# Chapter Four

- Utility

# What are we trying to do?

- We have a sense of the bundles from which to choose
- We want to know which bundle to choose

# What do we have?

## Preference Relations

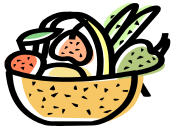
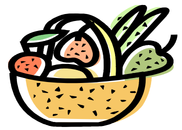
- x $\succ$ y: x is preferred strictly to y.
- x $\sim$ y: x and y are equally preferred.
- x $\succsim$ y: x is preferred at least as much as is y.

# A More Useful Description: Utility Functions

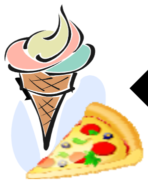
- A preference relation that is complete, reflexive, transitive and continuous can be represented by a continuous utility function.
- Continuity means that small changes to a consumption bundle cause only small changes to the preference level.

# Utility Functions

- A utility function U(x) represents a preference relation $\succsim$ if and only if:

 $\succ$  $\iff$ **U(****) > U(**

$$x' \prec x'' \iff U(x') < U(x'')$$

 $\sim$  $\iff$ **U(****) = U(****)**

# Utility Functions

- Utility is an ordinal (i.e. ordering) concept.
- *E.g*. if U(x) = 6 and U(y) = 2 then bundle x is strictly preferred to bundle y.

# Utility Functions & Indiff. Curves

- Consider the bundles (4,1), (2,3) and (2,2).
- Suppose (2,3) $\succ$ (4,1) $\sim$ (2,2).
- Assign to these bundles any numbers that preserve the preference ordering;
  *e.g.* U(2,3) = 6 > U(4,1) = U(2,2) = 4.
- Call these numbers utility levels.

# Utility Functions & Indiff. Curves

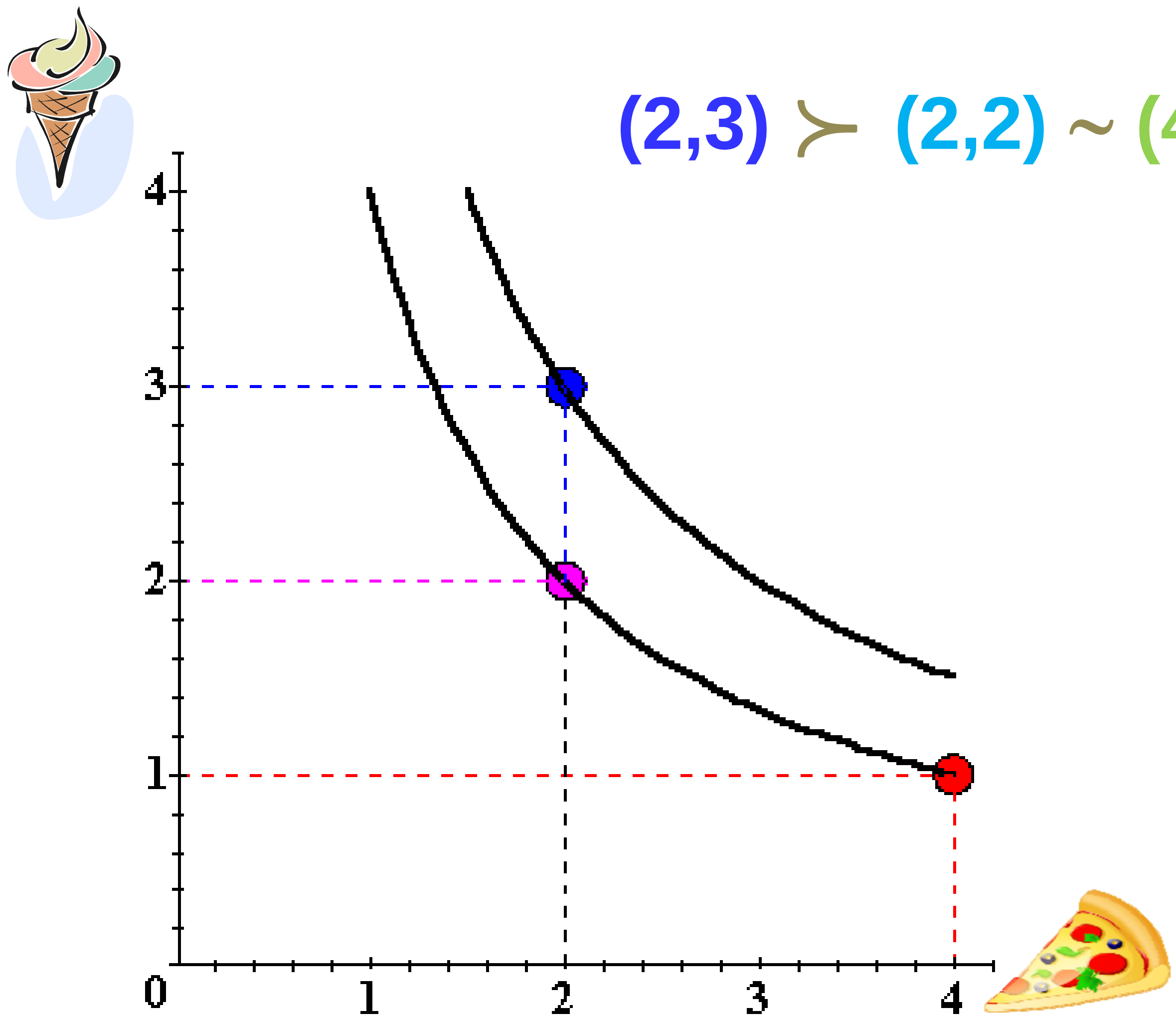
- An indifference curve contains equally preferred bundles.
- Equal preference $\Rightarrow$ same utility level.
- Therefore, all bundles in an indifference curve have the same utility level.

# Utility Functions & Indiff. Curves

- So the bundles (4,1) and (2,2) are in the indiff. curve with utility level $U \equiv 4$
- But the bundle (2,3) is in the indiff. curve with utility level $U \equiv 6$.
- On an indifference curve diagram, this preference information looks as follows:

# Utility Functions & Indiff. Curves

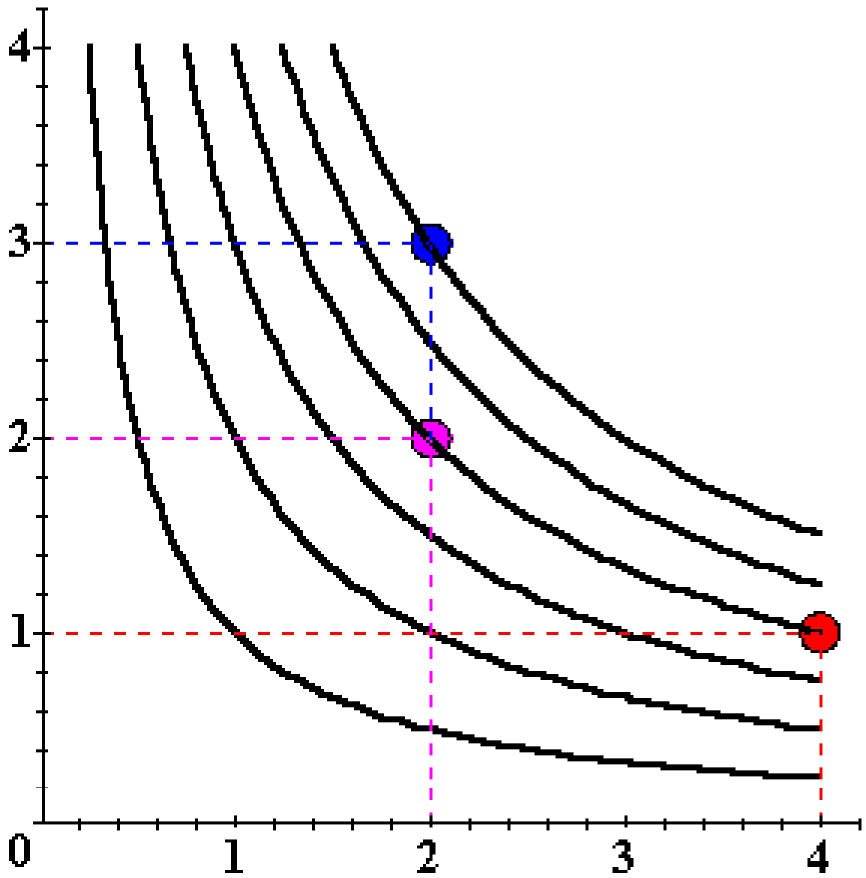
$(2,3) \succ (2,2) \sim (4,1)$



# Utility Functions & Indiff. Curves

- Comparing more bundles will create a larger collection of all indifference curves and a better description of the consumer’s preferences.

# Utility Functions & Indiff. Curves



# Utility Functions & Indiff. Curves

- Comparing all possible consumption bundles gives the complete collection of the consumer's indifference curves, each with its assigned utility level.
- This complete collection of indifference curves completely represents the consumer's preferences.

# Utility Functions & Indiff. Curves

- The collection of all indifference curves for a given preference relation is an indifference map.
- An indifference map is equivalent to a utility function.

# Utility Functions

- There is no unique utility function representation of a preference relation.
- Suppose $U(x_1,x_2) = x_1x_2$ represents a preference relation.
- Again consider the bundles (4,1), (2,3) and (2,2).

# Utility Functions

- $U(x_1,x_2) = x_1x_2$, so

  $U(2,3) = 6 > U(4,1) = U(2,2) = 4$;

  that is, $(2,3) \succ (4,1) \sim (2,2)$.

# Utility Functions

- $U(x_1,x_2) = x_1x_2 \Rightarrow (2,3) \succ (4,1) \sim (2,2)$.
- Define $V = U^2$.

# Utility Functions

- $U(x_1,x_2) = x_1x_2$ $\Rightarrow$ $(2,3) \succ (4,1) \sim (2,2)$.
- Define $V = U^2$.
- Then $V(x_1,x_2) = x_1^2x_2^2$ and
  $V(2,3) = 36 > V(4,1) = V(2,2) = 16$
  so again
  $(2,3) \succ (4,1) \sim (2,2)$.
- V preserves the same order as U and so represents the same preferences.

# Utility Functions

- $U(x_1,x_2) = x_1x_2 \Rightarrow (2,3) \succ (4,1) \sim (2,2)$.
- Define $W = 2U + 10$.

# Utility Functions

- $U(x_1,x_2) = x_1x_2$ $\Rightarrow$ $(2,3) \succ (4,1) \sim (2,2)$.
- Define $W = 2U + 10$.
- Then $W(x_1,x_2) = 2x_1x_2+10$ so $W(2,3) = 22 > W(4,1) = W(2,2) = 18$. Again, $(2,3) \succ (4,1) \sim (2,2)$.
- W preserves the same order as U and V and so represents the same preferences.

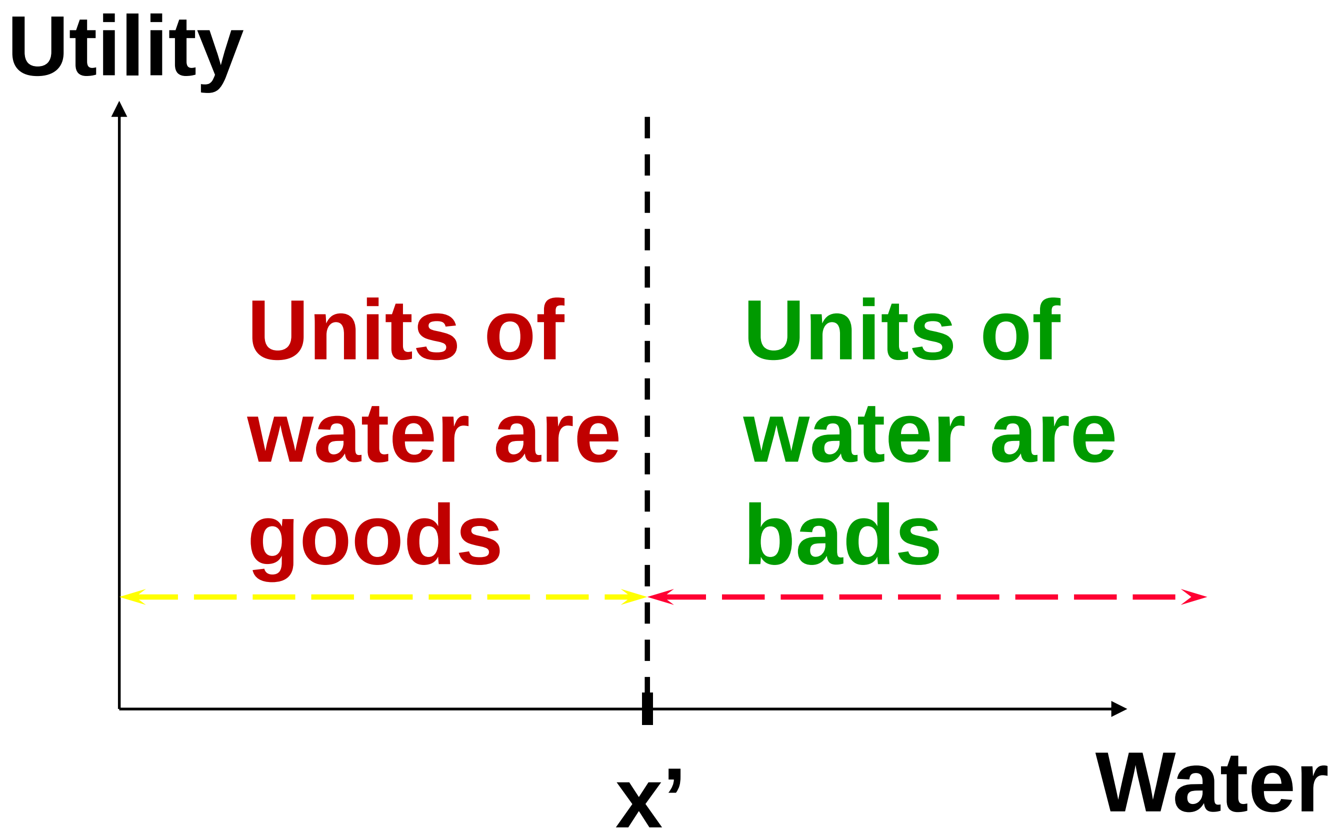
# Utility Functions

- If
  - U is a utility function that represents a preference relation $\succsim$ and
  - f is a strictly increasing function,
- then V = f(U) is also a utility function representing $\succsim$

# Goods, Bads and Neutrals

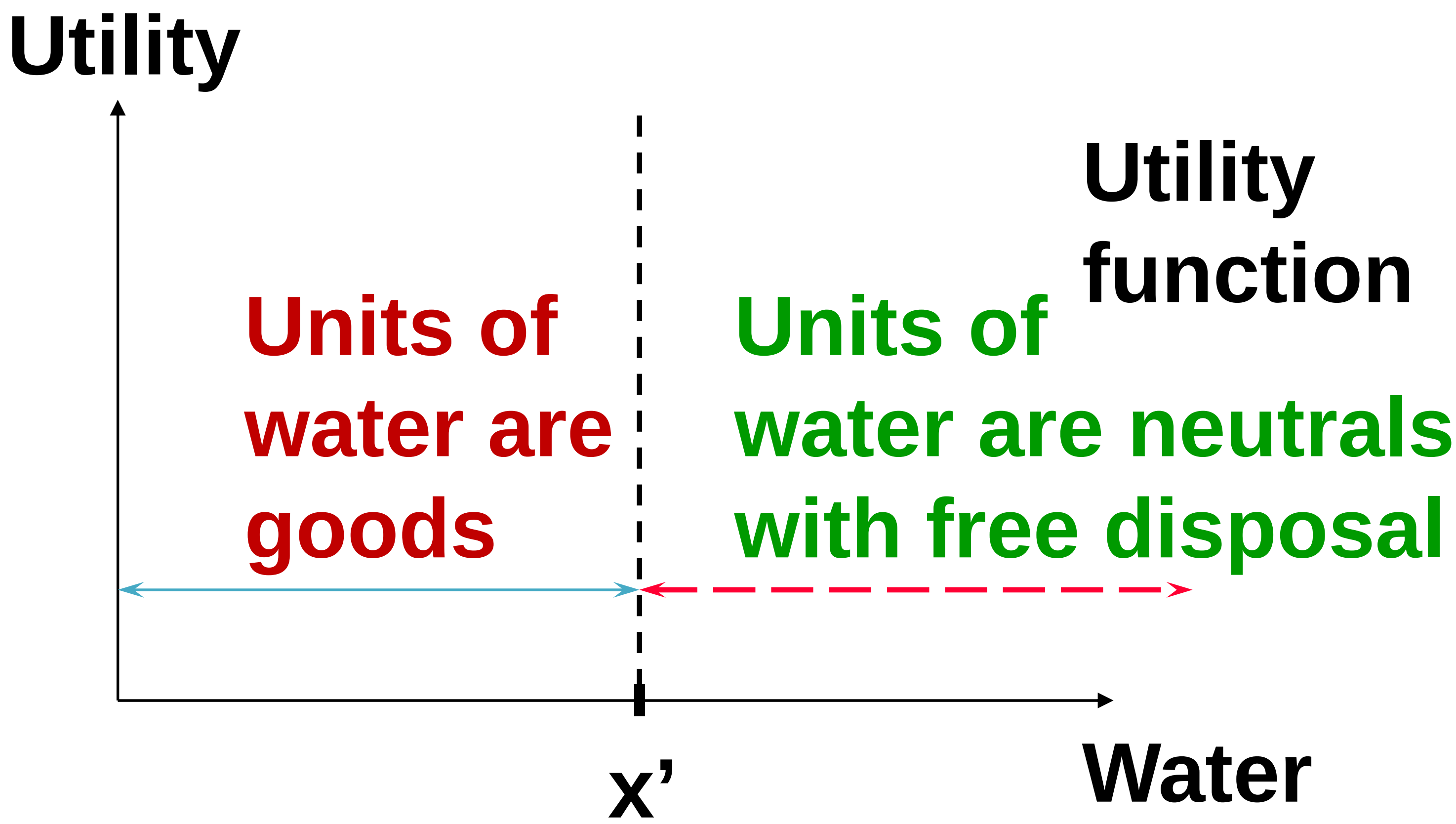
- A good is a commodity unit which increases utility (gives a more preferred bundle).
- A bad is a commodity unit which decreases utility (gives a less preferred bundle).
- A neutral is a commodity unit which does not change utility (gives an equally preferred bundle).

# Goods, Bads and Neutrals




**Around x' units, a little extra water is a neutral.**

# Goods, Bads and Neutrals




**Around x’ units, a little extra water is a neutral.**

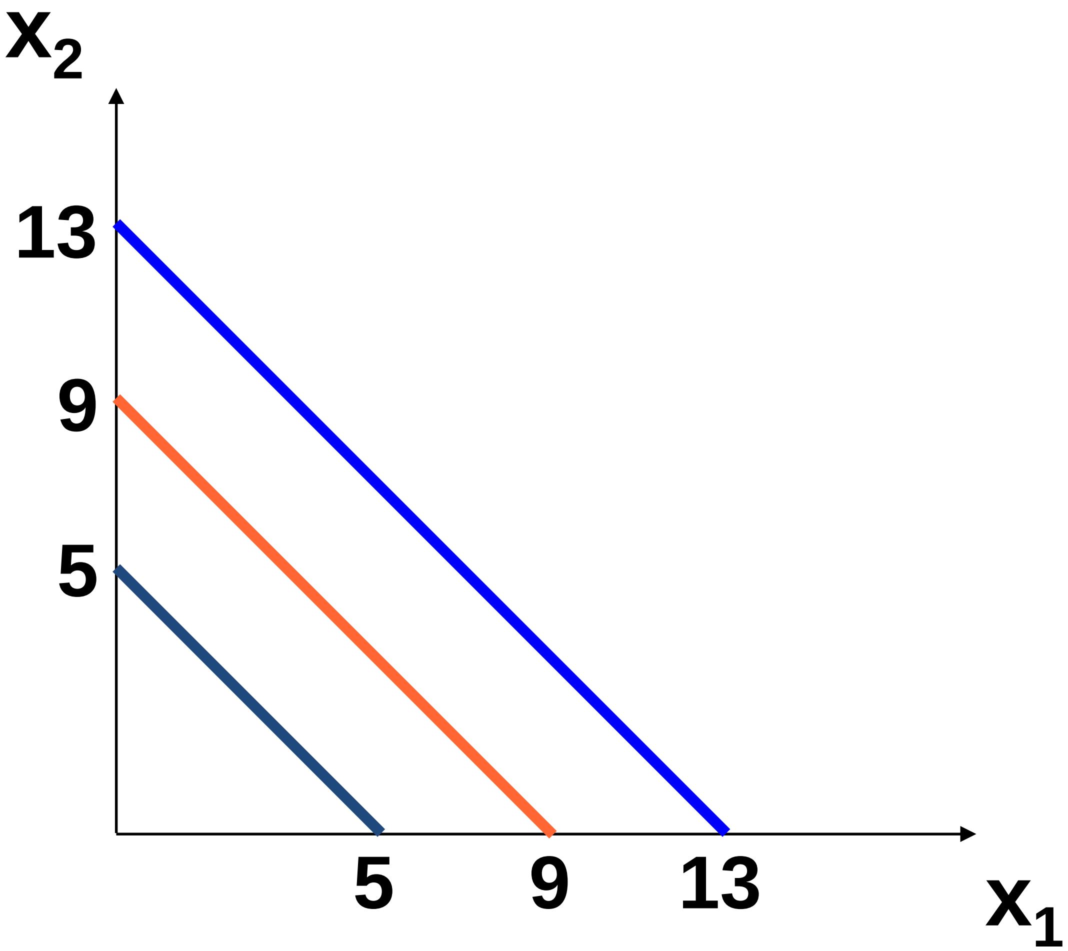
# Some Other Utility Functions and Their Indifference Curves

- Instead of $U(x_1,x_2) = x_1x_2$ consider

$$V(x_1,x_2) = x_1 + x_2.$$

What do the indifference curves for this "perfect substitution" utility function look like?

# Perfect Substitution Indifference Curves




**All are linear and parallel.**

# Some Other Utility Functions and Their Indifference Curves

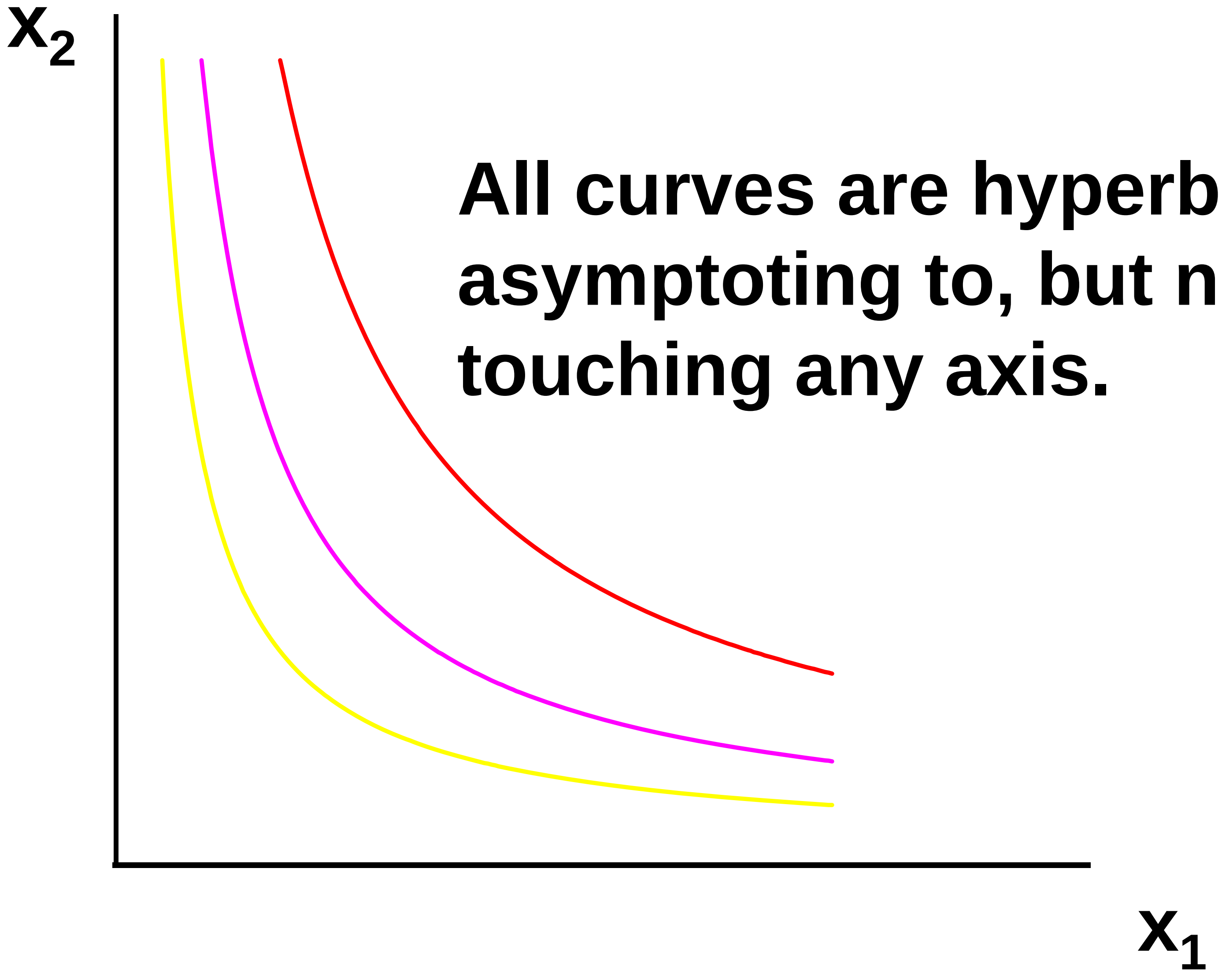
- Any utility function of the form

$$U(x_1,x_2) = x_1^{a}\, x_2^{b}$$

  with a > 0 and b > 0 is called a Cobb-Douglas utility function.

- *E.g.* $U(x_1,x_2) = x_1^{1/2}\, x_2^{1/2}$ (a = b = 1/2)
  $V(x_1,x_2) = x_1\, x_2^{3}$ (a = 1, b = 3)

# Cobb-Douglas Indifference Curves




**All curves are hyperbolic, asymptoting to, but never touching any axis.**

# Marginal Utilities

- Marginal means "incremental".
- The marginal utility of commodity i is the rate-of-change of total utility as the quantity of commodity i consumed changes; *i.e.*

$$MU_i = \frac{\partial U}{\partial x_i}$$

# Marginal Utilities

- *E.g.* if $U(x_1,x_2) = x_1^{1/2} x_2^2$ then

$$MU_1 = \frac{\partial U}{\partial x_1} = \frac{1}{2} x_1^{-1/2} x_2^2$$

# Marginal Utilities

- *So,* if $U(x_1,x_2) = x_1^{1/2} x_2^2$ then

$$MU_1 = \frac{\partial U}{\partial x_1} = \frac{1}{2} x_1^{-1/2} x_2^2$$

$$MU_2 = \frac{\partial U}{\partial x_2} = 2 x_1^{1/2} x_2$$

# Marginal Utilities and Marginal Rates-of-Substitution

$$\frac{dx_2}{dx_1} = -\frac{\dfrac{\partial U}{\partial x_1}}{\dfrac{\partial U}{\partial x_2}}$$

# Marg. Utilities & Marg. Rates-of-Substitution; An example

- Suppose $U(x_1,x_2) = x_1x_2$. Then

$$\frac{\partial U}{\partial x_1} =$$

$$\frac{\partial U}{\partial x_2} =$$

# Marg. Utilities & Marg. Rates-of-Substitution; An example