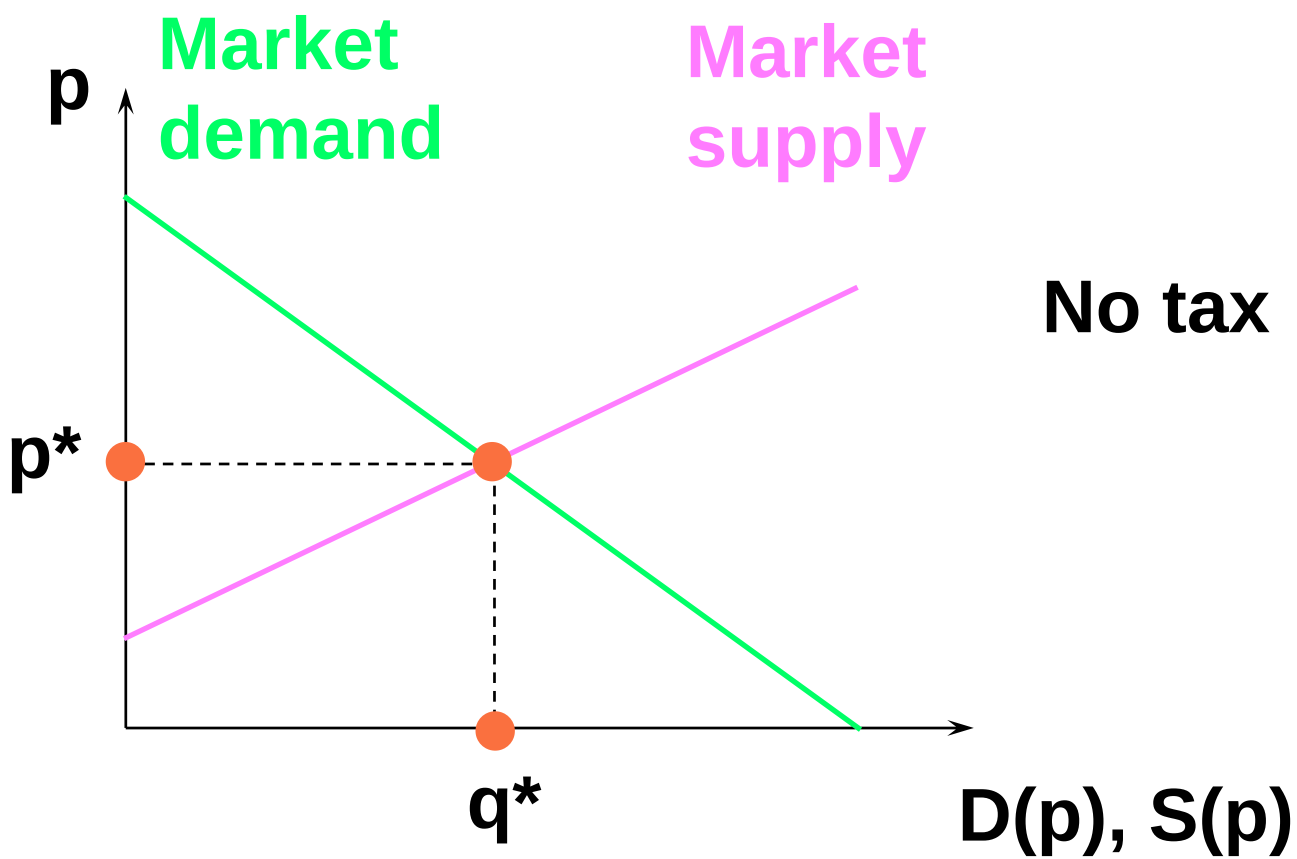
Quantity Taxes & Market Equilibrium



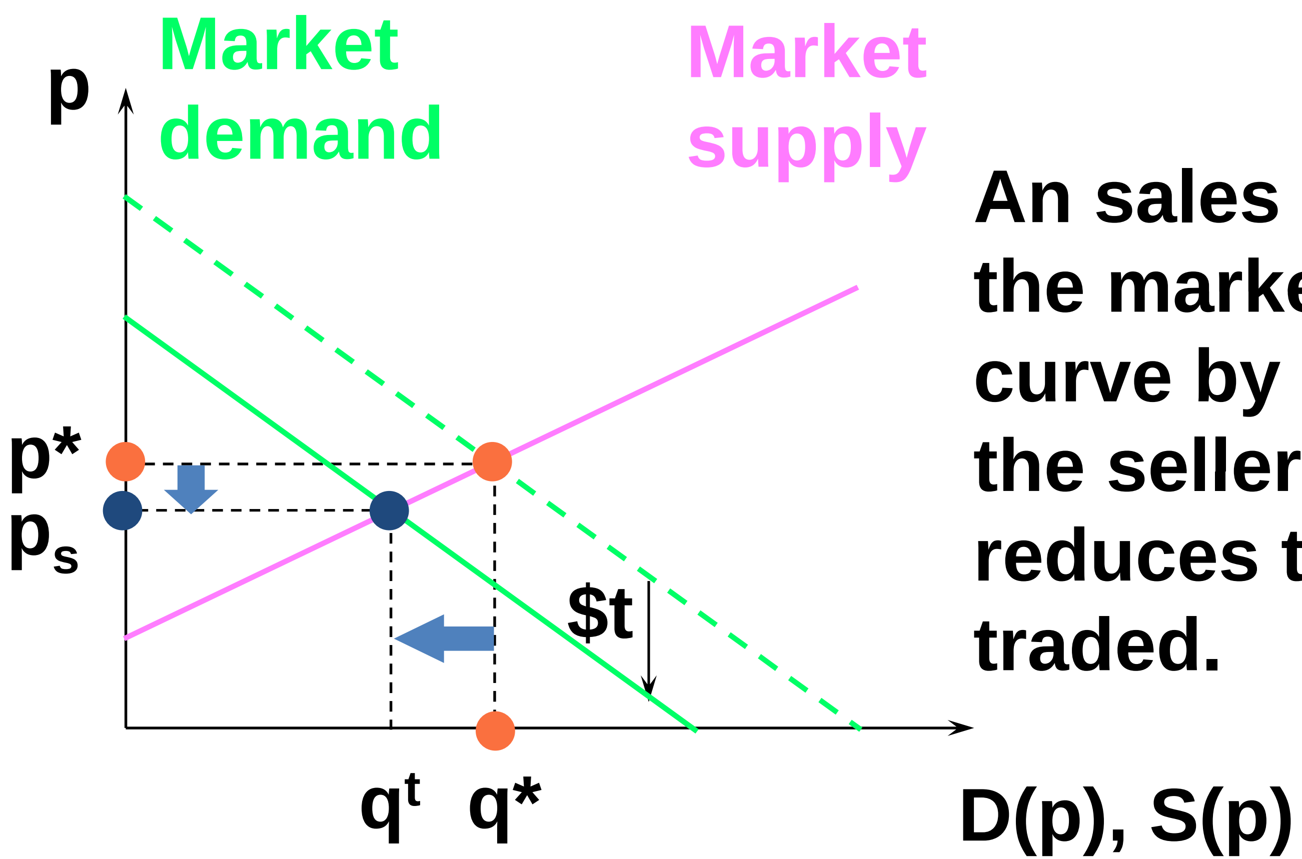
An excise tax raises the market supply curve by \$t, raises the buyers' price and lowers the quantity traded.

And sellers receive only $p_s = p_b - t$.

Quantity Taxes & Market Equilibrium

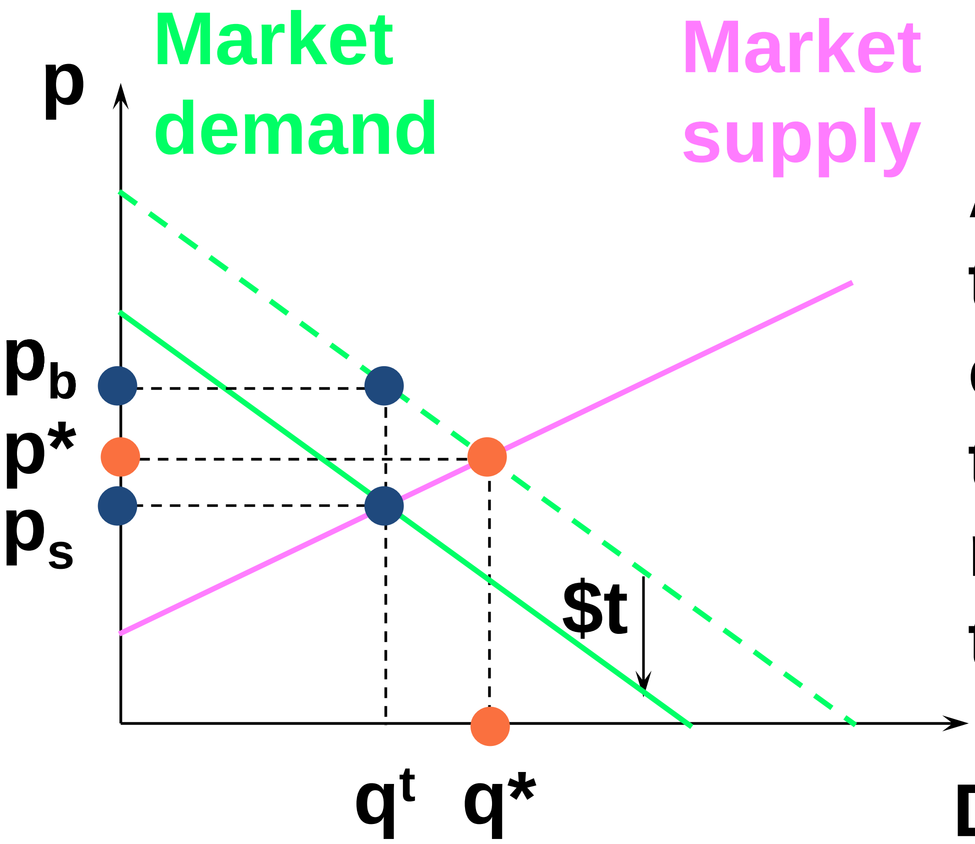


Quantity Taxes & Market Equilibrium



An sales tax lowers the market demand curve by $t, lowers the sellers' price and reduces the quantity traded.

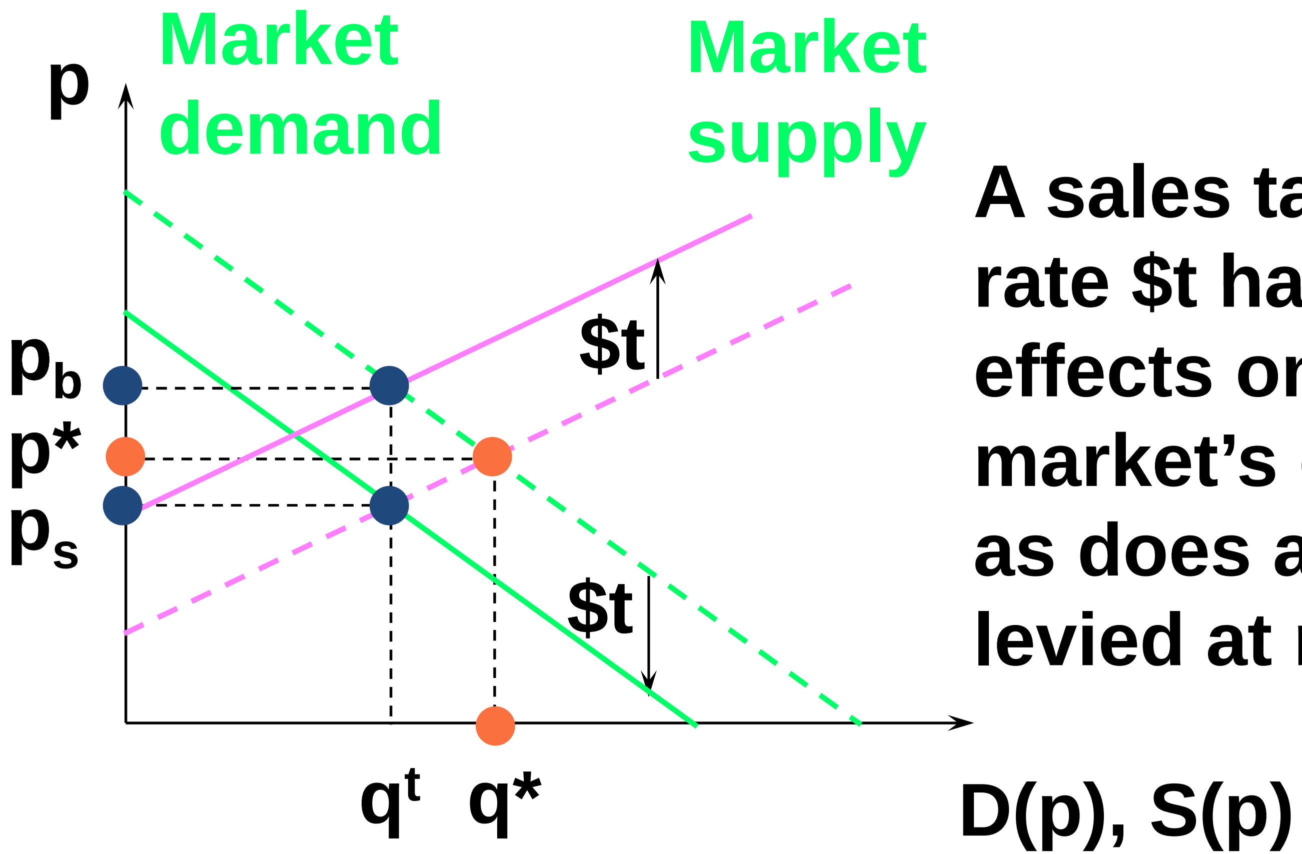
Quantity Taxes & Market Equilibrium



An sales tax lowers the market demand curve by \$t, lowers the sellers' price and reduces the quantity traded.

And buyers pay $p_b = p_s + t$.

Quantity Taxes & Market Equilibrium

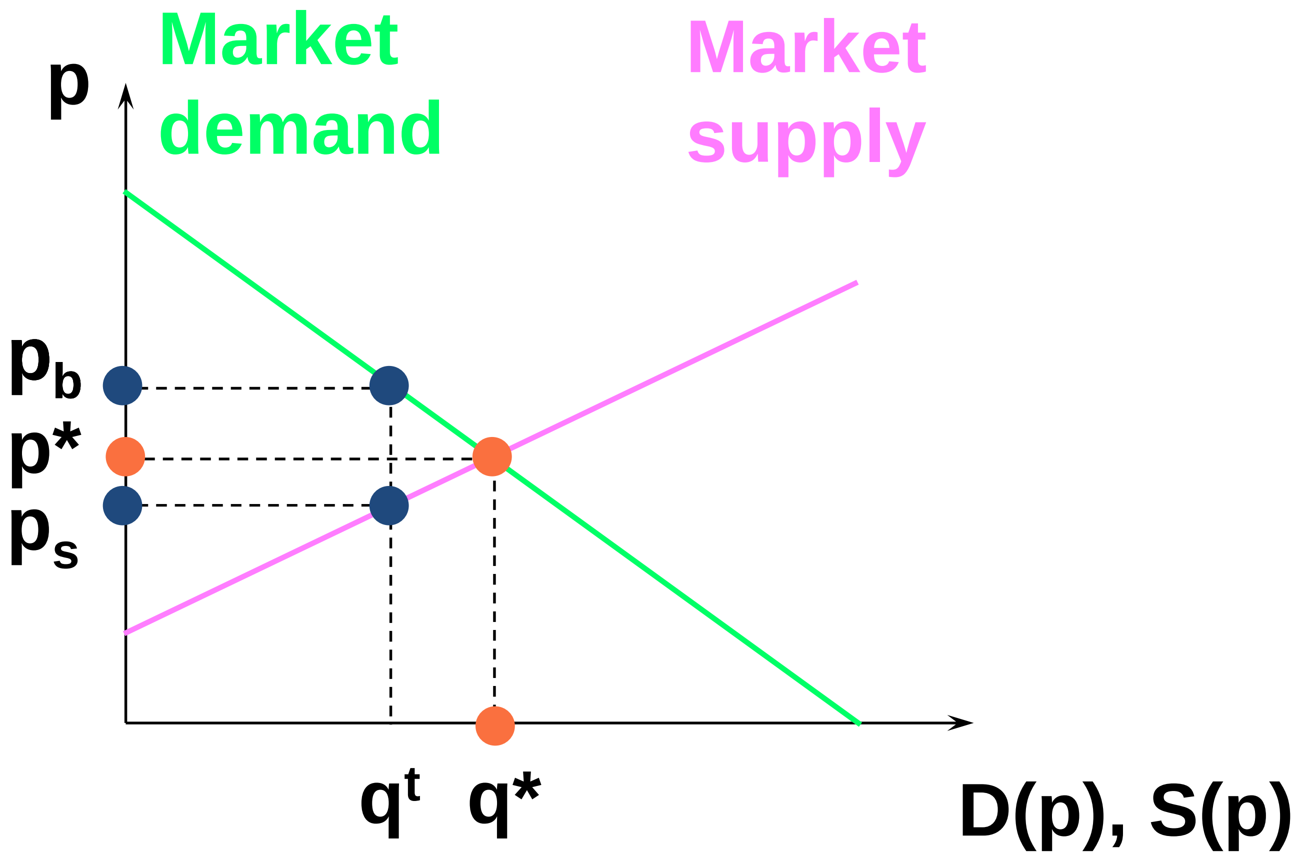


A sales tax levied at rate $t has the same effects on the market's equilibrium as does an excise tax levied at rate $t.

Quantity Taxes & Market Equilibrium

- Who pays the tax of \$t per unit traded?
- The division of the \$t between buyers and sellers is the incidence of the tax.

Quantity Taxes & Market Equilibrium



Quantity Taxes & Market Equilibrium

- E.g. suppose the market demand and supply curves are linear.

$$D(p_b) = a - bp_b$$

$$S(p_s) = c + dp_s$$

Quantity Taxes & Market Equilibrium

$$D(p_b) = a - bp_b \text{ and } S(p_s) = c + dp_s.$$

With the tax, the market equilibrium satisfies

$$p_b = p_s + t \quad \text{and} \quad D(p_b) = S(p_s) \quad \text{so}$$

$$p_b = p_s + t \quad \text{and} \quad a - bp_b = c + dp_s.$$

Substituting for p_b gives

$$a - b(p_s + t) = c + dp_s \Rightarrow p_s = \frac{a - c - bt}{b + d}.$$

Quantity Taxes & Market Equilibrium

$$p_s = \frac{a - c - bt}{b + d} \quad \text{and} \quad p_b = p_s + t \quad \text{give}$$

$$p_b = \frac{a - c + dt}{b + d}$$

The quantity traded at equilibrium is

$$q^t = D(p_b) = S(p_s)$$

$$= a - bp_b = \frac{ad + bc - bdt}{b + d}.$$

Quantity Taxes & Market Equilibrium

$$p_s = \frac{a - c - bt}{b + d}$$

$$q^t = \frac{ad + bc - bdt}{b + d}$$

$$p_b = \frac{a - c + dt}{b + d}$$

As $t \to 0$, p_s and $p_b \to \dfrac{a - c}{b + d} = p^*$, the equilibrium price if there is no tax ($t = 0$) and $q^t \to \dfrac{ad + bc}{b + d}$ the quantity traded at equilibrium when there is no tax.

Quantity Taxes & Market Equilibrium

$$p_s = \frac{a - c - bt}{b + d}$$

$$q^t = \frac{ad + bc - bdt}{b + d}$$

$$p_b = \frac{a - c + dt}{b + d}$$

Quantity Taxes & Market Equilibrium

$$p_s = \frac{a - c - bt}{b + d}$$

$$q^t = \frac{ad + bc - bdt}{b + d}$$

$$p_b = \frac{a - c + dt}{b + d}$$

The tax paid per unit by the buyer is

$$p_b - p^* = \frac{a - c + dt}{b + d} - \frac{a - c}{b + d} = \frac{dt}{b + d}.$$

Quantity Taxes & Market Equilibrium

$$p_s = \frac{a - c - bt}{b + d}$$

$$q^t = \frac{ad + bc - bdt}{b + d}$$

$$p_b = \frac{a - c + dt}{b + d}$$

The tax paid per unit by the buyer is

$$p_b - p^* = \frac{a - c + dt}{b + d} - \frac{a - c}{b + d} = \frac{dt}{b + d}.$$

The tax paid per unit by the seller is

$$p^* - p_s = \frac{a - c}{b + d} - \frac{a - c - bt}{b + d} = \frac{bt}{b + d}.$$

Quantity Taxes & Market Equilibrium

$$p_s = \frac{a - c - bt}{b + d}$$

$$q^t = \frac{ad + bc - bdt}{b + d}$$

$$p_b = \frac{a - c + dt}{b + d}$$

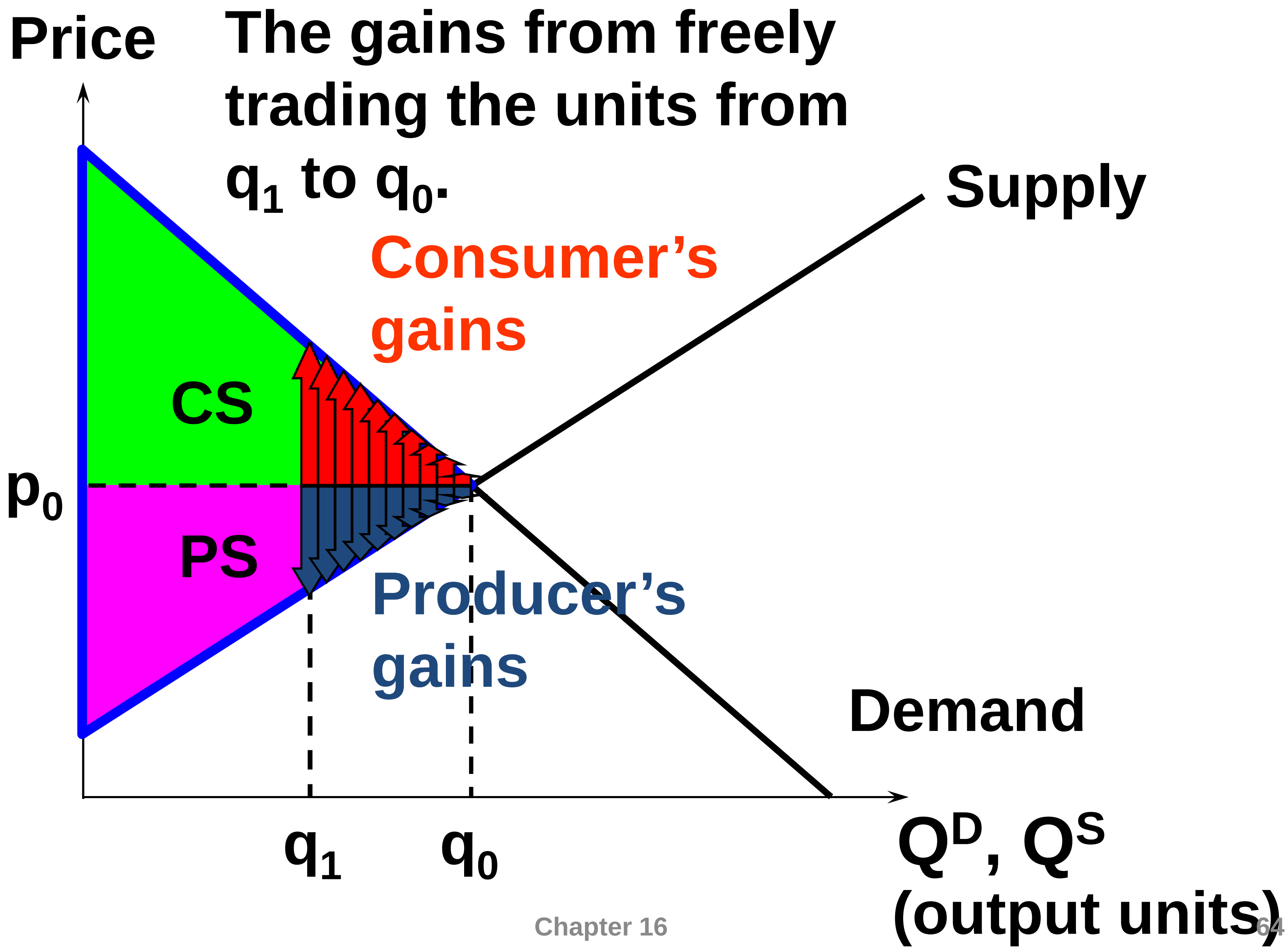
The total tax paid (by buyers and sellers combined) is

$$T = tq^t = t\frac{ad + bc - bdt}{b + d}.$$

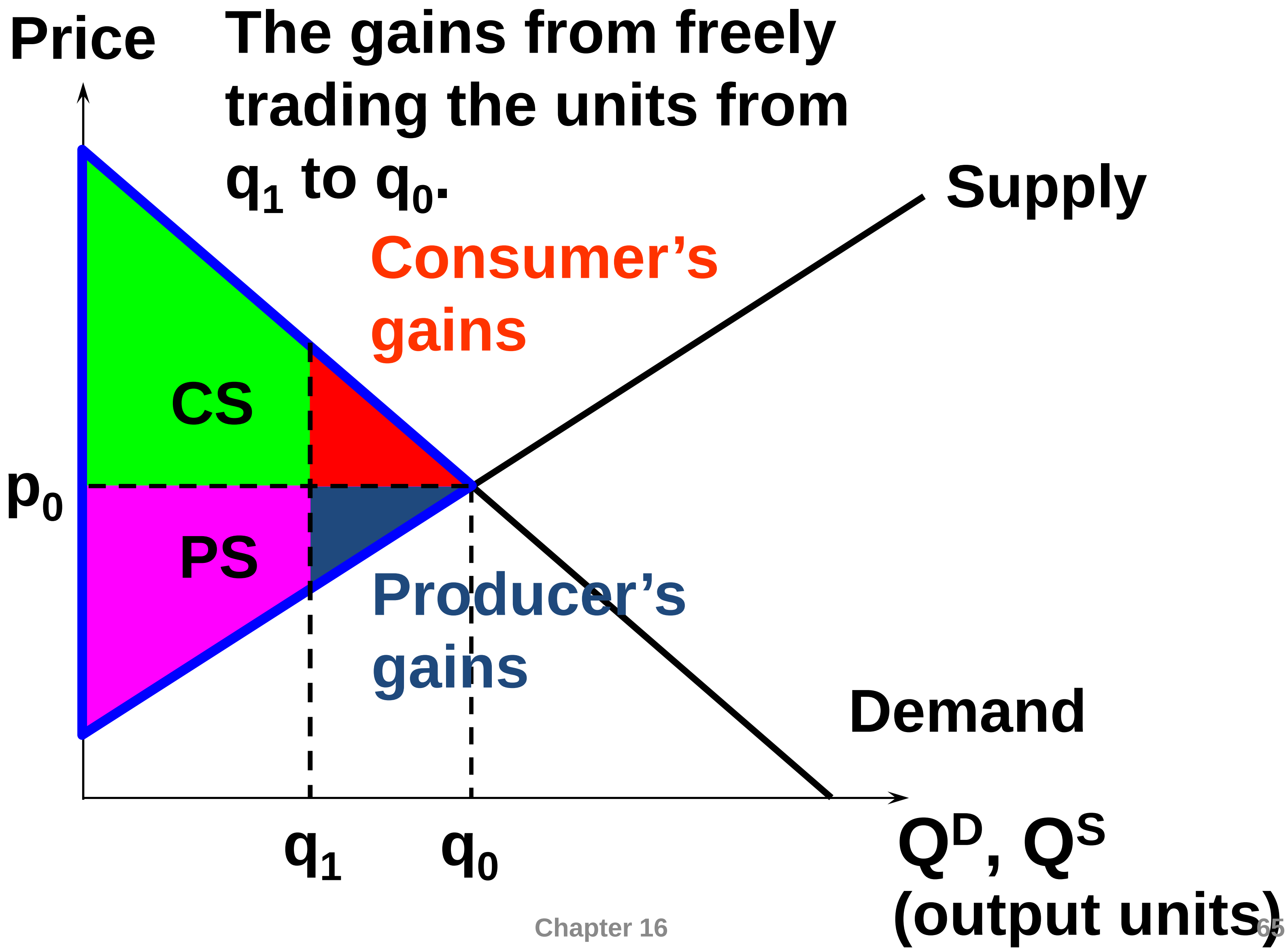
Benefit-Cost Analysis

- Convenient to use Consumer’s Surplus and Producer’s Surplus to measure the effects of government policies

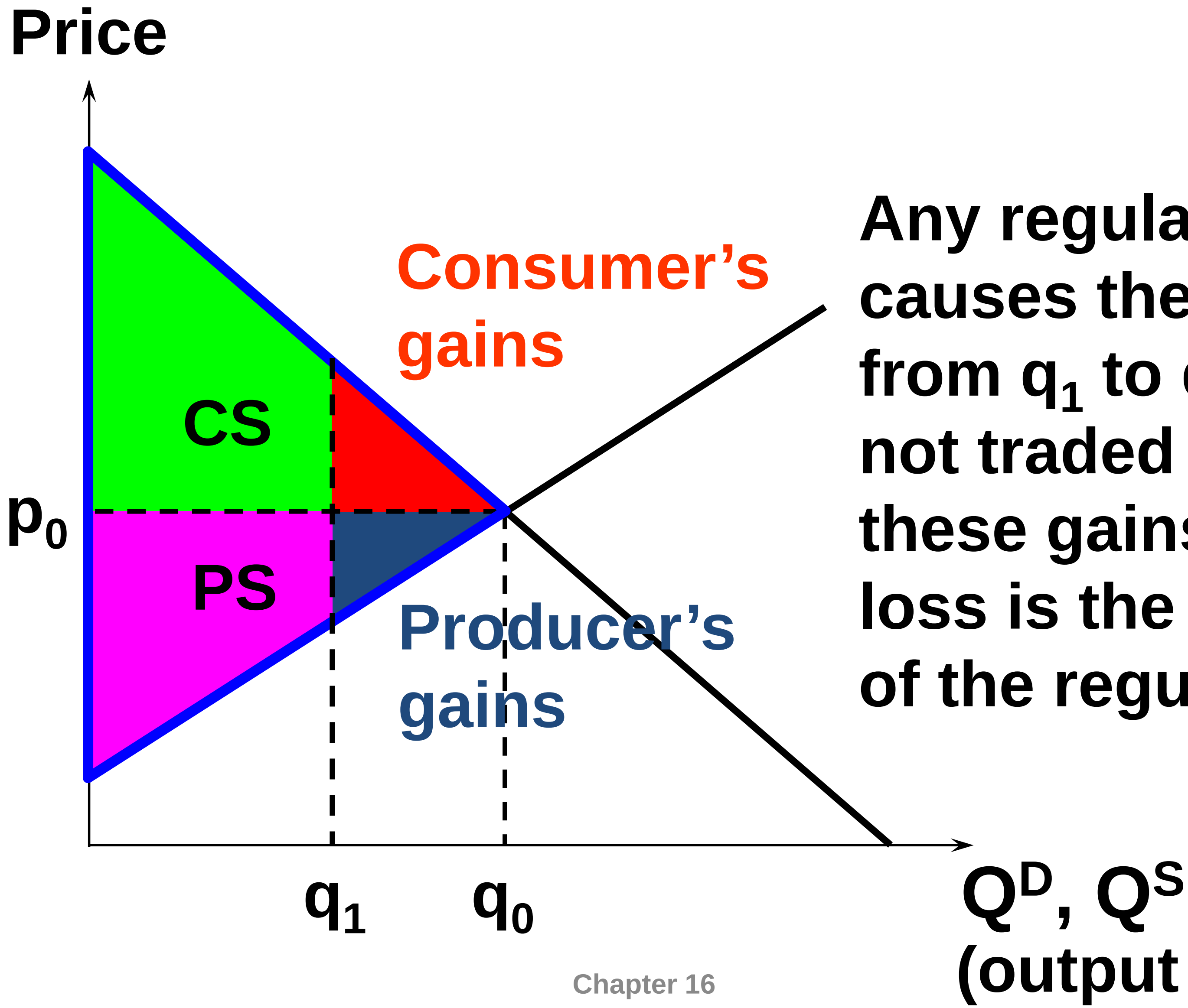
Benefit-Cost Analysis



Benefit-Cost Analysis



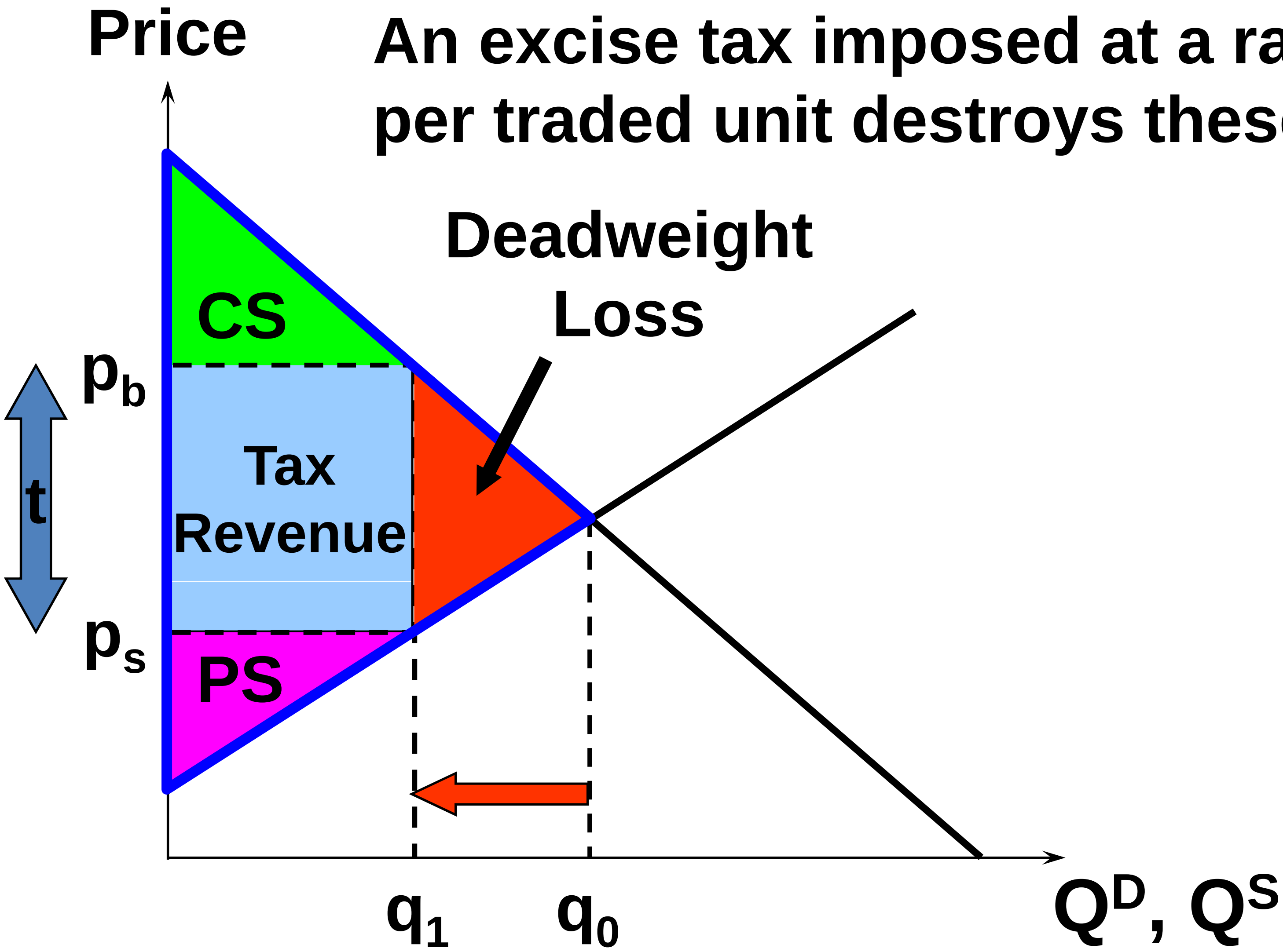
Benefit-Cost Analysis



Any regulation that causes the units from q_1 to q_0 to be not traded destroys these gains. This loss is the net cost of the regulation.

Benefit-Cost Analysis: Excise Tax

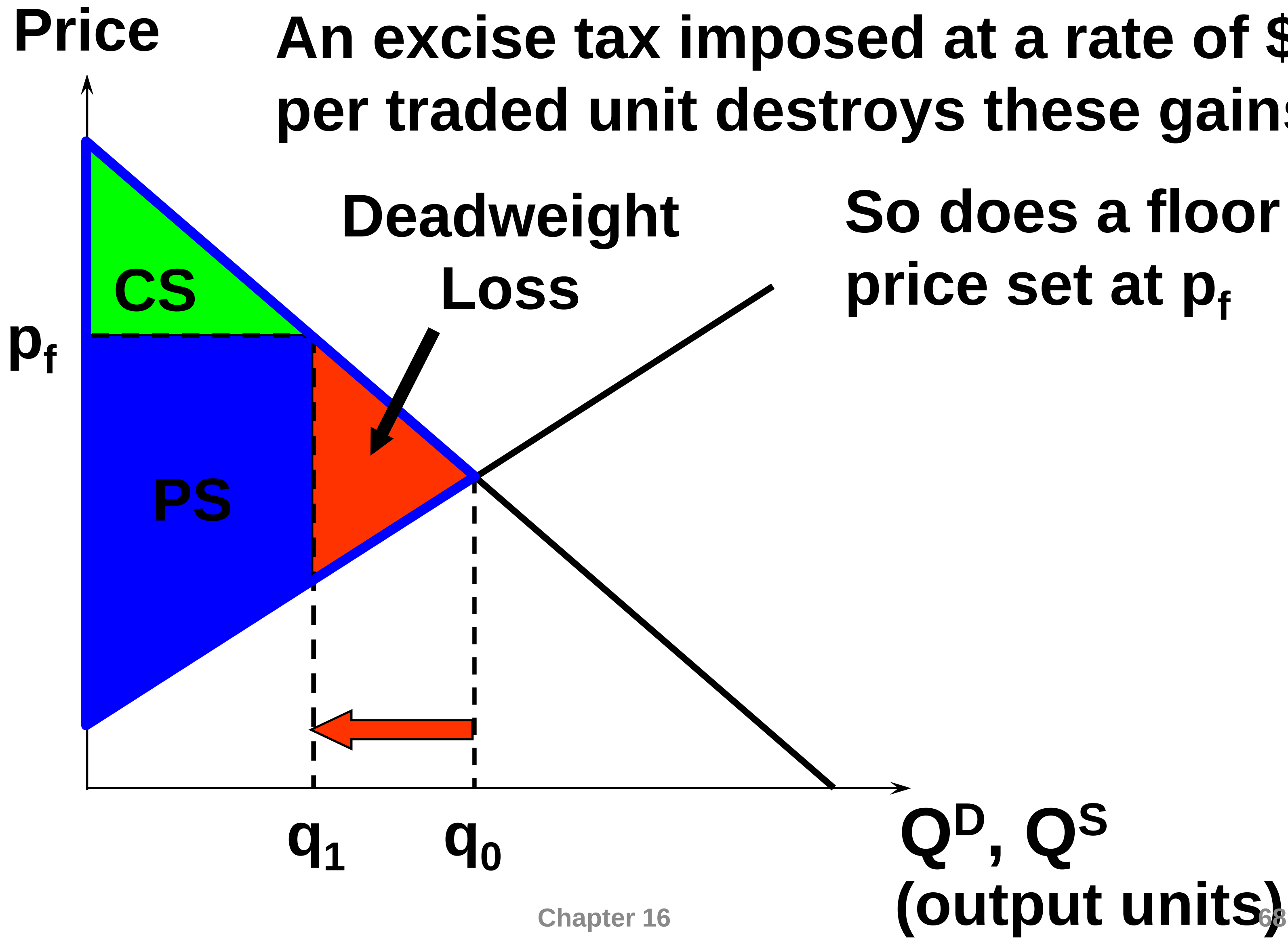
An excise tax imposed at a rate of $t per traded unit destroys these gains.



Benefit-Cost Analysis: Floor Price

An excise tax imposed at a rate of $t per traded unit destroys these gains.

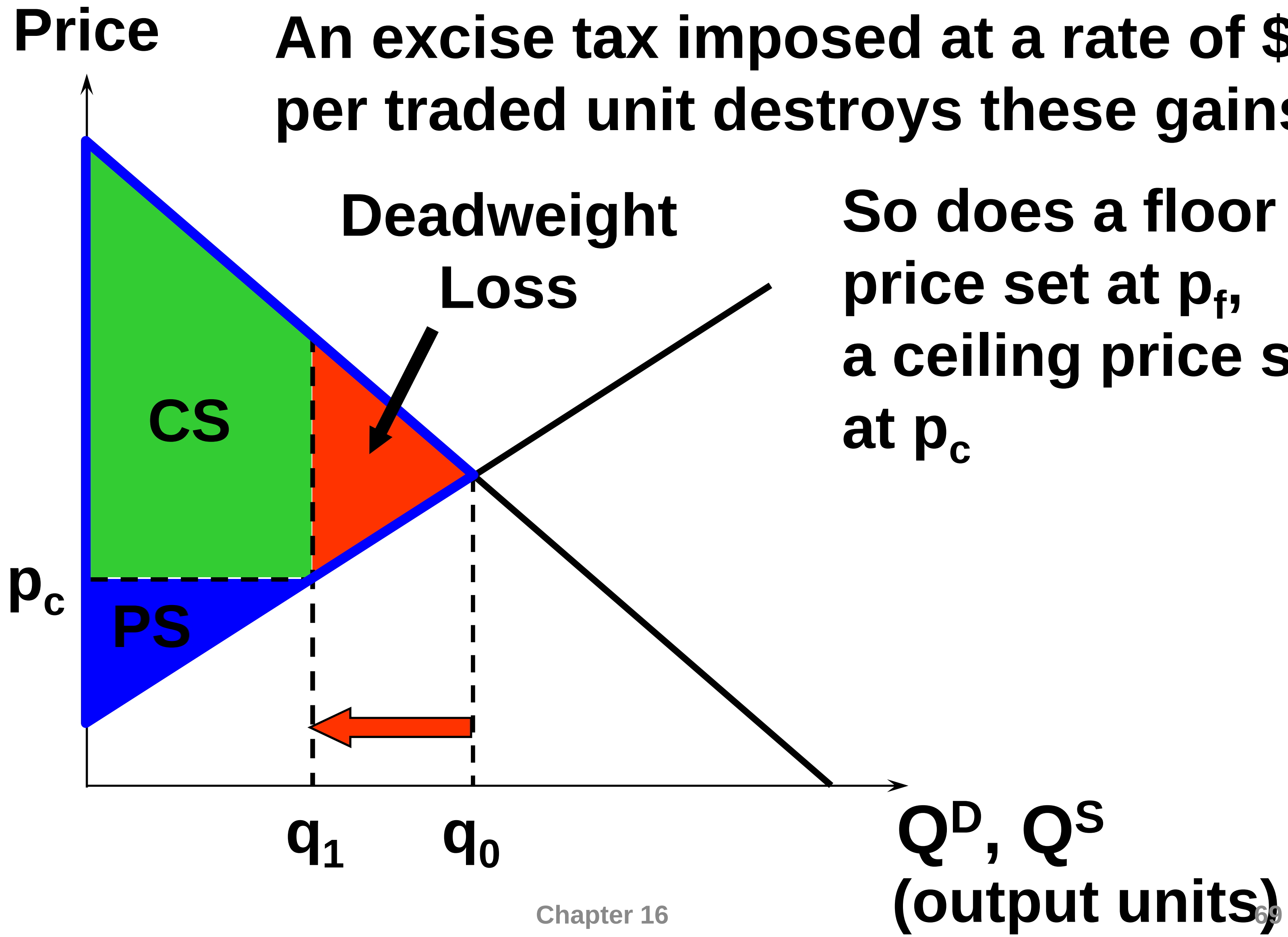
So does a floor price set at p_f



Benefit-Cost Analysis: Ceiling Price

An excise tax imposed at a rate of $t per traded unit destroys these gains.

So does a floor price set at p_f, a ceiling price set at p_c



Elasticity

Elasticities

- Elasticity measures the “sensitivity” of one variable with respect to another.
- The elasticity of variable X with respect to variable Y is

$$\varepsilon_{x,y} = \frac{\%\Delta x}{\%\Delta y}.$$

Economic Applications of Elasticity

Economists use elasticities to measure the sensitivity of

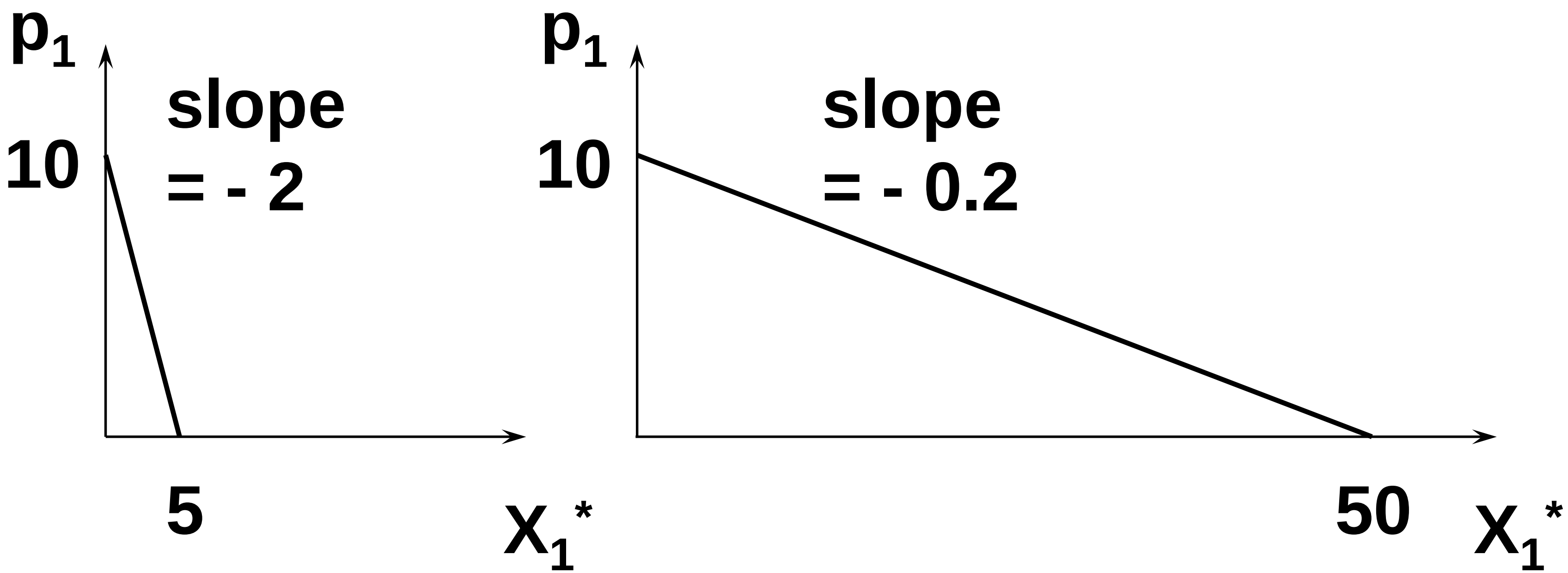
- quantity demanded of commodity i with respect to the price of commodity i: Own-price elasticity

- demand for commodity i with respect to the price of commodity j: Cross-price elasticity

Own-Price Elasticity of Demand

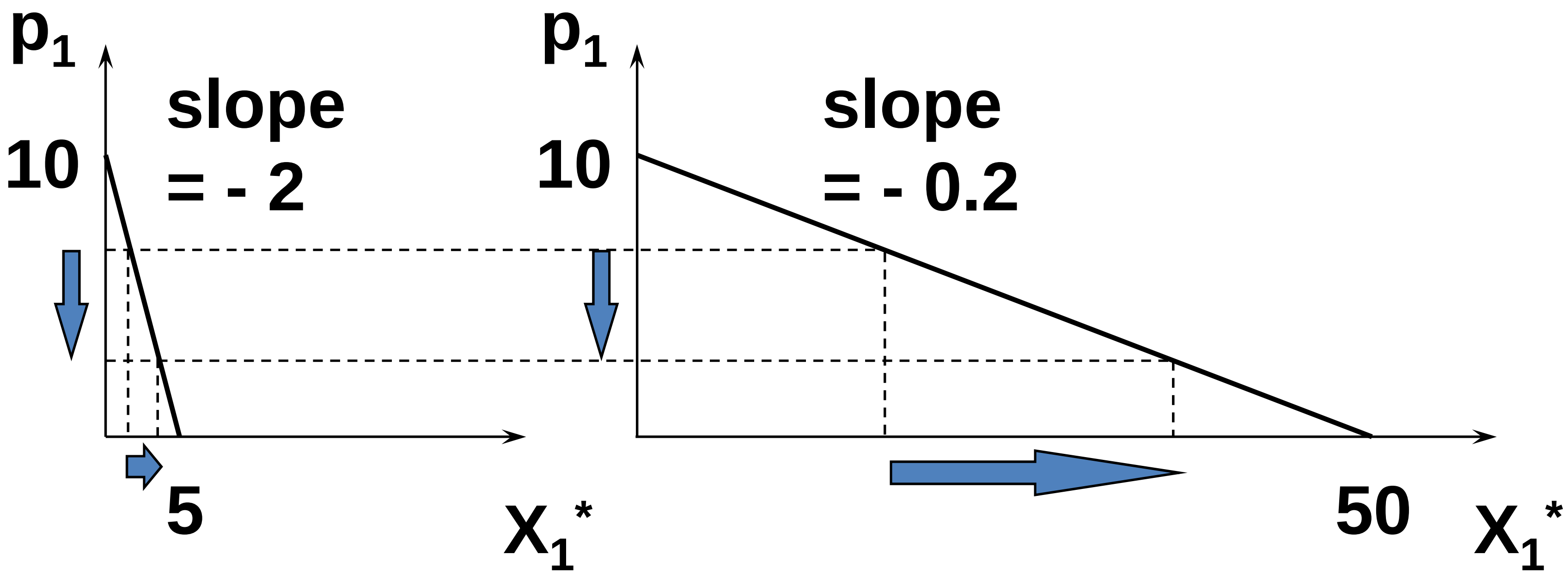
- Q: Why not use a demand curve's slope to measure the sensitivity of quantity demanded to a change in price?

Own-Price Elasticity of Demand



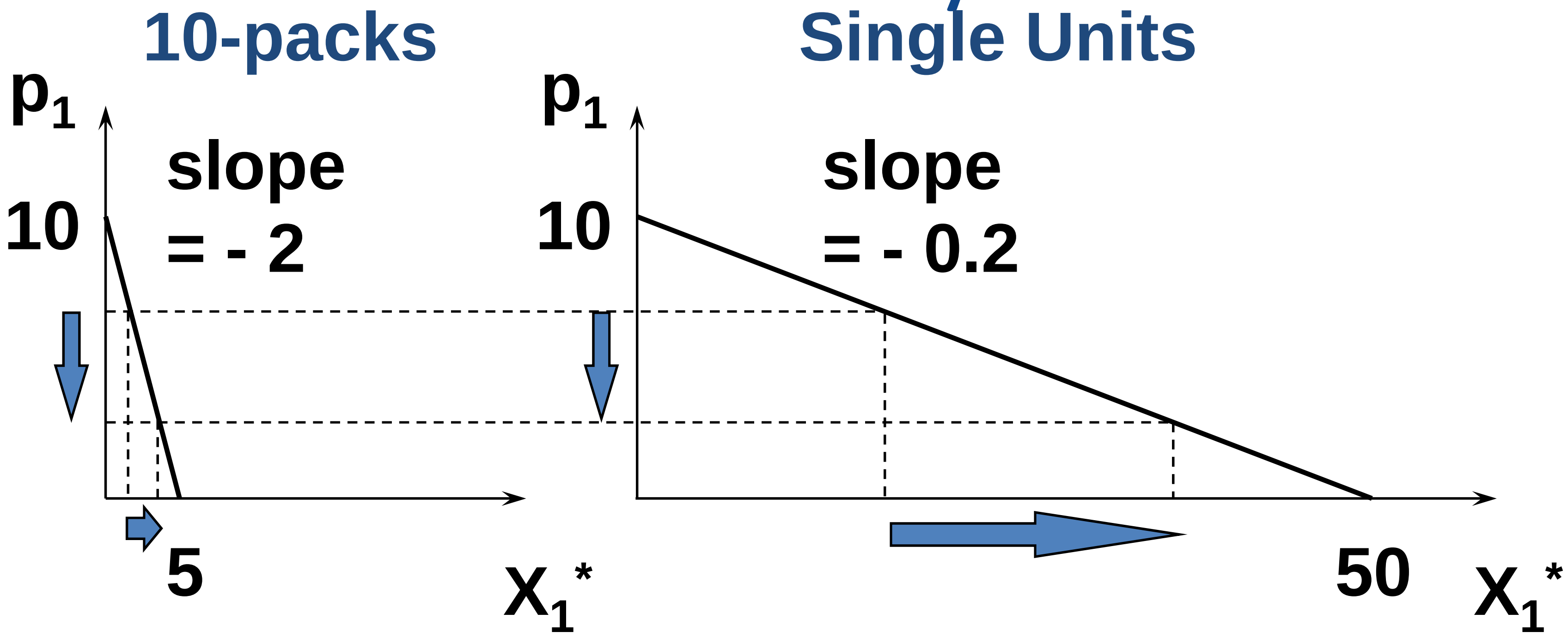
In which case is the quantity demanded X_1^* more sensitive to changes to p_1?

Own-Price Elasticity of Demand



In which case is the quantity demanded X_1^* more sensitive to changes to p_1?

Own-Price Elasticity of Demand



In which case is the quantity demanded X_1* more sensitive to changes to p_1?

Own-Price Elasticity of Demand

- It would be silly if “sensitivity” depends upon the (arbitrary) units of measurement for quantity

Own-Price Elasticity of Demand

$$\varepsilon_{x_1^*,p_1} = \frac{\%\Delta x_1^*}{\%\Delta p_1}$$

is a ratio of percentages and so has no units of measurement.
Hence own-price elasticity of demand is a sensitivity measure that is independent of units of measurement.

Own-Price Elasticity

$$\varepsilon_{X_i^*,p_i} = \frac{p_i}{X_i^*} \times \frac{dX_i^*}{dp_i}$$

E.g. Suppose $p_i = a - bX_i$.
Then $X_i = (a-p_i)/b$ and

$$\frac{dX_i^*}{dp_i} = -\frac{1}{b}.$$ **Therefore,**

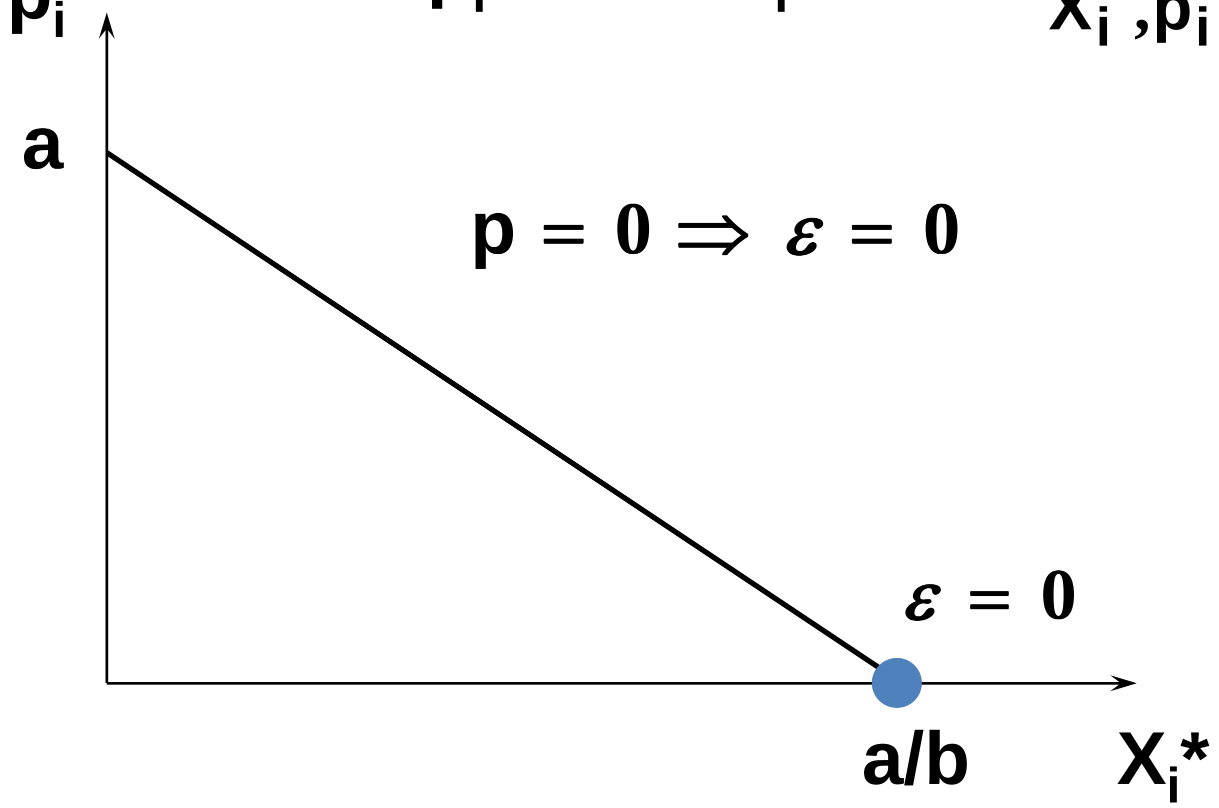
$$\varepsilon_{X_i^*,p_i} = \frac{p_i}{(a-p_i)/b} \times \left(-\frac{1}{b}\right) = -\frac{p_i}{a-p_i}.$$

Point Own-Price Elasticity

$$p_i = a - bX_i^*$$

$$\varepsilon_{X_i^*, p_i} = -\frac{p_i}{a - p_i}$$

$$p = 0 \Rightarrow \varepsilon = 0$$

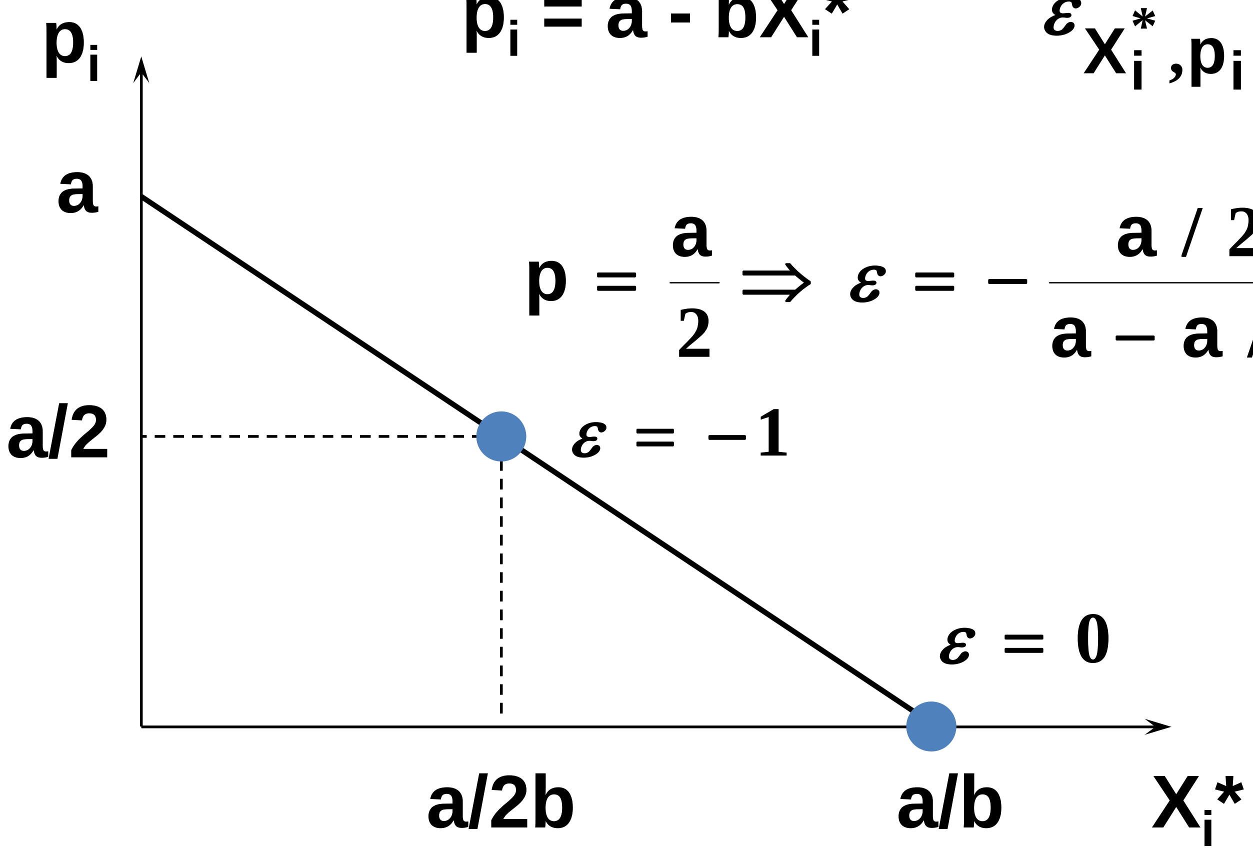


Point Own-Price Elasticity

$$p_i = a - bX_i^*$$

$$\varepsilon_{X_i^*, p_i} = -\frac{p_i}{a - p_i}$$

$$p = \frac{a}{2} \Rightarrow \varepsilon = -\frac{a/2}{a - a/2} = -1$$

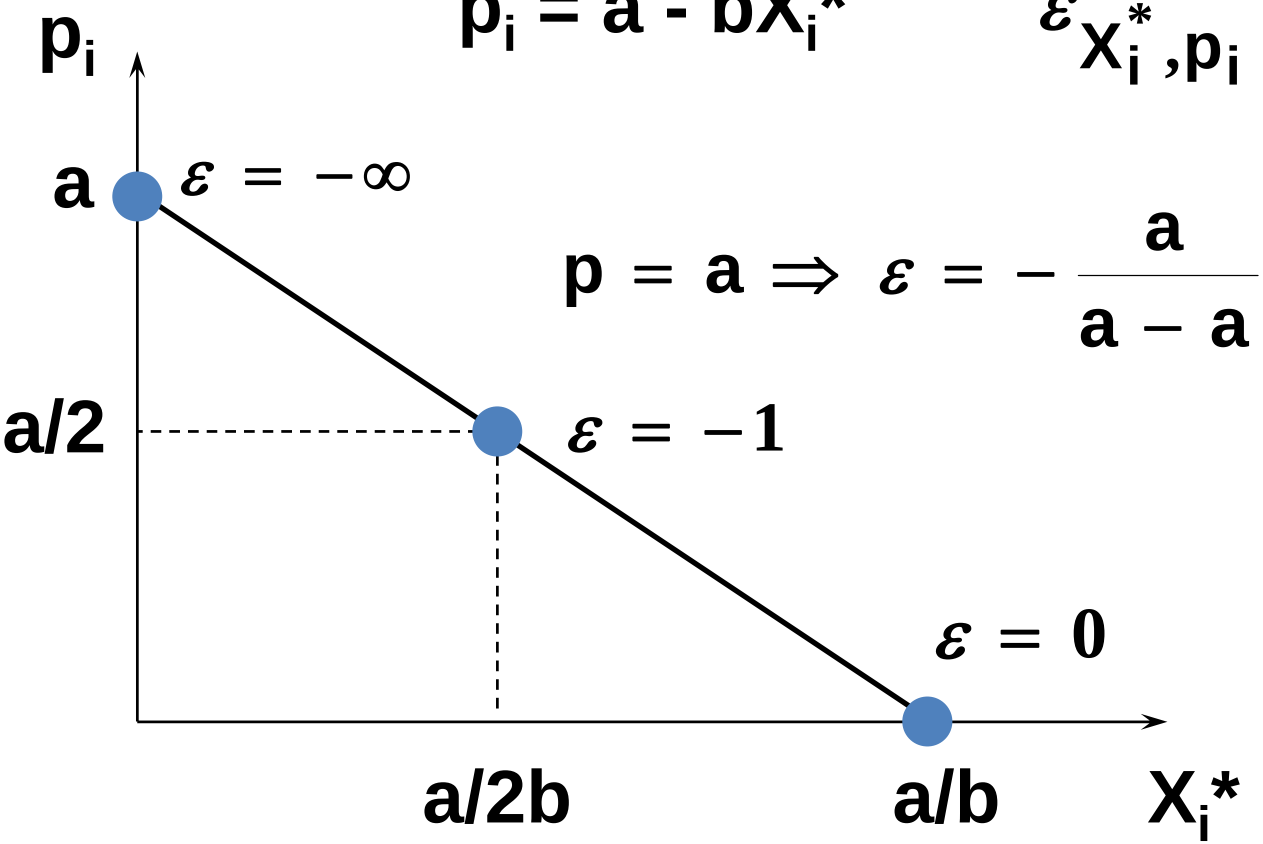


Point Own-Price Elasticity

$$p_i = a - bX_i^*$$

$$\varepsilon_{X_i^*, p_i} = -\frac{p_i}{a - p_i}$$

$$p = a \Rightarrow \varepsilon = -\frac{a}{a - a} = -\infty$$



Point Own-Price Elasticity

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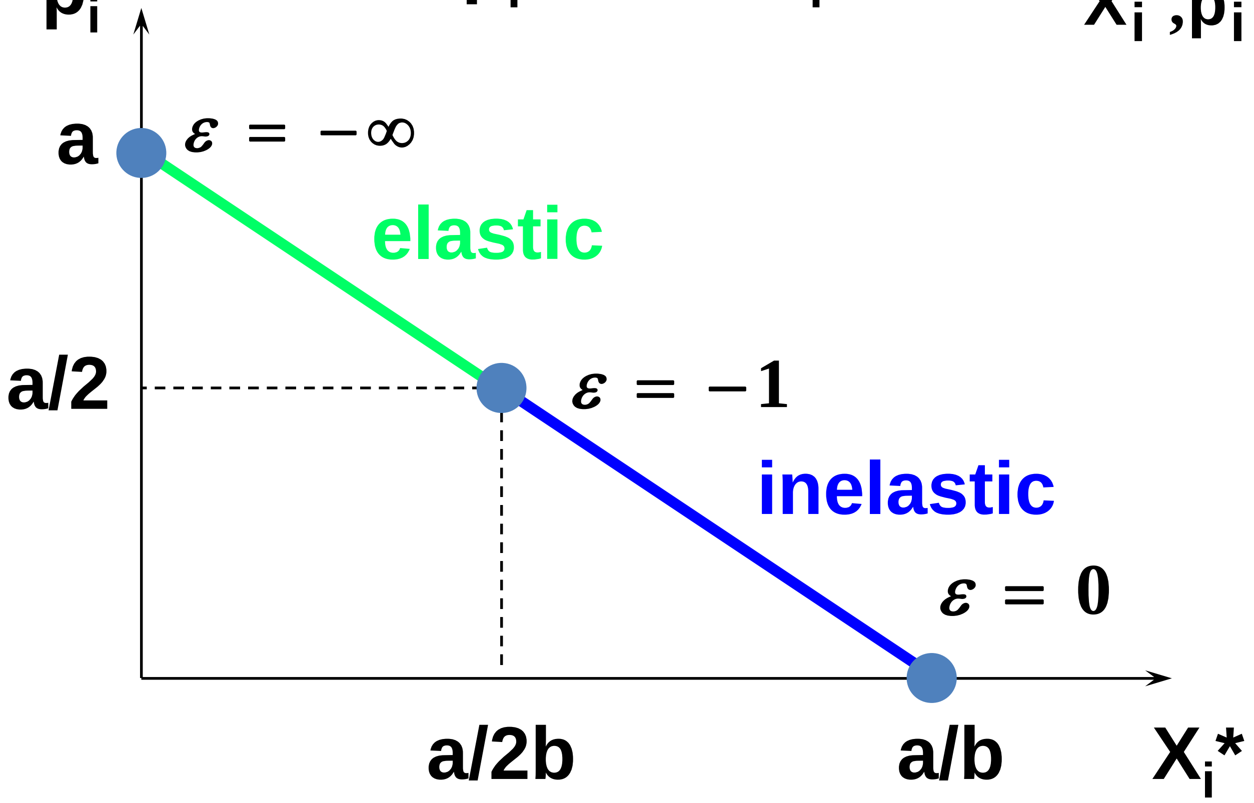


Revenue and Own-Price Elasticity of Demand

- If raising a commodity's price causes a large decrease in quantity demanded, then sellers' revenues fall.
- Hence own-price elastic demand causes sellers' revenues to fall as price rises.

Revenue and Own-Price Elasticity of Demand

Sellers' revenue is $R(p) = p \times X^*(p).$

So $$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$

$$= X^*(p)\left[1 + \frac{p}{X^*(p)}\frac{dX^*}{dp}\right]$$

$$= X^*(p)[1 + \varepsilon].$$

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^{*}(p)[1+\varepsilon]$$

so if $\varepsilon = -1$ **then** $\frac{dR}{dp} = 0$

and a change to price does not alter sellers' revenue.

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)[1+\varepsilon]$$

but if $-1 < \varepsilon \leq 0$ **then** $\frac{dR}{dp} > 0$

and a price increase raises sellers' revenue.

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)\left[1+\varepsilon\right]$$

And if $\varepsilon < -1$ **then** $\dfrac{dR}{dp} < 0$

and a price increase reduces sellers' revenue.

Revenue and Own-Price Elasticity of Demand

In summary:

Own-price inelastic demand; $-1 < \varepsilon \le 0$
price rise causes rise in sellers' revenue.

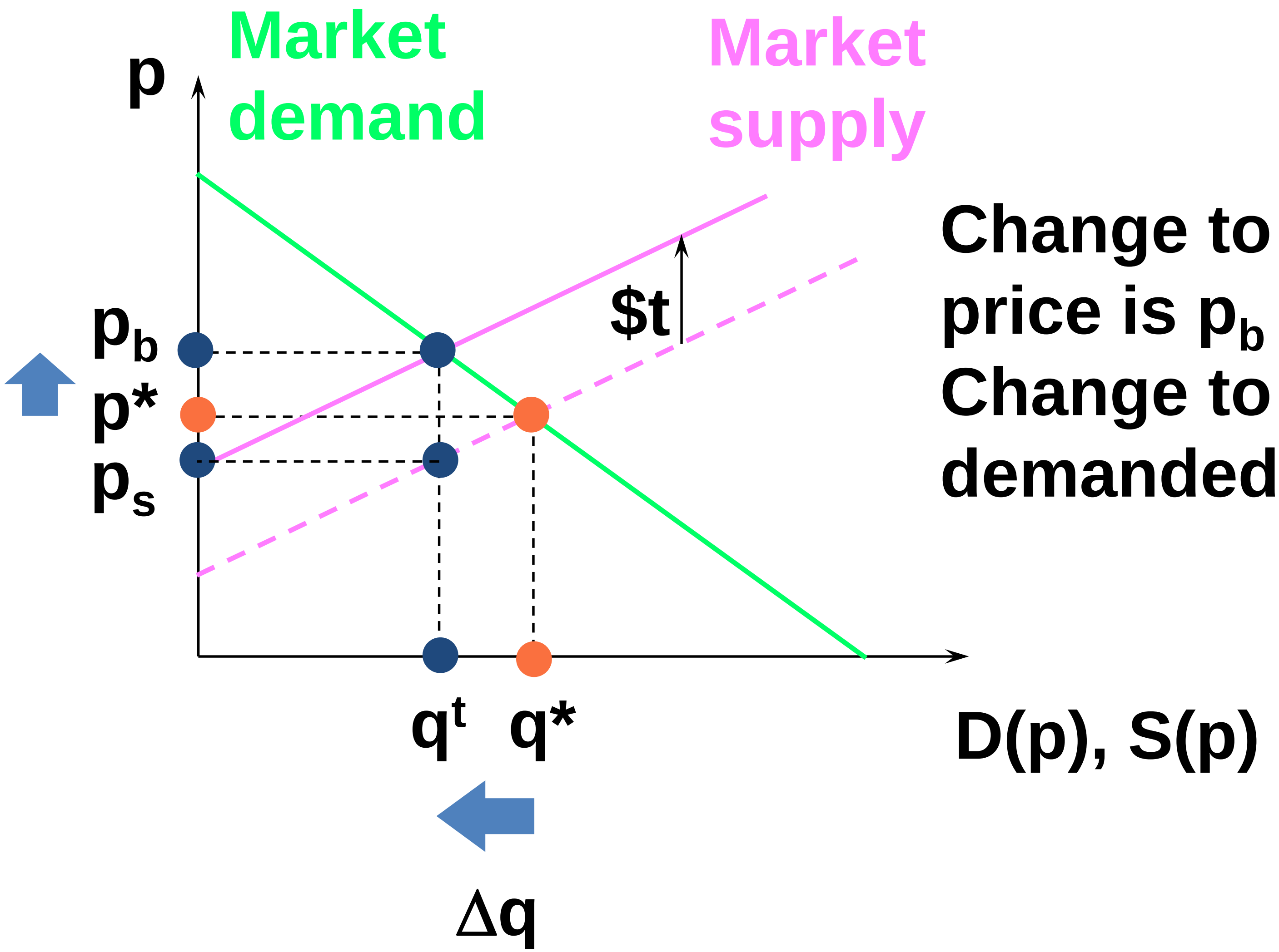
Own-price unit elastic demand; $\varepsilon = -1$
price rise causes no change in sellers' revenue.

Own-price elastic demand; $\varepsilon < -1$
price rise causes fall in sellers' revenue.

Tax Incidence and Own-Price Elasticities

- The incidence of a quantity tax depends upon the own-price elasticities of demand and supply.

Tax Incidence and Own-Price Elasticities



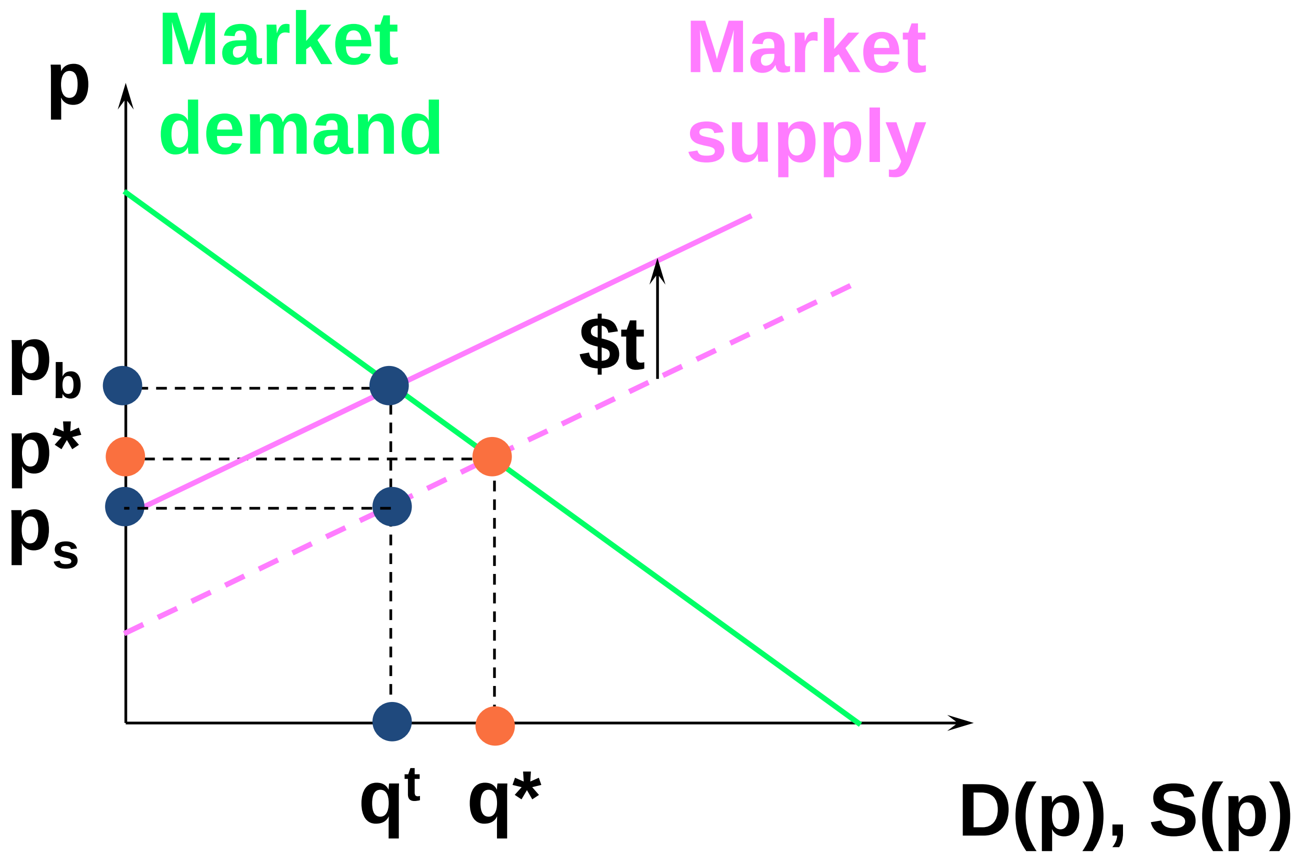
Change to buyers' price is p_b - p*.
Change to quantity demanded is Δq.

Tax Incidence and Own-Price Elasticities

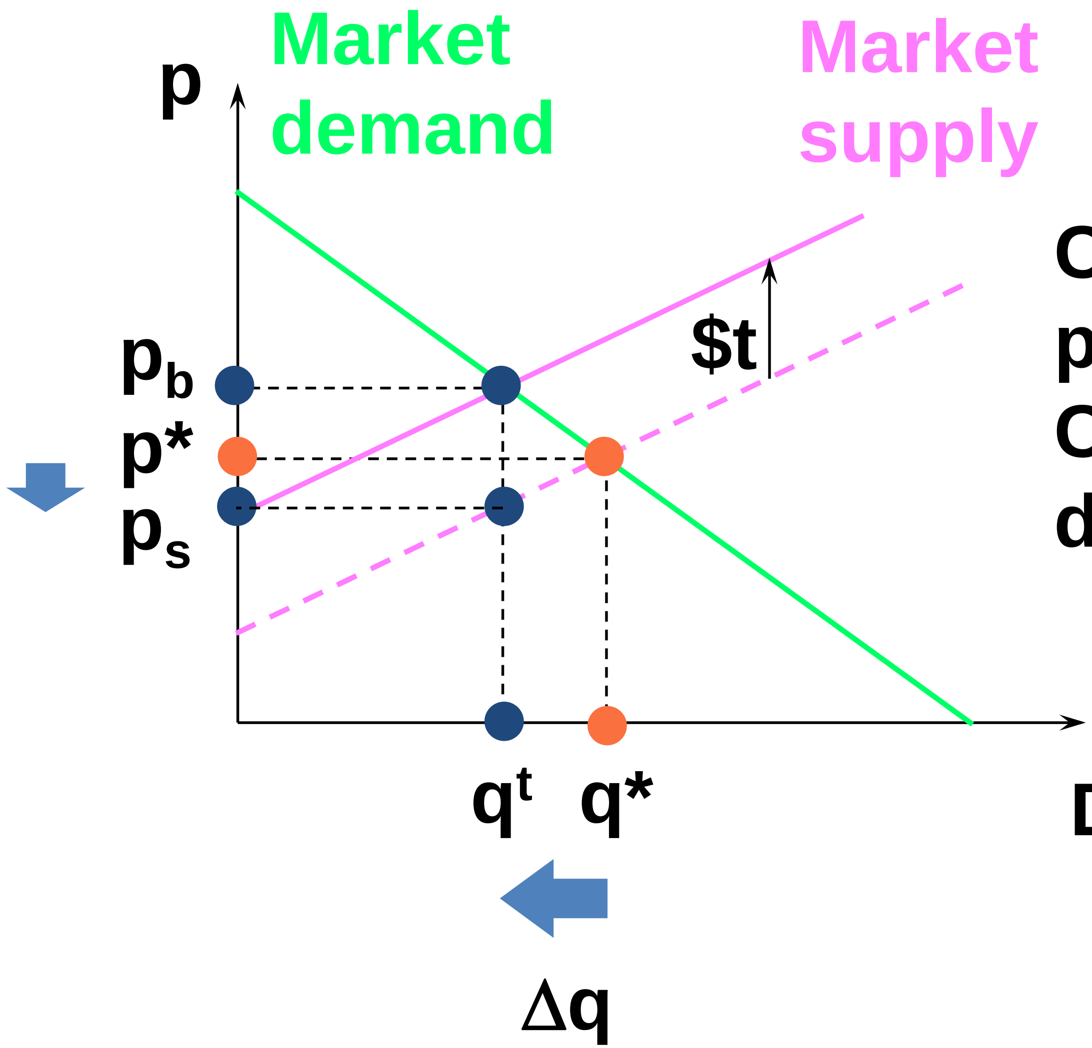
Around p = p* the own-price elasticity of demand is approximately

$$\varepsilon_D \approx \frac{\dfrac{\Delta q}{q^*}}{\dfrac{p_b - p^*}{p^*}} \quad \Rightarrow \quad p_b - p^* \approx \frac{\Delta q \times p^*}{\varepsilon_D \times q^*}.$$

Tax Incidence and Own-Price Elasticities



Tax Incidence and Own-Price Elasticities



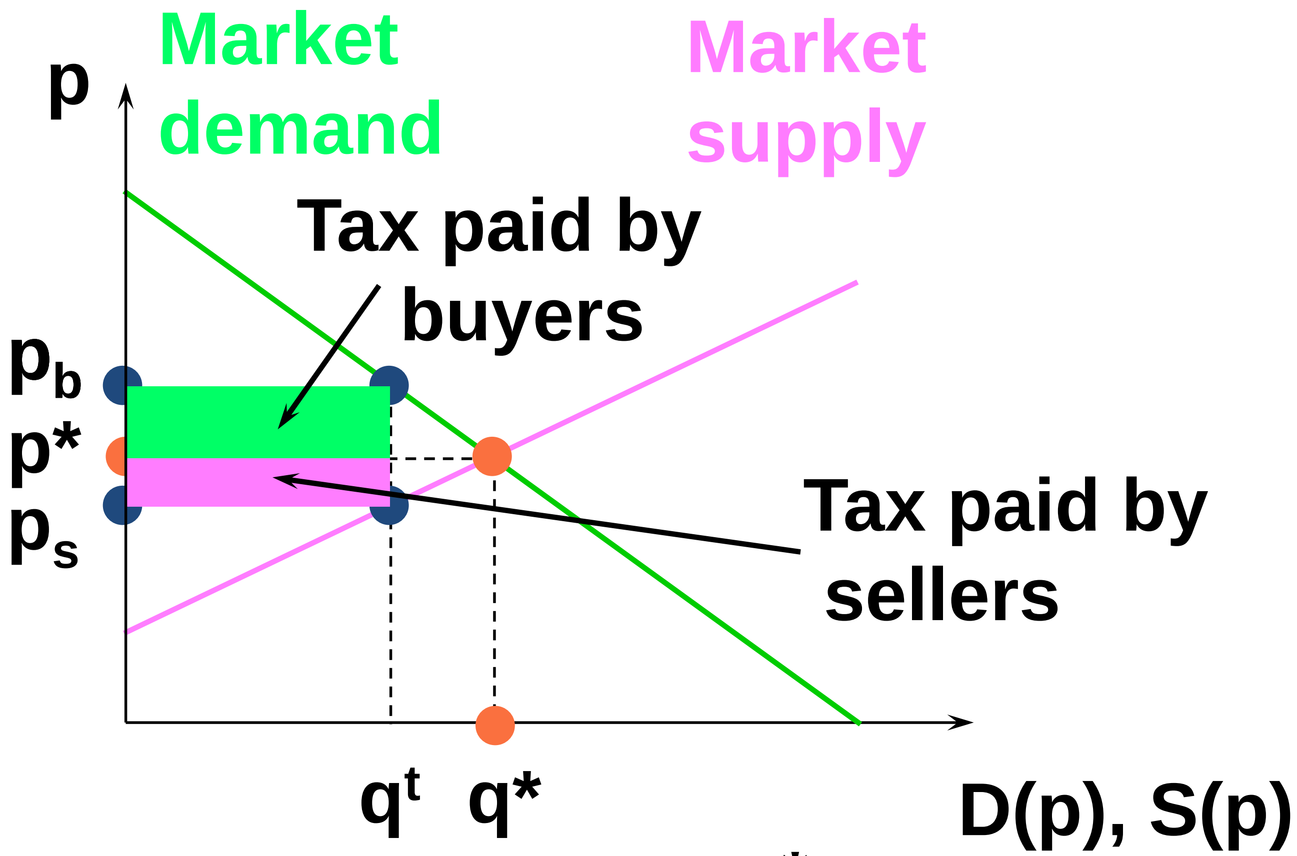
Change to sellers' price is p_s - p*.
Change to quantity demanded is Δq.

Tax Incidence and Own-Price Elasticities

Around p = p* the own-price elasticity of supply is approximately

$$\varepsilon_S \approx \frac{\dfrac{\Delta q}{q^*}}{\dfrac{p_s - p^*}{p^*}} \quad \Rightarrow \quad p_s - p^* \approx \frac{\Delta q \times p^*}{\varepsilon_S \times q^*}.$$

Tax Incidence and Own-Price Elasticities



$$\text{Tax incidence} = \frac{p_b - p^*}{p^* - p_s}.$$

Tax Incidence and Own-Price Elasticities

$$\textbf{Tax incidence} = \frac{p_b - p^*}{p^* - p_s}.$$

$$p_b - p^* \approx \frac{\Delta q \times p^*}{\varepsilon_D \times q^*}. \qquad p_s - p^* \approx \frac{\Delta q \times p^*}{\varepsilon_S \times q^*}.$$

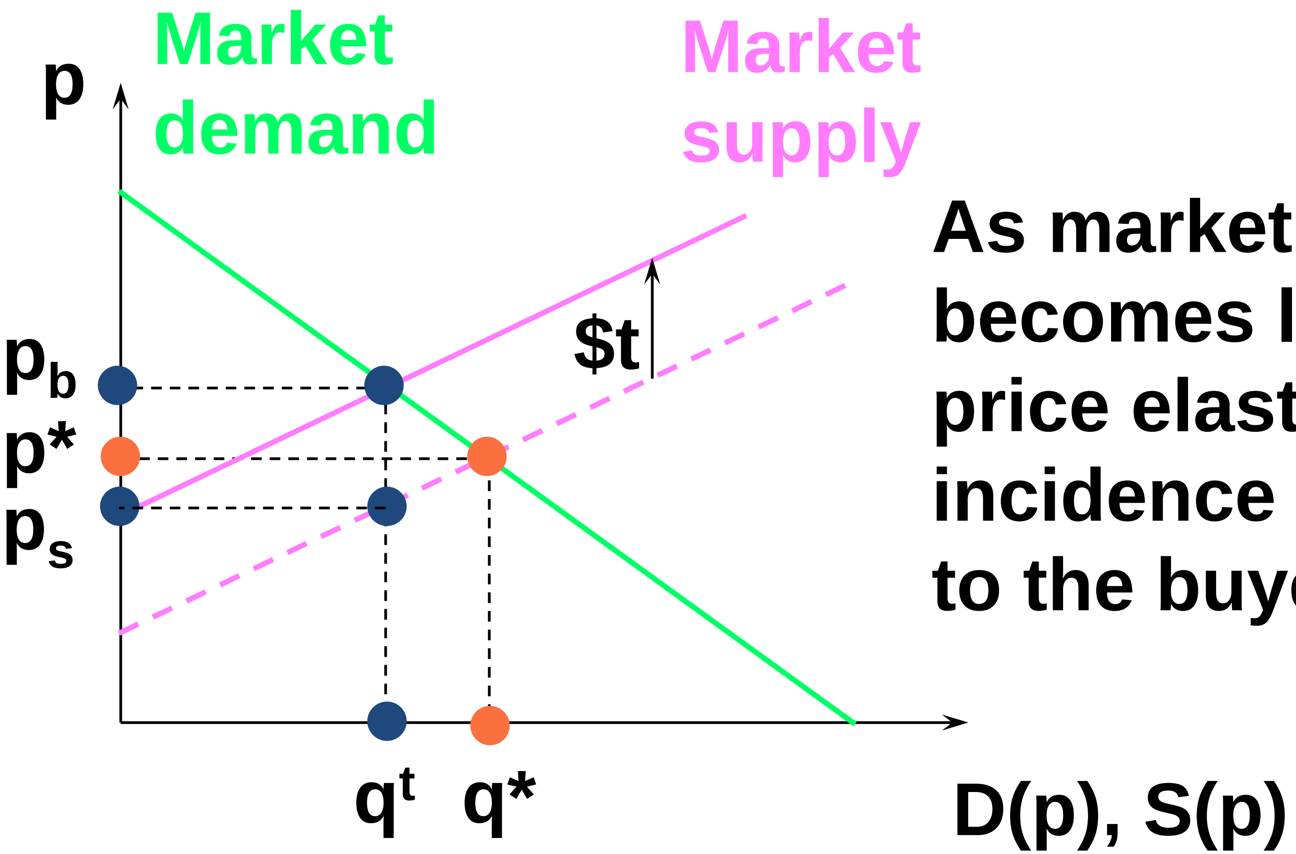
So
$$\frac{p_b - p^*}{p^* - p_s} \approx -\frac{\varepsilon_S}{\varepsilon_D}.$$

Tax Incidence and Own-Price Elasticities

Tax incidence is $$\frac{p_b - p^*}{p^* - p_s} \approx -\frac{\varepsilon_S}{\varepsilon_D}.$$

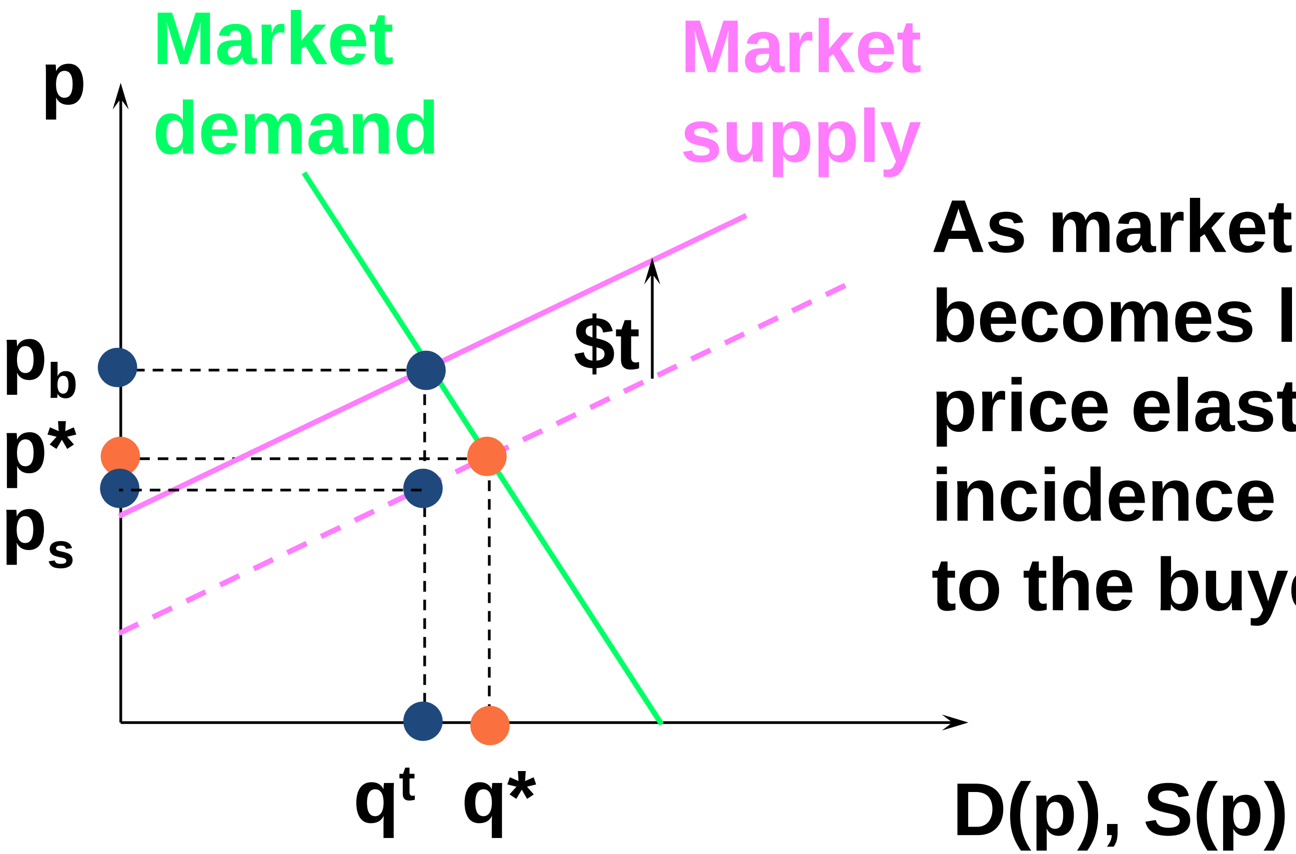
The fraction of a \$t quantity tax paid by buyers rises as supply becomes more own-price elastic or as demand becomes less own-price elastic.

Tax Incidence and Own-Price Elasticities



As market demand becomes less own-price elastic, tax incidence shifts more to the buyers.

Tax Incidence and Own-Price Elasticities



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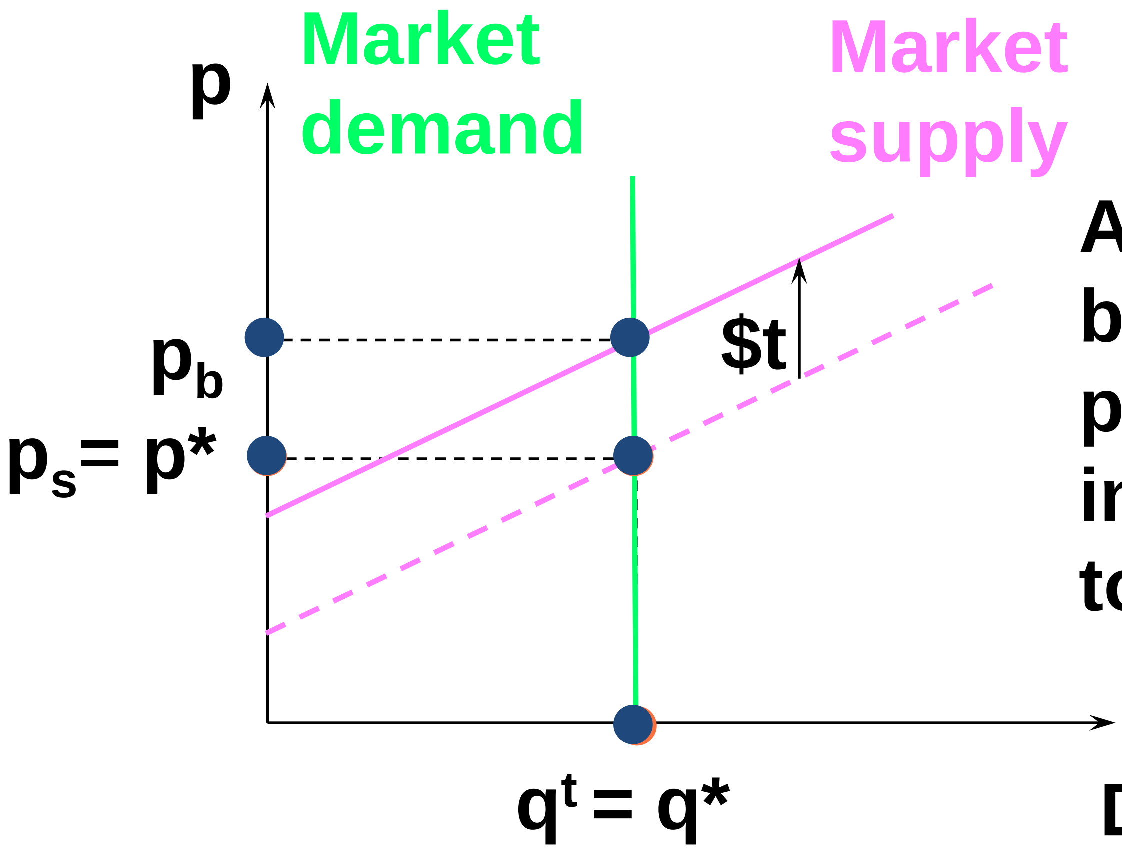
Tax Incidence and Own-Price Elasticities



As market demand becomes less own-price elastic, tax incidence shifts more to the buyers.

When $\varepsilon_D = 0$, buyers pay the entire tax, even though it is levied on the sellers.

Tax Incidence and Own-Price Elasticities

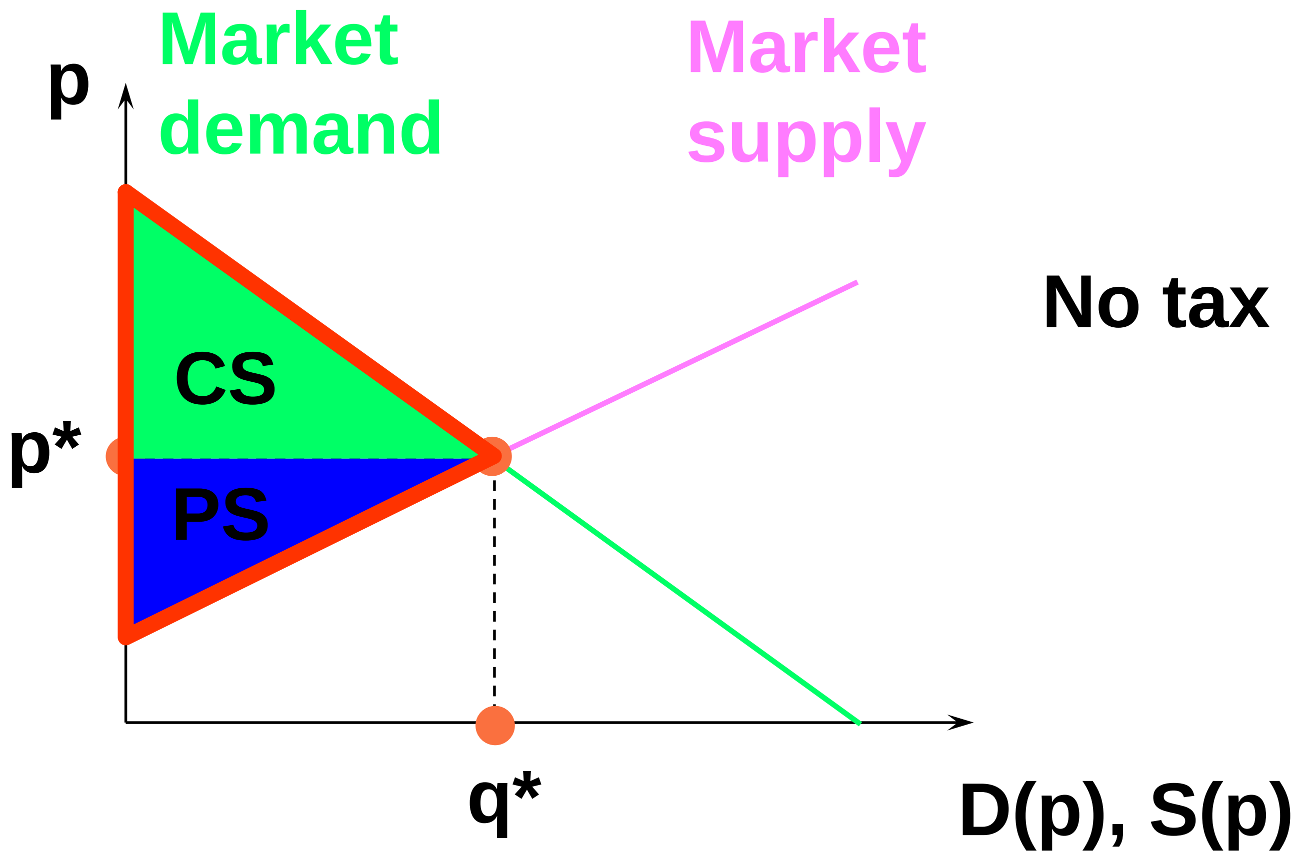
Tax incidence is $$\frac{p_b - p^*}{p^* - p_s} \approx -\frac{\varepsilon_S}{\varepsilon_D}.$$

Similarly, the fraction of a \$t quantity tax paid by sellers rises as supply becomes less own-price elastic or as demand becomes more own-price elastic.

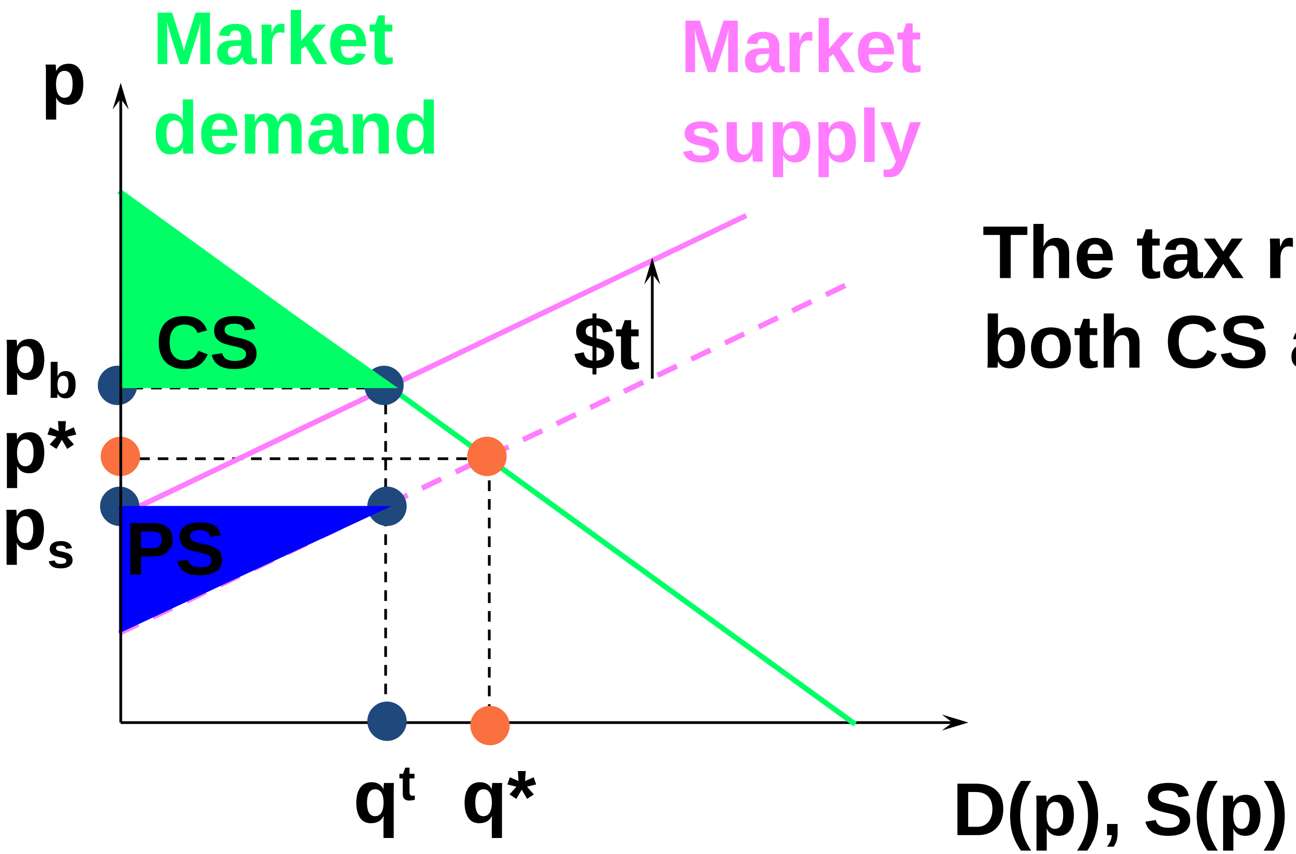
Deadweight Loss and Own-Price Elasticities

- A quantity tax imposed on a competitive market reduces the quantity traded and so reduces gains-to-trade (*i.e.* the sum of Consumers' and Producers' Surpluses).
- The lost total surplus is the tax's deadweight loss, or excess burden.

Deadweight Loss and Own-Price Elasticities

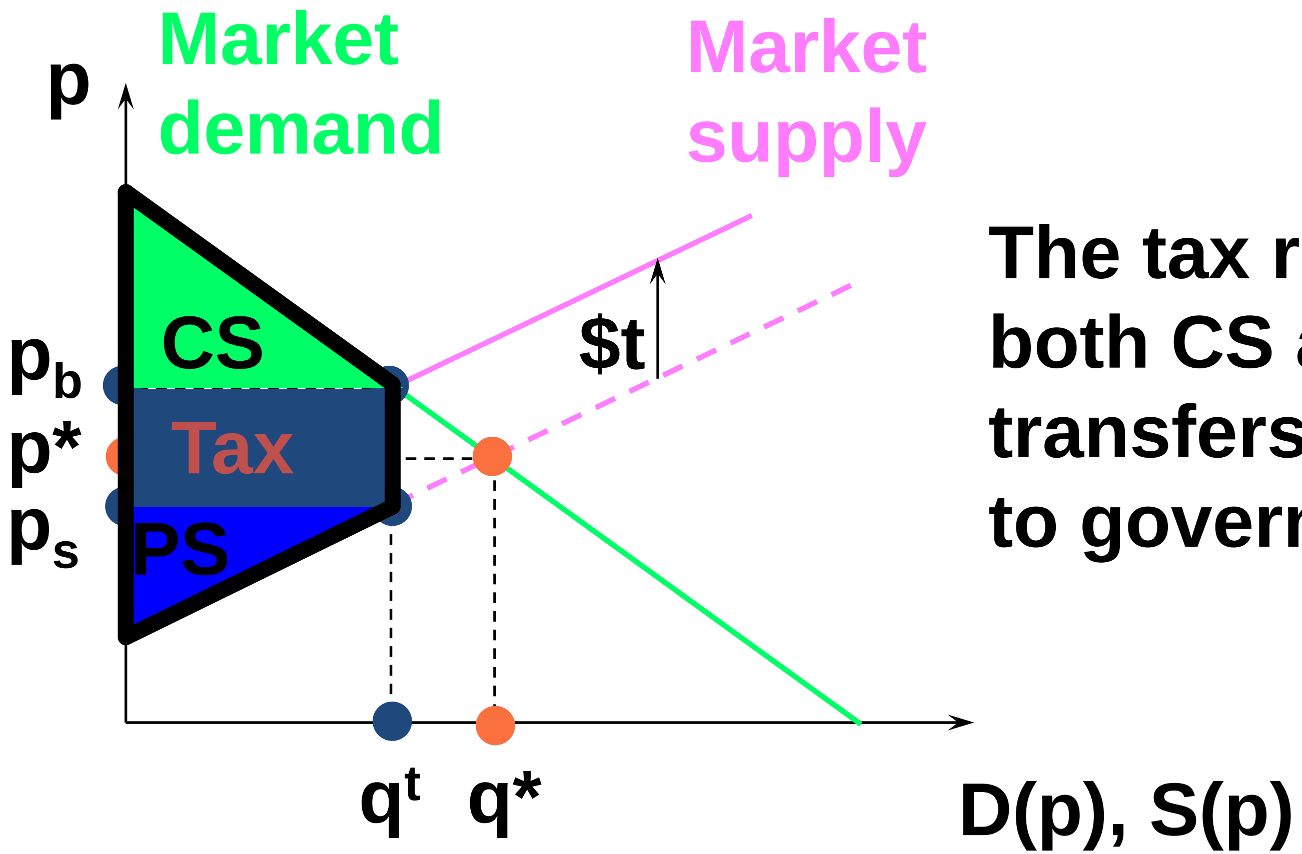


Deadweight Loss and Own-Price Elasticities



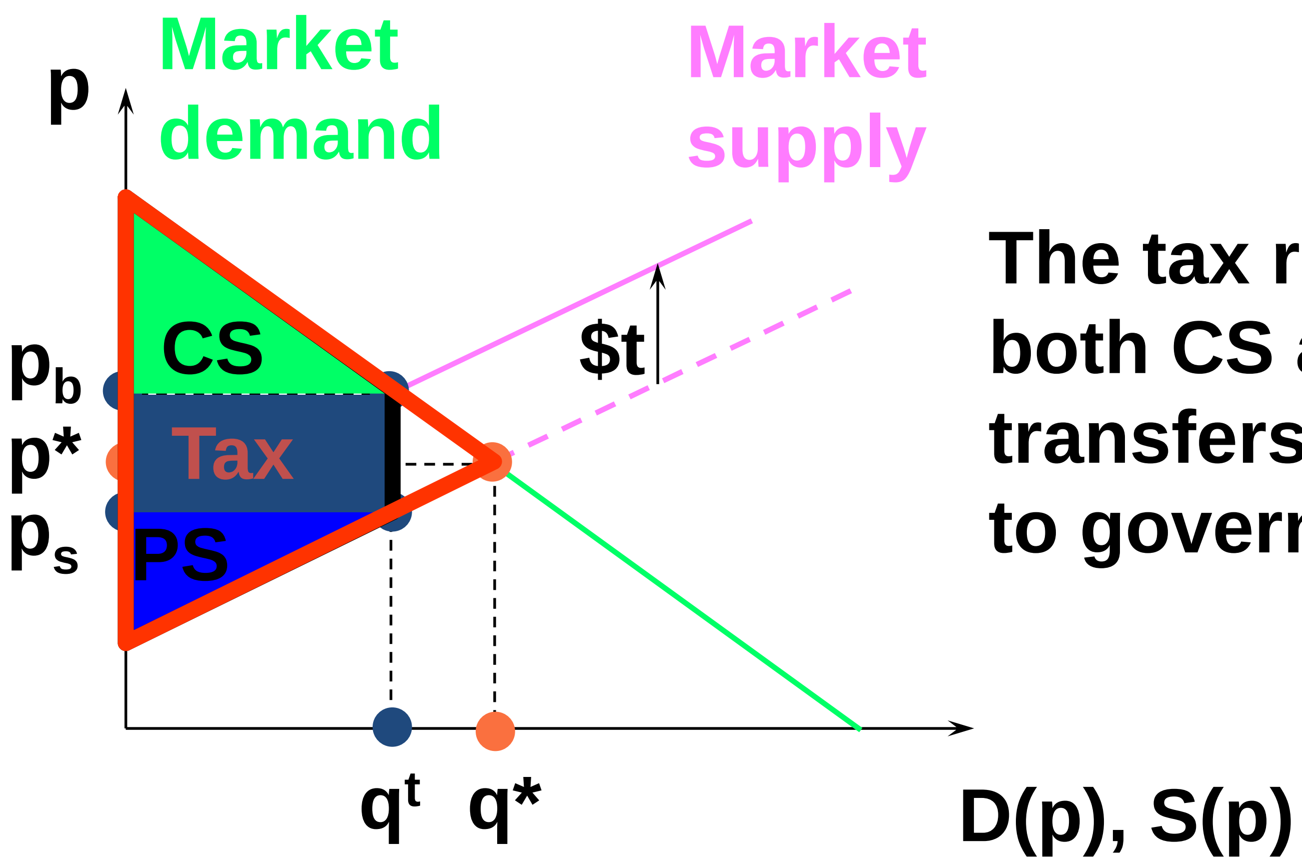
The tax reduces both CS and PS

Deadweight Loss and Own-Price Elasticities



The tax reduces both CS and PS, transfers surplus to government

Deadweight Loss and Own-Price Elasticities



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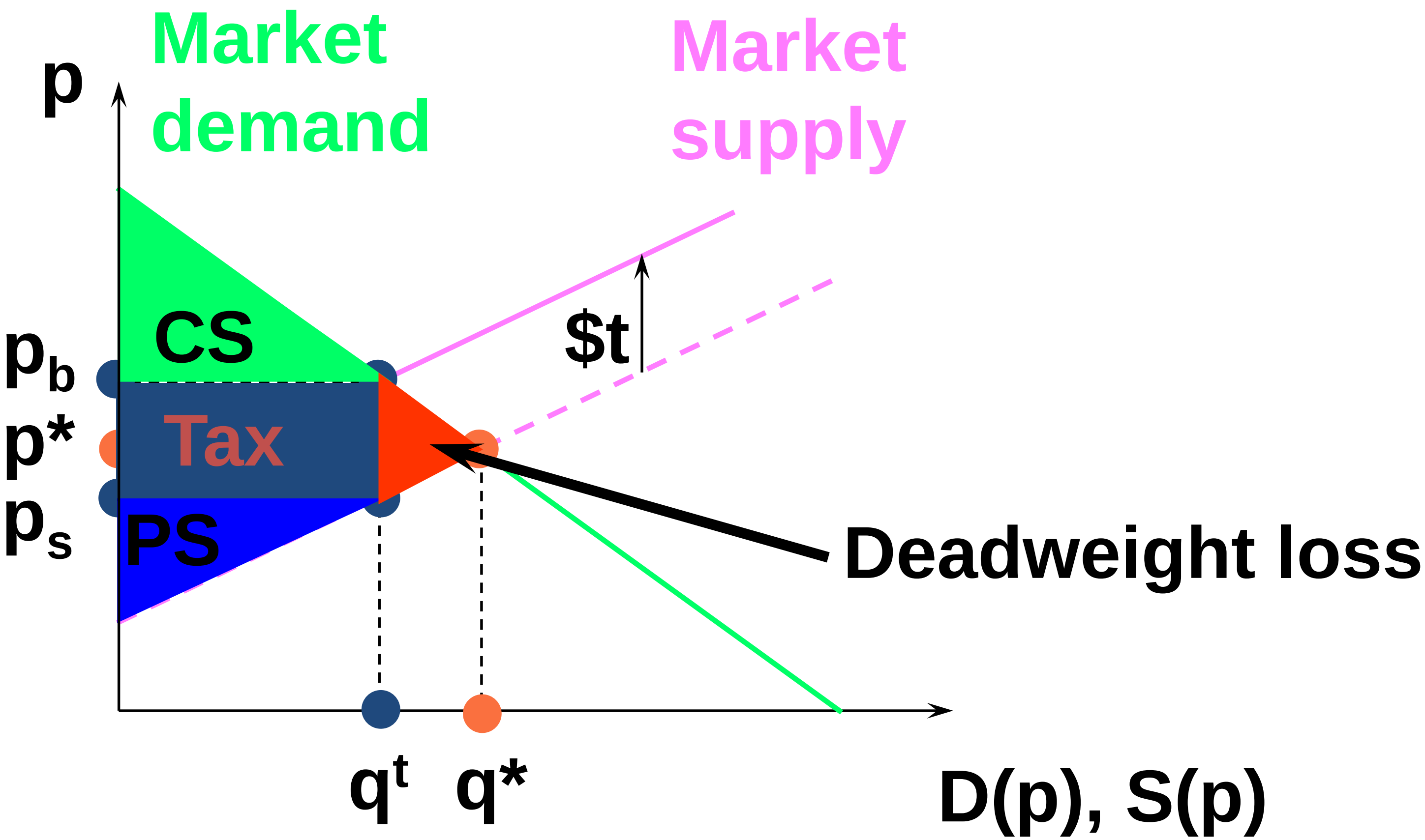
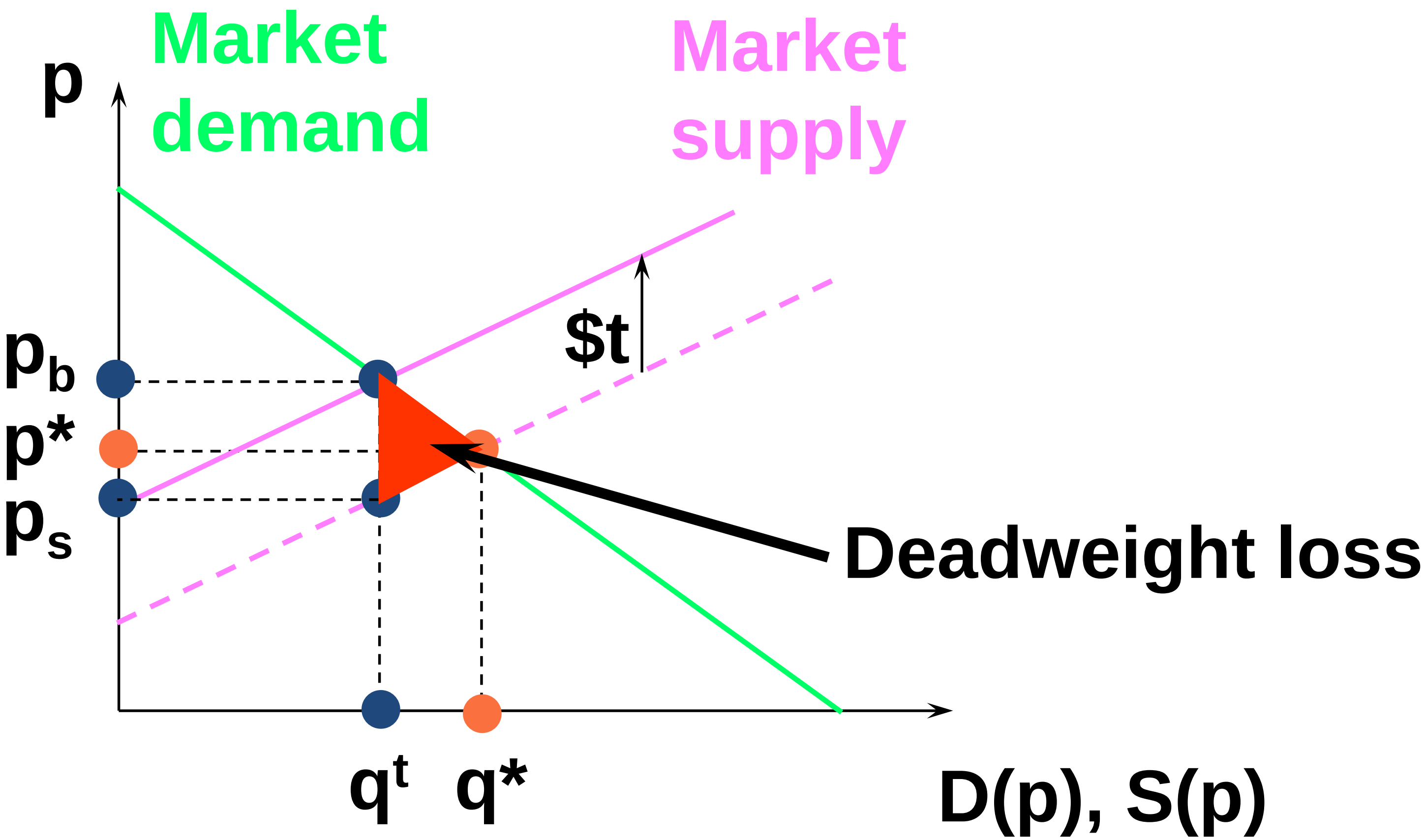
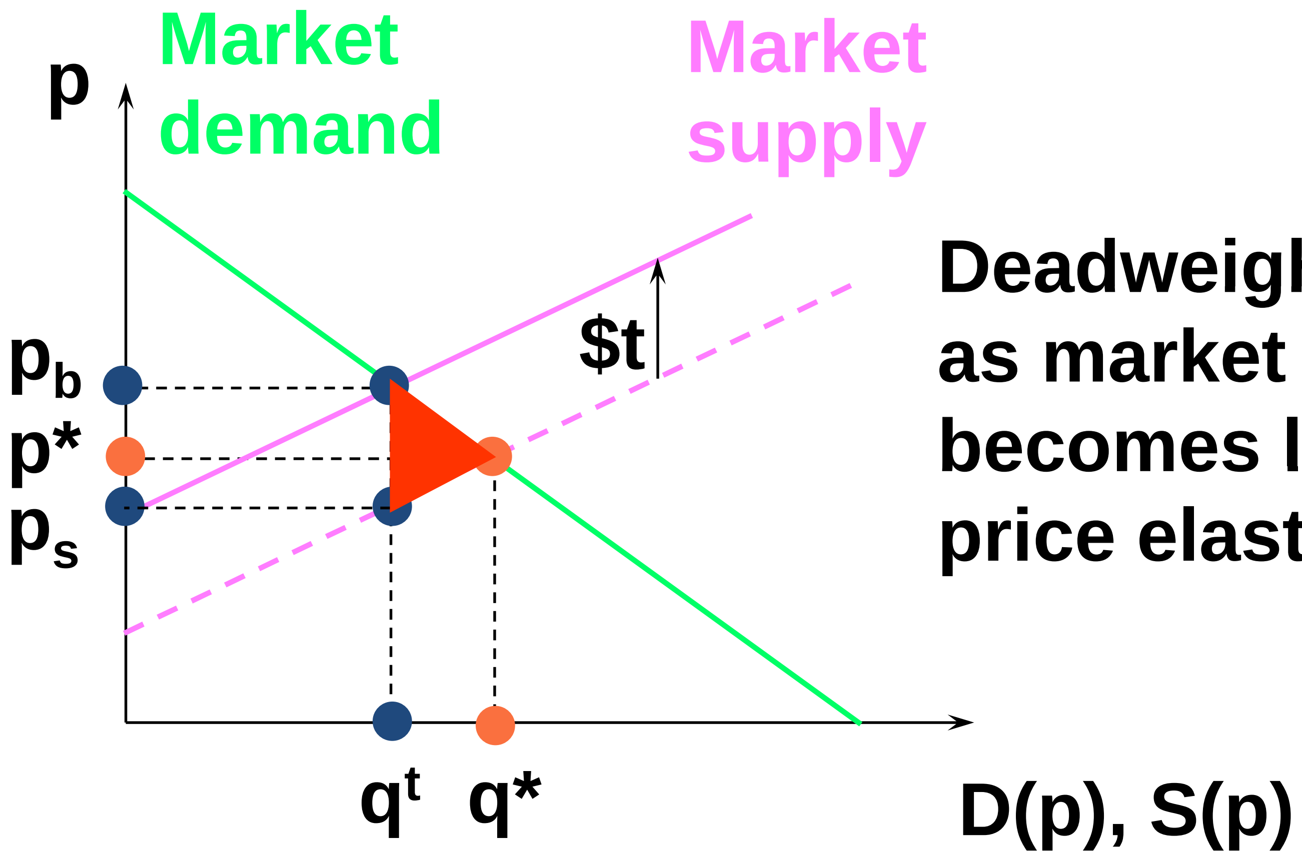
Deadweight Loss and Own-Price Elasticities



Deadweight Loss and Own-Price Elasticities

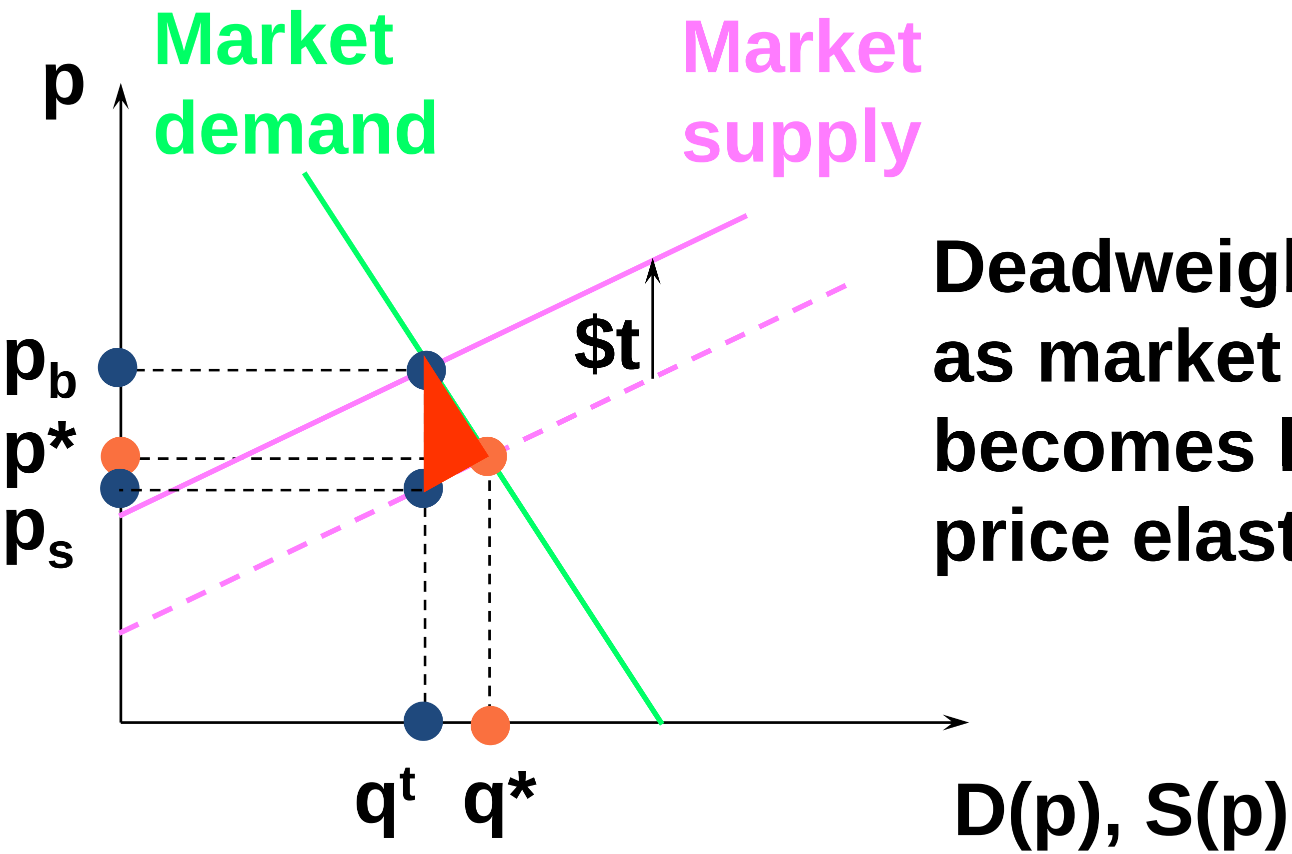


Deadweight Loss and Own-Price Elasticities



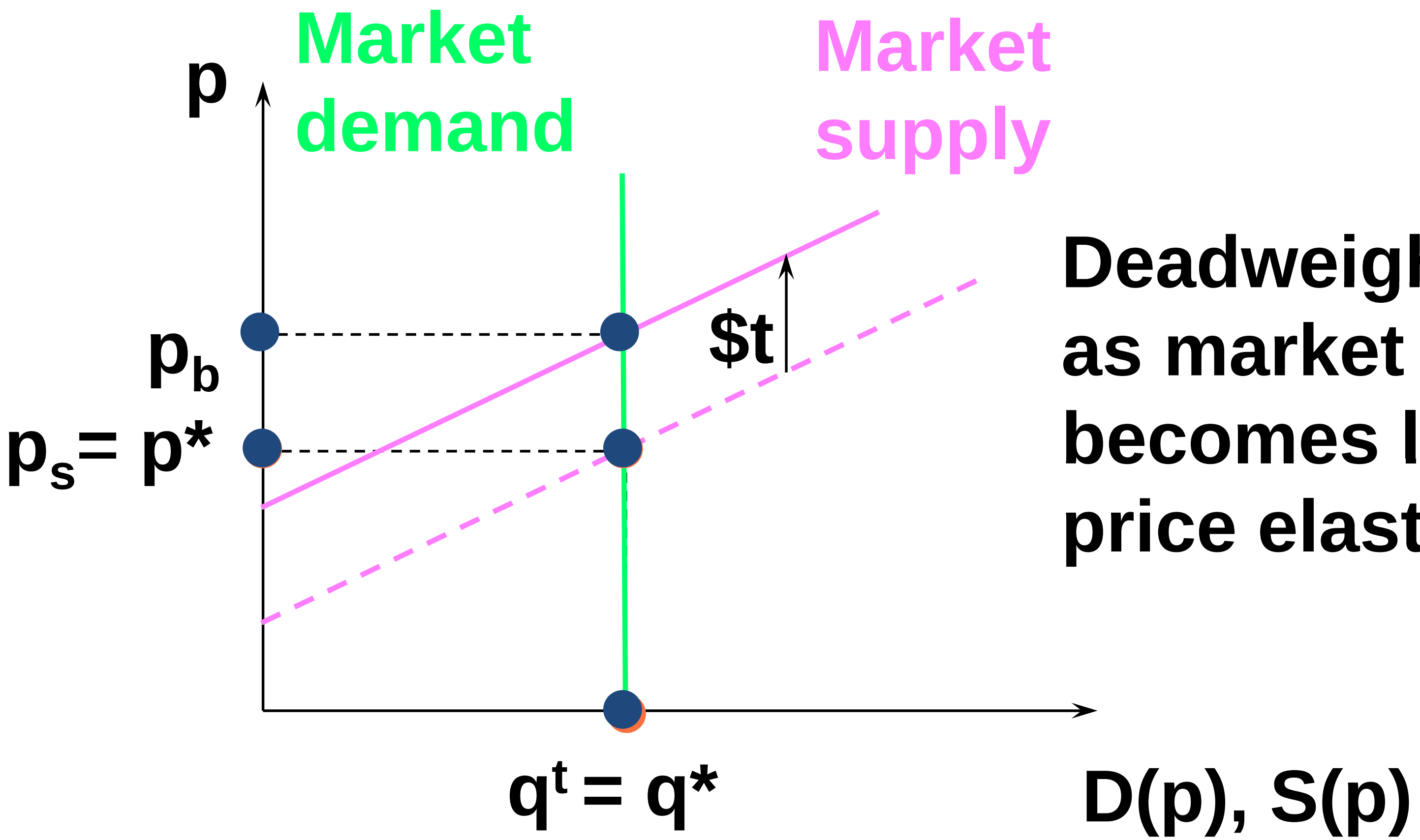
Deadweight loss falls as market demand becomes less own-price elastic.

Deadweight Loss and Own-Price Elasticities



Deadweight loss falls as market demand becomes less own-price elastic.

Deadweight Loss and Own-Price Elasticities



Deadweight loss falls as market demand becomes less own-price elastic.

When $\varepsilon_D = 0$, the tax causes no deadweight loss.

Deadweight Loss and Own-Price Elasticities

- Deadweight loss due to a quantity tax rises as either market demand or market supply becomes more own-price elastic.
- If either $\varepsilon_D = 0$ or $\varepsilon_S = 0$ then the deadweight loss is zero.